Chapter 1
Basic Concepts

Radiation Dosimetry I

Text: H.E Johns and J.R. Cunningham,
The physics of radiology, 4th ed.

Units

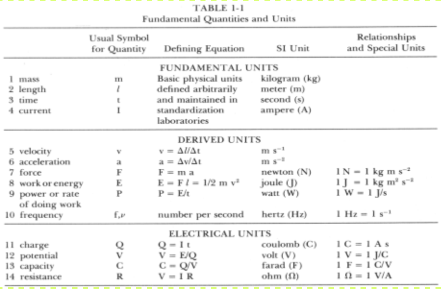
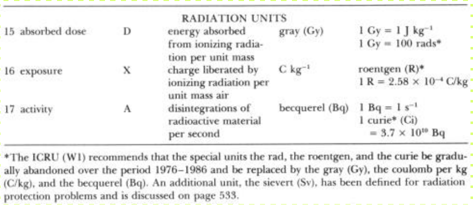


TABLE 1-1
Fundamental Quantities and Units

	Usual Symbol for Quantity	Defining Equation	SI Unit	Relationships and Special Units
		FUNDAMENTAL UNITS		
1 mass	m	Basic physical units defined arbitrarily and maintained in standardization laboratories	kilogram (kg)	
2 length	l		meter (m)	
3 time	t		second (s)	
4 current	I		ampere (A)	
		DERIVED UNITS		
5 velocity	v	$v = \Delta l/\Delta t$	$m\ s^{-1}$	
6 acceleration	a	$a = \Delta v/\Delta t$	$m\ s^{-2}$	
7 force	F	F = m a	newton (N)	$1\ N = 1\ kg\ m\ s^{-2}$
8 work or energy	E	$E = F\,l = 1/2\ m\ v^2$	joule (J)	$1\ J = 1\ kg\ m^2\ s^{-2}$
9 power or rate of doing work	P	P = E/t	watt (W)	1 W = 1 J/s
10 frequency	f,ν	number per second	hertz (Hz)	$1\ Hz = 1\ s^{-1}$
		ELECTRICAL UNITS		
11 charge	Q	Q = I t	coulomb (C)	1 C = 1 A s
12 potential	V	V = E/Q	volt (V)	1 V = 1 J/C
13 capacity	C	C = Q/V	farad (F)	1 F = 1 C/V
14 resistance	R	V = I R	ohm (Ω)	1 Ω = 1 V/A

- Special unit of energy: electron volt eV
- 1 eV=1.602×10^{-19} C x 1 volt= 1.602×10^{-19} J

Units



		RADIATION UNITS		
15 absorbed dose	D	energy absorbed from ionizing radiation per unit mass	gray (Gy)	$1\ Gy = 1\ J\ kg^{-1}$ 1 Gy = 100 rads*
16 exposure	X	charge liberated by ionizing radiation per unit mass air	$C\ kg^{-1}$	roentgen (R)* $1\ R = 2.58 \times 10^{-4}$ C/kg
17 activity	A	disintegrations of radioactive material per second	becquerel (Bq)	$1\ Bq = 1\ s^{-1}$ 1 curie* (Ci) $= 3.7 \times 10^{10}$ Bq

*The ICRU (W1) recommends that the special units the rad, the roentgen, and the curie be gradually abandoned over the period 1976–1986 and be replaced by the gray (Gy), the coulomb per kg (C/kg), and the becquerel (Bq). An additional unit, the sievert (Sv), has been defined for radiation protection problems and is discussed on page 533.

- Absorbed dose: describes energy deposition in water phantom, detector, patient, etc.
- Exposure: describes the ability of radiation to ionize air, used for energies < 3 MeV (below typical radiotherapy range)
- Activity: describes radioactive isotopes; 1 Ci is the activity of 1 g of radium

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Example 1

- A current of 2 µA must flow into 100 nf capacitor for how many seconds to produce a 50 V potential difference across the capacitor?

A. 1
B. 2
[✓ C.] 2.5
D. 5
E. 25

$$C = \frac{Q}{V} = \frac{I \cdot t}{V}$$

$$t = \frac{CV}{I} = \frac{100\times10^{-9}\,\mathrm{f}\cdot 50\mathrm{V}}{2\times10^{-6}\,\mathrm{A}} = 2.5\,\mathrm{s}$$

Example 2

- 2-cc-volume ionization chamber is placed in a radiation field of 100 R/s. What is the current generated in amperes (ρ_{air} = 0.0013 g/cm^3)?

A. 5.1 x 10 $^{-11}$
B. 6.3 x 10 $^{-10}$
C. 5.1 x 10 $^{-9}$
[✓ D.] 6.7 x 10 $^{-8}$
E. 5.1 x 10 $^{-7}$

$$100\ \mathrm{R/s} = 100 \cdot 2.58\times10^{-4}\ \mathrm{C/(kg\cdot s)} = 2.58\times10^{-2}\mathrm{C/(kg\cdot s)}$$

$$m_{air} = V_{air}\rho_{air} = 2\ \mathrm{cm}^3 \cdot 0.0013\,\mathrm{g/cm}^3 = 0.0026\,\mathrm{g} = 2.6\times10^{-6}\mathrm{kg}$$

$$I = 2.58\times10^{-2}\mathrm{C/(kg\cdot s)} \cdot 2.6\times10^{-6}\mathrm{kg} = 6.71\times10^{-8}\ \mathrm{A}$$

Atoms

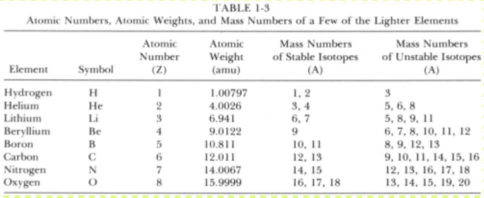


TABLE 1-3
Atomic Numbers, Atomic Weights, and Mass Numbers of a Few of the Lighter Elements

Element	Symbol	Atomic Number (Z)	Atomic Weight (amu)	Mass Numbers of Stable Isotopes (A)	Mass Numbers of Unstable Isotopes (A)
Hydrogen	H	1	1.00797	1, 2	3
Helium	He	2	4.0026	3, 4	5, 6, 8
Lithium	Li	3	6.941	6, 7	5, 8, 9, 11
Beryllium	Be	4	9.0122	9	6, 7, 8, 10, 11, 12
Boron	B	5	10.811	10, 11	8, 9, 12, 13
Carbon	C	6	12.011	12, 13	9, 10, 11, 14, 15, 16
Nitrogen	N	7	14.0067	14, 15	12, 13, 16, 17, 18
Oxygen	O	8	15.9999	16, 17, 18	13, 14, 15, 19, 20

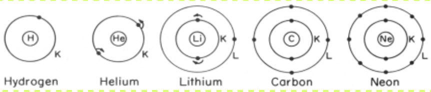
- Atomic number Z: number of electrons (protons)
- Mass number A: total number of protons + neutrons
- Atomic mass: Carbon 12 has atomic mass 12.0000 amu (6 protons + 6 neutrons)
- Typical notation:

$${}^{A}{}_{Z}\,\mathrm{Element}$$

Atoms

- *Isotopes*: have the same number of protons, but different number of neutrons
  - Same atomic number Z, and number of electrons
  - Same chemical properties
  - Different mass number A
- *Isotones*: have the same number of neutrons (A-Z)
- *Isobars*: have the same atomic mass A (total number of protons + neutrons)
- There is redundancy in full notation ${}^{A}_{Z}X$
  - Atomic number Z determines the element X
- *Isomers*: the same A and Z, different nuclear energy state (stable vs. metastable, or excited); notation: ${}^{A}_{Z}{}^{m}X$

Atomic energy levels



- Electron orbits have defined energies (levels)
- The innermost is K (up to two electrons with opposite spins), next is L (up to eight electrons), etc.
- Filled outer shell – chemically inert atom

Atomic energy levels

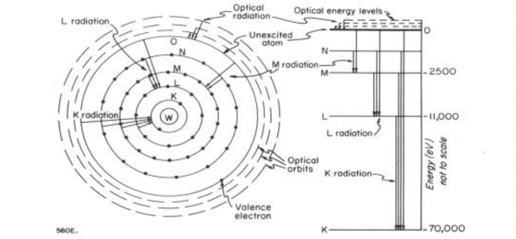


Figure 1-2. Schematic diagram of the tungsten atom showing the shells on the left and an energy level diagram on the right. The energy scale in eV is not drawn to scale. X-radiation arises through transitions of electrons to the K, L, and M shells. Optical radiation arises by transitions of the valence electron from optical orbits to the O shell.

X-rays arise from transitions to K, L, M levels (eV to keV energy range)

Nucleus and its energy levels

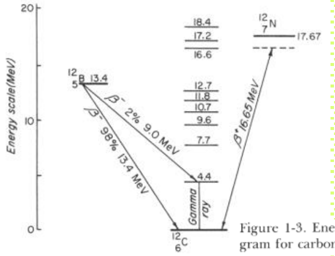


Figure 1-3. Energy level diagram for carbon 12.

- System of nuclear energy levels
- Nuclear transitions produce photons (γ-rays) and particles in MeV energy range

Mass and energy

- Photon energy: E = $h\nu = hc/\lambda$
- Mass-to-energy conversion: E = mc^2
- Relativistic mass:

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

- Rest mass m_0
- Kinetic energy: K.E. = $mc^2 - m_0c^2$

Mass and energy

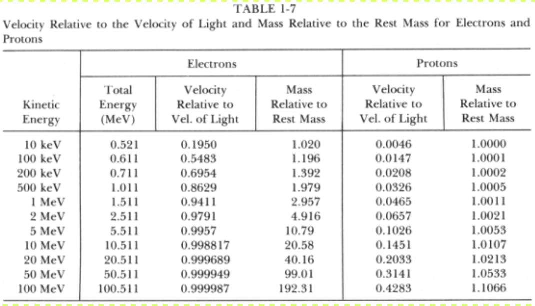


TABLE 1-7

Velocity Relative to the Velocity of Light and Mass Relative to the Rest Mass for Electrons and Protons

Kinetic Energy	Electrons			Protons	
	Total Energy (MeV)	Velocity Relative to Vel. of Light	Mass Relative to Rest Mass	Velocity Relative to Vel. of Light	Mass Relative to Rest Mass
10 keV	0.521	0.1950	1.020	0.0046	1.0000
100 keV	0.611	0.5483	1.196	0.0147	1.0001
200 keV	0.711	0.6954	1.392	0.0208	1.0002
500 keV	1.011	0.8629	1.979	0.0326	1.0005
1 MeV	1.511	0.9411	2.957	0.0465	1.0011
2 MeV	2.511	0.9791	4.916	0.0657	1.0021
5 MeV	5.511	0.9957	10.79	0.1026	1.0053
10 MeV	10.511	0.998817	20.58	0.1451	1.0107
20 MeV	20.511	0.999689	40.16	0.2033	1.0213
50 MeV	50.511	0.999949	99.01	0.3141	1.0533
100 MeV	100.511	0.999987	192.31	0.4283	1.1066

Mass-to-energy conversion factors:
1 electron mass = 0.511 MeV
1 amu = 931.5 MeV

Example 3

From the following table of particle rest masses, calculate the gamma energy emitted when a proton captures a neutron to create a deuteron. 1 amu corresponds to the rest mass energy of 931.5 MeV

particle rest mass.	**amu**
Proton	1.00727
Neutron	1.00866
Deuteron	2.01355

A. 1.875 MeV
B. 2.02 MeV
[✓ C.] 2.22 MeV
D. 2.38 MeV
E. 4.03 MeV

$$E_\gamma = E_{rmP} + E_{rmN} - E_{rmD} =$$
$$(1.00727 + 1.00866 - 2.01355) \times 931.5\ \mathrm{MeV} =$$
$$0.00238 \times 931.5\ \mathrm{MeV} = 2.21697\,\mathrm{MeV} \approx 2.22\,\mathrm{MeV}$$

Example 4

- Find the velocity of an electron accelerated through the potential difference of 5 MeV.

$$E = m_0 c^2 \cdot \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right)$$

$$5\,\mathrm{MeV} = 0.511\,\mathrm{MeV} \cdot \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right)$$

$$v^2/c^2 \sim 1 - 0.01 = 0.99$$

$$v/c \sim 0.995$$

$$v \sim 0.995 \cdot 3 \times 10^8 \ \ \mathrm{m/c}$$

Exponential behavior

$$\frac{dN}{dt} = -\lambda N = -\frac{1}{t_a} N$$

$$N = N_0 e^{-\lambda t} = N_0 e^{-t/t_a} = N_0 2^{-t/t_h}$$

The sign determines the process: decay or growth λ

λ - transformation constant

t_a – average life; t_h – half-life;

$\lambda = 1/t_a = \ln 2/t_h = 0.693/t_h$

Exponential behavior

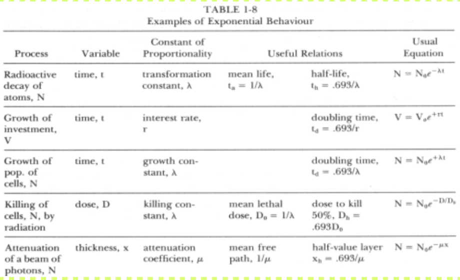


TABLE 1-8
Examples of Exponential Behaviour

Process	Variable	Constant of Proportionality	Useful Relations		Usual Equation
Radioactive decay of atoms, N	time, t	transformation constant, λ	mean life, $t_a = 1/\lambda$	half-life, $t_h = .693/\lambda$	$N = N_0 e^{-\lambda t}$
Growth of investment, V	time, t	interest rate, r		doubling time, $t_d = .693/r$	$V = V_0 e^{+rt}$
Growth of pop. of cells, N	time, t	growth constant, λ		doubling time, $t_d = .693/\lambda$	$N = N_0 e^{+\lambda t}$
Killing of cells, N, by radiation	dose, D	killing constant, λ	mean lethal dose, $D_0 = 1/\lambda$	dose to kill 50%, $D_h = .693 D_0$	$N = N_0 e^{-D/D_0}$
Attenuation of a beam of photons, N	thickness, x	attenuation coefficient, μ	mean free path, $1/\mu$	half-value layer $x_h = .693/\mu$	$N = N_0 e^{-\mu x}$

- If more than one process takes place: $$\frac{dN}{dt} = -(\lambda_1 + \lambda_2 + \cdots + \lambda_n) N$$

Exponential behavior

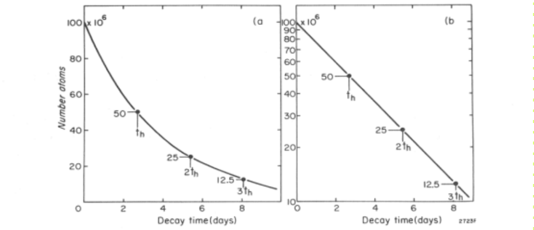


Figure 1-4. Graphs showing the exponential decay of a source of 10^8 atoms of ^{198}Au with half-life of 2.69 days. The vertical scale on the left is linear while the one on the right is logarithmic.

$$N = N_0 2^{-t/t_h}$$

Example 5

In the case of simultaneous physical decay and biological clearance, when T_p, is the physical half-life and T_b is the biological half-life, the effective half-life T_{eff} is generally equal to:

A. $T_p + T_b$
B. $1/T_p + 1/T_b$
C. T_p x T_b
[✓ D.] $1/(1/T_p + 1/T_b)$
E. $\sqrt{T_p \times T_b}$

$$\frac{dN}{dt} = -\frac{1}{t_{a.eff}} N = -\frac{1}{t_{a.p}} N - \frac{1}{t_{a.b}} N$$

$$t_{a.eff} = 1 \Big/ \left(\frac{1}{t_{a.p}} + \frac{1}{t_{a.b}} \right)$$

Since $T_{eff} = t_{a.eff} / \ln 2$

$$T_{eff} = 1 \Big/ \left(\frac{1}{T_p} + \frac{1}{T_b} \right)$$