Introduction

- Radiological physics studies ionizing radiation and its interaction with matter
- Began with discovery of x-rays, radioactivity and radium in 1890s
- Special interest is in the energy absorbed in matter
- Radiation dosimetry deals with quantitative determination of the energy absorbed in matter

Ionizing Radiation
Chapter 1
F.A. Attix, Introduction to Radiological Physics and Radiation Dosimetry

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Types and sources of ionizing radiation

- γ-rays: electromagnetic radiation (photons) emitted from a nucleus or in annihilation reaction
  - Practical energy range from 2.6 keV (Kα from electron capture in $^{37}$Ar) to 6.1 and 7.1 MeV (γ-rays from $^{16}$N)
- x-rays: electromagnetic radiation (photons) emitted by charged particles (characteristic or bremsstrahlung processes). Energies:
  - 0.1-20 kV "soft" x-rays
  - 20-120 kV diagnostic range
  - 120-300 kV orthovoltage x-rays
  - 300 kV-1 MV intermediate energy x-rays
  - 1 MV and up megavoltage x-rays

Types of interaction

- ICRU (The International Commission on Radiation Units and Measurements; established in 1925) terminology
- Directly ionizing radiation: by charged particles, delivering their energy to the matter directly through multiple Coulomb interactions along the track
- Indirectly ionizing radiation: by photons (x-rays or γ-rays) and neutrons, which transfer their energy to charged particles (two-step process)
Description of ionizing radiation fields

- To describe radiation field at a point P need to define non-zero volume around it
- Can use stochastic or non-stochastic physical quantities

Stochastic quantities

- Values occur randomly, cannot be predicted
- Radiation is random in nature, associated physical quantities are described by probability distributions
- Defined for finite domains (non-zero volumes)
- The expectation value of a stochastic quantity (e.g. number of x-rays detected per measurement) is the mean of its measured value for infinite number of measurements:
  \[ \overline{N} \rightarrow N_e \text{ for } n \rightarrow \infty \]

Stochastic quantities

- For a “constant” radiation field a number of x-rays observed at point P per unit area and time interval follows Poisson distribution
- For large number of events it may be approximated by normal (Gaussian) distribution, characterized by standard deviation \( \sigma \) or corresponding percentage standard deviation \( S \) for a single measurement
  \[ \sigma = \sqrt{N_e} \approx \sqrt{\overline{N}} \]
  \[ S = \frac{100\sigma}{N_e} = \frac{100}{\sqrt{N_e}} \approx \frac{100}{\sqrt{\overline{N}}} \]

Stochastic quantities

- For a given number of measurements \( n \) standard deviation is defined as
  \[ \sigma' = \frac{\sigma}{\sqrt{n}} = \frac{\sqrt{N_e}}{\sqrt{n}} \approx \frac{1}{\sqrt{n}} \]
  \[ S' = \frac{100\sigma'}{N_e} = \frac{100}{\sqrt{nN_e}} \approx \frac{100}{\sqrt{n}} \]
- \( \overline{N} \) will have a 68.3% chance of lying within interval \( \pm \sigma' \) of \( N_e \), 95.5% to be within \( \pm 2\sigma' \), and 99.7% to be within interval \( \pm 3\sigma' \). No experiment-related fluctuations

Stochastic quantities

- In practice one always uses a detector. An estimated precision (proximity to \( N_e \)) of any single random measurement \( N_i \)
  \[ \sigma \approx \left[ \frac{1}{n-1} \sum_{i=1}^{n} (N_i - \overline{N})^2 \right]^{1/2} \]
  \[ \overline{N} = \left( \sum_{i} N_i \right) / n \]
- Determined from the data set of \( n \) such measurements
Stochastic quantities

- An estimate of the precision (proximity to \( N_e \)) of the mean value \( \bar{N} \) measured with a detector \( n \) times

\[
\sigma' = \frac{\sigma}{\sqrt{n}}
\]

\[
\sigma' = \left[ \frac{1}{n(n-1)} \sum_{i=1}^{n} (N_i - \bar{N})^2 \right]^{1/2}
\]

- \( N_j \) is as correct as your experimental setup

Stochastic quantities: Example

- A \( \gamma \)-ray detector having 100% counting efficiency is positioned in a constant field, making 10 measurements of equal duration, \( \Delta t=100 \)s (exactly). The average number of rays detected ("counts") per measurement is \( 1.00 \times 10^5 \). What is the mean value of the count rate \( C \), including a statement of its precision (i.e., standard deviation)?

\[
\bar{C} = \frac{N}{\Delta t} = \frac{1.00 \times 10^5}{100} = 1.00 \times 10^3 \text{ c/s}
\]

\[
\sigma_C \approx \frac{\bar{C}}{\sqrt{n}} = \frac{1.00 \times 10^3}{\sqrt{10}} = 1 \text{ c/s}
\]

\[
\bar{C} = 1.00 \times 10^3 \pm 1 \text{ c/s}
\]

- Here the standard deviation is due entirely to the stochastic nature of the field, since detector is 100% efficient

Non-stochastic quantities

- For given conditions the value of non-stochastic quantity can, in principle, be calculated
- In general, it is a "point function" defined for infinitesimal volumes
  - It is a continuous and differentiable function of space and time; with defined spatial gradient and time rate of change
- Its value is equal to, or based upon, the expectation value of a related stochastic quantity, if one exists
  - In general does not need to be related to stochastic quantities, they are related in description of ionizing radiation

Non-stochastic quantities: Fluence

- A number of rays crossing an infinitesimal area surrounding point \( P \), define fluence as

\[
\Phi = \frac{dN}{da}
\]

- Units of \( \text{m}^2 / \text{s} \) or \( \text{cm}^2 / \text{s} \)

Description of radiation fields by non-stochastic quantities

- Fluence
- Flux Density (or Fluence Rate)
- Energy Fluence
- Energy Flux Density (or Energy Fluence Rate)

Non-stochastic quantities: Flux density (Fluence rate)

- An increment in fluence over an infinitesimally small time interval

\[
\varphi = \frac{d\Phi}{dt} = \frac{d}{dt} \left( \frac{dN}{da} \right)
\]

- Units of \( \text{m}^2 \text{s}^{-1} \) or \( \text{cm}^2 \text{s}^{-1} \)
- Fluence can be found through integration:

\[
\Phi(t_0, t_1) = \int_{t_0}^{t_1} \varphi(t) dt
\]
Non-stochastic quantities: Energy fluence

- For an expectation value $R$ of the energy carried by all the $N_e$ rays crossing an infinitesimal area surrounding point $P$, define energy fluence as

$$\Psi = \frac{dR}{da}$$

- Units of J m$^{-2}$ or erg cm$^{-2}$

- If all rays have energy $E$

$$R = E N_e$$

$$\Psi = E \Phi$$

Differential distributions

- More complete description of radiation field is often needed
- Generally, flux density, fluence, energy flux density, or energy fluence depend on all variables: $\theta$, $\beta$, or $E$
- Simpler, more useful differential distributions are those which are functions of only one of the variables

Differential distributions by energy and angle of incidence

- Differential flux density as a function of energy and angles of incidence: distribution $\varphi(\theta, \beta, E)$
- Typical units are m$^{-2}$ s$^{-1}$ sr$^{-1}$ keV$^{-1}$
- Integration over all variables gives the flux density:

$$\varphi = \int_0^{2\pi} \int_0^\pi \int_0^E \varphi(\theta, \beta, E) \sin \theta \, d\theta \, d\beta \, dE$$

Differential distributions: Energy spectra

- If a quantity is a function of energy only, such distribution is called the energy spectrum (e.g. $\varphi(E)$)
- Typical units are m$^{-2}$ s$^{-1}$ keV$^{-1}$ or cm$^{-2}$ s$^{-1}$ keV$^{-1}$
- Integration over angular variables gives flux density spectrum

$$\varphi(E) = \int_0^\pi \int_0^{2\pi} \varphi(\theta, \beta, E) \sin \theta \, d\theta \, d\beta$$

- Similarly, may define energy flux density $\psi(E)$

Differential distributions example: Energy spectrum

- A “flat” distribution of photon flux density
- Energy flux density spectrum is found by

$$\psi'(E) = E \varphi'(E)$$

Typically units for $E$ are joule or erg, so that $[\psi'] =$ J m$^{-2}$ s$^{-1}$ keV$^{-1}$

Example: Problem 1.8

An x-ray field at a point $P$ contains $7.5 \times 10^8$ photons/(m$^2$-sec-keV), uniformly distributed from 10 to 100 keV.

a) What is the photon flux density at $P$?

b) What would be the photon fluence in one hour?

c) What is the corresponding energy fluence, in J/m$^2$ and erg/cm$^2$?
Example: Problem 1.8
Energy spectrum of a flux density \( \phi(E) = 7.5 \times 10^8 \) photons/m\(^2\)-sec-keV

a) Photon flux density
\[
\Phi = \phi(E) \cdot (E_{\text{max}} - E_{\text{min}}) = 7.5 \times 10^8 \cdot 90 = 6.75 \times 10^{10} \text{ photons/m}^2\text{s}
\]
b) The photon fluence in one hour
\[
\Phi(t = 1 \text{ hour}) = \phi \cdot \Delta t = 6.75 \times 10^{10} \cdot 3600 = 2.43 \times 10^{14} \text{ photons/m}^2
\]
c) The corresponding energy fluence, in J/m\(^2\) and erg/cm\(^2\)
\[
\Psi = \Delta t \cdot \int \phi(E) \cdot E \left[ \Delta t \cdot \phi \cdot \frac{E^2}{2} \right] \frac{100}{E_{10}} = 3600 \cdot 7.5 \times 10^8 \cdot \frac{1}{2} \left( 100^2 - 10^2 \right) = 1.336 \times 10^{16} \text{ keV/m}^2 = 1.336 \times 10^{16} \cdot 1.602 \times 10^{-16} = 2.14 \text{ J/m}^2 = 2.14 \times 10^3 \text{ erg/cm}^2
\]

Differential distributions: Angular distributions
- Full differential distribution integrated over energy leaves only angular dependence
- Often the field is symmetrical with respect to a certain direction, then only dependence on polar angle \( \theta \) or azimuthal angle \( \beta \)

Summary
- Types and sources of ionizing radiation
  - \( \gamma \)-rays, x-rays, fast electrons, heavy charged particles, neutrons
- Description of ionizing radiation fields
  - Due random nature of radiation: expectation values and standard deviations
  - Non-stochastic quantities: fluence, flux density, energy fluence, energy flux density, differential distributions

Quantities for Describing the Interaction of Ionizing Radiation with Matter

Chapter 2

F.A. Attix, Introduction to Radiological Physics and Radiation Dosimetry

Introduction
- Need to describe interactions of ionizing radiation with matter
- Special interest is in the energy absorbed in matter, absorbed dose – delivered by directly ionizing radiation
- Two-step process for indirectly ionizing radiation involves kerma and absorbed dose

Definitions
- Most of the definitions are by ICRU
  - ICRU Report 33, Radiation quantities and units, 1980
  - Revised several times (the latest: ICRU Report 85 Fundamental quantities and units for ionizing radiation, 2011)
- Energy transferred by indirectly ionizing radiation leads to the definition of kerma
- Energy imparted by ionizing radiation leads to the definition of absorbed dose
- Energy carried by neutrinos is ignored
Energy transferred

• $\varepsilon_{tr}$ - energy transferred in a volume V to charged particles by indirectly ionizing radiation (photons and neutrons)
• Radiant energy $R$ – the energy of particles emitted, transferred, or received, excluding rest mass energy
• $Q$ - energy delivered from rest mass in V (positive if $m \rightarrow E$, negative for $E \rightarrow m$)

Kerma

• Kerma $K$ is the energy transferred to charged particles per unit mass
  \[ K = \frac{d(\varepsilon_{tr})}{dm} = \frac{d\varepsilon_{tr}}{dm} \]
  - Includes radiative losses by charged particles (bremsstrahlung or in-flight annihilation of positron)
  - Excludes energy passed from one charged particle to another
• Units: 1 Gy = 1 J/kg = 10^2 rad = 10^4 erg/g

Relation of kerma to energy fluence for photons

• For mono-energetic photon of energy $E$ and medium of atomic number $Z$, relation is through the mass energy-transfer coefficient:
  \[ K = \Psi \left( \frac{\mu_{\varepsilon}}{\rho} \right)_{E,Z} \]
• For a spectrum of energy fluence $\Psi'(E)$
  \[ K = \int_{E=0}^{E=E_{max}} \Psi'(E) \left( \frac{\mu_{\varepsilon}}{\rho} \right)_{E,Z} dE \]

Energy-transfer coefficient

• Linear energy-transfer coefficient $\mu_{\varepsilon}$, units of m$^{-1}$ or cm$^{-1}$
• Mass energy-transfer coefficient $\left( \frac{\mu_{\varepsilon}}{\rho} \right)_{E,Z}$, units of m$^2$/kg or cm$^2$/g
• Set of numerical values, tabulated for a range of photon energies, Appendix D.3

Relation of kerma to fluence for neutrons

• Neutron field is usually described in terms of fluence rather than energy fluence
• Kerma factor is tabulated instead of kerma (units are rad cm$^2$/neutron, Appendix F)
  \[ (F_n)_{E,Z} = \left( \frac{\mu_{\varepsilon}}{\rho} \right)_{E,Z} \cdot E \]
• For mono-energetic neutrons
  \[ K = \Phi \cdot (F_n)_{E,Z} \]
Components of Kerma

• Energy received by charged particles may be spent in two ways
  – Collision interactions – local dissipation of energy, ionization and excitation along electron track
  – Radiative interactions, such as bremsstrahlung or positron annihilation, carry energy away from the track

• Kerma may be subdivided in two components, collision and radiative:
  \[ K = K_c + K_r \]
• When kerma is due to neutrons, resulting charged particles are much heavier,
  \[ K = K_c \]

Collision Kerma

• Subtracting radiant energy emitted by charged particles from energy transferred results in net energy transferred locally
  \[ e_{\text{net}} = e_{\text{in}} - R'_\text{c} = (R_{\text{in}})_{\text{net}} - R'_\text{c} + \sum Q \]
• Now collision kerma can be defined
  \[ K_c = \frac{de_{\text{net}}}{dm} \]

Mass energy-absorption coefficient

• Since collision kerma represents energy deposited (absorbed) locally, introduce mass energy-absorption coefficient. For mono-energetic photon beam
  \[ K_e = \psi \left( \frac{\mu_e}{\rho} \right)_{\text{e,Z}} \]
• Depends on materials present along particle track before reaching point P

Absorbed dose

• Energy imparted by ionizing radiation to matter of mass m in volume V
  \[ \varepsilon = (R_{\text{un}})_{\text{net}} - \sum Q \]
due to uncharged
  \[ \varepsilon = (R_{\text{un}})_{\text{net}} + (R_{\text{ch}})_{\text{net}} + \sum Q \]
due to charged
• Absorbed dose is defined as
  \[ D = \frac{d\varepsilon}{dm} \]
• Units: 1 Gy = 1 J/kg = 10^2 rad = 10^4 erg/g
Absorbed dose

- $D$ represents the energy per unit mass which remains in the matter at $P$ to produce any effects attributable to radiation.
- The most important quantity in radiological physics.
- Absorbed dose rate:

$$dD = d \frac{dD}{dt} = d \left( \frac{dE}{dm} \right)$$

Example 1

- Positron transferred excess kinetic energy to photons after annihilation.
- Positron transfers excess kinetic energy $T_3$ to photons after annihilation.
- It generates radiative loss from charged-particle kinetic energy.
- Affects $\epsilon$ and $\epsilon''_\nu$ by subtraction of $T_3$.

$$\epsilon = h\nu_1 - (h\nu_2 + h\nu_3 + T') + 0$$

$$\epsilon''_\nu = h\nu_1 - h\nu_2 + 0 = T$$

$$\epsilon''_\nu = h\nu_1 - h\nu_2 - h\nu_3 + 0$$

$$= T - h\nu_3$$

Example 2

- Positron has no excess kinetic energy to transfer to photons after annihilation.

$$\epsilon = \epsilon''_\nu = h\nu_1 - 1.022 \text{ MeV} = T_1 + T_2$$

$$\sum Q = h\nu_1 - 2m_e c^2 + 2m_e c^2 = h\nu_1$$

Example 3

- Positron transfers excess kinetic energy $T_3$ to photons after annihilation.
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$$\sum Q = h\nu_1 - 2m_e c^2 + 2m_e c^2 = h\nu_1$$

$$R'_n = \frac{T_3}{T}$$

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$$\sum Q = h\nu_1 - 2m_e c^2 + 2m_e c^2 = h\nu_1$$

$$dQ = \frac{dQ}{dm}$$

- $dQ$ is absolute value of the total charge of the ions of one sign produced in air when all electrons liberated by photons in air of mass $dm$ are completely stopped in air.

- Ionization from the absorption of radiative loss of kinetic energy by electrons is not included.

### Exposition

- Historically, was introduced before kerma and dose, measured in roentgen (R).
- Defined as a quotient.

$$X = \frac{dQ}{dm}$$

- $dQ$ is absolute value of the total charge of the ions of one sign produced in air when all electrons liberated by photons in air of mass $dm$ are completely stopped in air.

- Ionization from the absorption of radiative loss of kinetic energy by electrons is not included.
Exposure

- Exposure is the ionization equivalent of the collision kerma in air for x and γ-rays
- Introduce mean energy expended in a gas per ion pair formed, \( \bar{W} \), constant for each gas, independent of incoming photon energy
- For dry air
  \[
  \bar{W}_{\text{air}} \times 10^{-19} \text{J/eV} = 33.97 \text{ J/C}
  \]

Relation of exposure to energy fluence

- Exposure at a point due to energy fluence of mono-energetic photons
  \[
  X = \psi \left( \frac{\mu_{\text{en}}}{\rho} \right)_{E_{\text{air}}} \left( \frac{e}{W} \right)_{\text{air}}
  \]
  \[
  (K_e)_{\text{air}} \left( \frac{e}{W} \right)_{\text{air}} = (K_e)_{\text{air}} / 33.97
  \]
- Units of [X]=C/kg in SI

Units of exposure

- The roentgen R is the customary unit
- The roentgen is defined as exposure producing in air one unit of esu of charge per 0.001293 g of air irradiated by the photons. Conversion
  \[
  1R = \frac{1 \text{ esu}}{0.001293 \text{g}} \times \frac{1 \text{ C}}{2.998 \times 10^9 \text{ esu}} \times \frac{1 \text{ g}}{1 \text{ kg}}
  \]
  \[
  = 2.58 \times 10^{-4} \text{ C/kg}
  \]
  \[
  1 \text{ C/kg} = 3876 \text{ R}
  \]

Significance of exposure

- Energy fluence is proportional to exposure for any given photon energy or spectrum
- Due to similarity in effective atomic number
  - Air can be made a tissue equivalent medium with respect to energy absorption – convenient in measurements
  - Collision kerma in muscle per unit of exposure is nearly independent of photon energy

Ratio of mass energy-absorption coefficients for muscle/air and water/air are nearly constant (within <5%) for energies from 4keV to 10 MeV

Ratio of mass energy-absorption coefficients for bone/air and acrylic/air are nearly constant for energies above 100keV
**Significance of exposure**

- X-ray field at a point can be characterized by means of exposure regardless of whether there is air actually located at this point
- It implies that photon energy fluence at that point is such that it would produce exposure of a stated value
- Same is applicable to kerma or collision kerma, except that reference medium (not necessarily air) has to be specified

**Radiation protection quantities**

- Quality factor Q – weighting factor to be applied to absorbed dose to provide an estimate of the relative human hazard of ionizing radiation
- It is based on relative biological effectiveness (RBE) of a particular radiation source
- Q is dimensionless

**Radiation protection quantities**

- Higher-density charged particle tracks (higher collision stopping power) are more damaging per unit dose

**Radiation protection quantities**

- Dose equivalent $H$, is defined as
  \[ H \equiv DQN \]
- Here D – dose, Q- quality factor, N-product of modifying factors (currently=1)
- Units of H:
  - **Sv**, if dose is expressed in J/kg
  - **rem**, if dose is in rad (10^{-2} J/kg)

**Summary**

- Quantities describing the interaction of ionizing radiation with matter
  - Kerma, components of kerma
  - Absorbed dose
  - Exposure
- Relationship with fluence and energy fluence
- Quantities for use in radiation protection
  - Quality factor Q
  - Dose equivalent H