Chapter 1

Basic Concepts

Radiation Dosimetry I


Units

<table>
<thead>
<tr>
<th>Units</th>
<th>Fundamental Quantities and Units</th>
<th>SI Unit</th>
<th>Relationships and Special Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 mass</td>
<td>m</td>
<td>Basic physical units: kilogram (kg)</td>
<td>kg</td>
</tr>
<tr>
<td>2 length</td>
<td>l</td>
<td>kilometer (km)</td>
<td>m</td>
</tr>
<tr>
<td>3 rate</td>
<td>r</td>
<td>and measured as</td>
<td>m/s</td>
</tr>
<tr>
<td>4 electric</td>
<td>e</td>
<td>specialization</td>
<td>ampere (A)</td>
</tr>
</tbody>
</table>

5 velocity | v | Basic physical units | m/s | |
6 concentration | c | Basic physical units | m^-3 | |
7 time | T | sec (s) | s | |
8 mass or energy | E | Basic physical units | J | |
9 pressure | P | Basic physical units | Pa | 1 atm = 101,325 Pa |
10 density | D | Basic physical units | kg/m^3 | |

11 charge | Q | coulomb (C) | C | 1 C = 1.602 x 10^-19 A s |
12 potential | V | volt (V) | V | 1 V = 1 J/C |
13 Isotopes | C | curie (Ci) | Ci | 1 Ci = 3.7 x 10^10 Bq |
14 radiance | R | radian (rad) | rad | 1 rad = 1.002 x 10^-2 C/kg s |

Example 1

- A current of 2 μA must flow into a 100 nF capacitor for how many seconds to produce a 50 V potential difference across the capacitor?

A. 1uria
B. 2
C. 2.5
D. 5
E. 25

\[ C = \frac{Q}{V} = \frac{I}{V} \]
\[ t = \frac{CV}{I} = \frac{100 \times 10^{-9} \times 50 \text{ V}}{2 \times 10^{-6} \text{ A}} = 2.5 \text{ s} \]

Example 2

- A 2-cc-volume ionization chamber is placed in a radiation field of 100 R/s. What is the current generated in amperes (ρ = 0.0013 g/cm³)?

A. 5.1 x 10^-11
B. 6.3 x 10^-10
C. 5.1 x 10^-9
D. 6.7 x 10^-9
E. 5.1 x 10^-8

\[ \rho = V_{air} \rho_{air} = \frac{2 \text{ cm}^3 \times 0.0013 \text{ g/cm}^3}{0.000266 \text{ g} = 2.6 \times 10^{-6} \text{ kg}} \]
\[ I = 2.58 \times 10^{-3} \text{ C/(kg s)} \times 2.6 \times 10^{-6} \text{ kg} = 6.71 \times 10^{-8} \text{ A} \]

Atoms

<table>
<thead>
<tr>
<th>Atomic Numbers</th>
<th>Atomic Weights (amu)</th>
<th>Mass Numbers of Stable Isotopes (A)</th>
<th>Mass Numbers of Unstable Isotopes (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen</td>
<td>H</td>
<td>1</td>
<td>1.007827</td>
</tr>
<tr>
<td>Helium</td>
<td>He</td>
<td>2</td>
<td>4.0026</td>
</tr>
<tr>
<td>Lithium</td>
<td>Li</td>
<td>3</td>
<td>6.941</td>
</tr>
<tr>
<td>Beryllium</td>
<td>Be</td>
<td>4</td>
<td>9.0122</td>
</tr>
<tr>
<td>Boron</td>
<td>B</td>
<td>5</td>
<td>10.811</td>
</tr>
<tr>
<td>Carbon</td>
<td>C</td>
<td>6</td>
<td>12.011</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>N</td>
<td>7</td>
<td>14.0067</td>
</tr>
<tr>
<td>Oxygen</td>
<td>O</td>
<td>8</td>
<td>15.9994</td>
</tr>
</tbody>
</table>

- Atomic number Z: number of electrons (protons)
- Mass number A: total number of protons + neutrons
- Atomic mass: Carbon 12 has atomic mass 12.0000 amu (6 protons + 6 neutrons)
- Typical notation: gclement
Atoms

- **Isotopes**: have the same number of protons, but different number of neutrons
  - Same atomic number $Z$, and number of electrons
  - Same chemical properties
  - Different mass number $A$
- **Isotones**: have the same number of neutrons ($A-Z$)
- **Isobars**: have the same atomic mass $A$ (total number of protons + neutrons)
- There is redundancy in full notation ${}_{Z}^{A}X$
  - Atomic number $Z$ determines the element $X$
  - Isomers: the same $A$ and $Z$, different nuclear energy state (stable vs. metastable, or excited); notation: ${}_{Z}^{A}X$

Atomic energy levels

![Atomic energy levels](image)

- Electron orbits have defined energies (levels)
- The innermost is K (up to two electrons with opposite spin), next is L (up to eight electrons), etc.
- Filled outer shell – chemically inert atom

Mass and energy

- Photon energy: $E = h\nu = hc/\lambda$
- Mass-to-energy conversion: $E = mc^2$
- Relativistic mass: $m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$
- Rest mass $m_0$
- Kinetic energy: $K.E. = mc^2 - m_0c^2$

Nucleus and its energy levels

![Nucleus and its energy levels](image)

- System of nuclear energy levels
- Nuclear transitions produce photons ($\gamma$-rays) and particles in MeV energy range

Mass and energy

<table>
<thead>
<tr>
<th>Electron</th>
<th>Proton</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kinetic Energy (MeV)</td>
<td>Total Energy (MeV)</td>
</tr>
<tr>
<td>10 keV</td>
<td>0.521</td>
</tr>
<tr>
<td>100 keV</td>
<td>0.611</td>
</tr>
<tr>
<td>200 keV</td>
<td>0.711</td>
</tr>
<tr>
<td>500 keV</td>
<td>1.011</td>
</tr>
<tr>
<td>1 MeV</td>
<td>1.511</td>
</tr>
<tr>
<td>2 MeV</td>
<td>3.511</td>
</tr>
<tr>
<td>5 MeV</td>
<td>5.511</td>
</tr>
<tr>
<td>10 MeV</td>
<td>10.511</td>
</tr>
<tr>
<td>20 MeV</td>
<td>20.511</td>
</tr>
<tr>
<td>50 MeV</td>
<td>50.511</td>
</tr>
<tr>
<td>100 MeV</td>
<td>100.511</td>
</tr>
</tbody>
</table>

Mass-to-energy conversion factors:
- 1 electron mass = 0.511 MeV
- 1 amu = 931.5 MeV
Example 3
From the following table of particle rest masses, calculate the gamma energy emitted when a proton captures a neutron to create a deuteron. 1 amu corresponds to the rest mass energy of 931.5 MeV.

<table>
<thead>
<tr>
<th>particle rest mass</th>
<th>amu</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proton</td>
<td>1.00727</td>
</tr>
<tr>
<td>Neutron</td>
<td>1.00866</td>
</tr>
<tr>
<td>Deuteron</td>
<td>2.01355</td>
</tr>
</tbody>
</table>

A. 1.875 MeV  
B. 2.02 MeV  
C. 2.22 MeV  
D. 2.38 MeV  
E. 4.03 MeV

\[ E = m_u c^2 \left( \sqrt{1 - \beta^2} - 1 \right) \]

5 MeV = 0.511 MeV \( \times \) 9.31 MeV

\[ \frac{v}{c} \approx 1 - 0.01 = 0.99 \\
\frac{v}{c} \approx 0.995 \\
v \approx 0.995 \times 3 \times 10^8 \text{ m/s} \]

Example 4
• Find the velocity of an electron accelerated through the potential difference of 5 MeV.

\[ KE = 5 \text{ MeV} \]

Exponential behavior

\[ \frac{dN}{dt} = -\lambda N = -\frac{1}{t_a} N \]

\[ N = N_0 e^{-\lambda t} = N_0 e^{-\frac{t}{t_a}} = N_0 2^{-t/t_h} \]

The sign determines the process: decay or growth \( \lambda \)

\( t_a \) – average life; \( t_b \) – half-life;

\( \lambda = \frac{1}{t_a} = \ln 2/t_b = 0.693/t_b \)

Example 5
In the case of simultaneous physical decay and biological clearance, when \( T_p \) is the physical half-life and \( T_b \) is the biological half-life, the effective half-life \( T_{eff} \) is generally equal to:

A. \( T_p + T_b \)
B. \( 1/T_p + 1/T_b \)
C. \( T_p \times T_b \)
D. \( 1/(T_p \times T_b) \)
E. \( \sqrt{T_p \times T_b} \)

\( J_{eff} = \sqrt{\left( \frac{1}{T_p} + \frac{1}{T_b} \right)} \)

Since \( T_{eff} = t_{eff} / \ln 2 \)

\[ T_{eff} = \sqrt{\frac{1}{T_p} + \frac{1}{T_b}} \]
Chapter 2
The Production and Properties of X Rays

Radiation Dosimetry I

http://www.utoledo.edu/med/depts/radther

X-ray tube design

- Filament is heated, releasing electrons via thermionic emission ($V_f \sim 10V$, $I_f \sim 4A$, resulting in $T>2000^\circ C$)
- X rays are produced by high-speed electrons bombarding the target
- Typically < 1% of energy is converted to x-rays; the rest is heat

X-ray tube current

- Electron cloud near the filament creates space charge region, opposing the release of additional electrons
- Increase in tube voltage increases tube current; limited by filament emission
- High filament currents and tube voltage of 40 to 140 kV must be used

X-ray tube: power source

- The source of electrical power is usually ac (easier to transmit through power lines)
- X-ray tubes are designed to operate at a single polarity: positive anode, negative cathode
- Need to manipulate available power source (suppress or rectify wrong polarity)
- The highest x-ray production efficiency can be achieved at a constant potential

Alternating currents and voltages

- Phase changes from 0 to 360° during the 1 cycle time of 1/60 s
- Negative wave is suppressed or rectified
- Averaging: $V_{ave} = \frac{1}{2} V_0$ or $V_{max} = \frac{1}{\sqrt{2}} V_0$
Rectification

Three phase units

- Need to increase pulse repetition rate to deliver high x-ray flux in a short period of time
- Three phase units: voltage between any pair of 3 wires

Example 1

- Which type of x-ray generator produces the highest effective tube voltage, assuming the peak voltage is applied across the tube?
  
  A. One-phase
  B. Three-phase
  C. Constant potential
  D. The effective voltage is the same for all types

  In C - effective voltage = peak voltage.

Example 2

Ratio of the turns in a transformer is \( N \). Given an input RMS (primary) voltage, what is the peak output (secondary) voltage?

Faraday's law:

\[
V_p = N_p \frac{dN}{dt} \quad \text{and} \quad V_s = N_s \frac{dN}{dt}
\]

\[
V_{\text{rms}} = V_p / \sqrt{2}
\]

Example 3

- What energy (kJ) is imparted to a rotating anode (0.25 kg) during a 2 s exposure that produced a temperature of 2500°C. Specific heat of tungsten is 0.035 kcal/kg°C, and 1 cal = 4.186 J

\[
E = cm \Delta T = 0.035 \times 4.186 \times 10^3 \times 0.25 \times 2.500 = 91.5 \times 10^3 = 91.5 \text{ kJ}
\]
Diagnostic x ray tubes

- X-rays that are emitted from the target travel through different thickness of cathode material
- **Heel effect**: radiation intensity toward the cathode side of the x-ray tube is higher than on the anode side
- Cathode is typically mounted over the thicker part of the patient to balance the amount of transmitted photons on the imager

Rating of diagnostic tubes

- Focal spot loading determines the maximum permissible exposure: there is a maximum power that can be tolerated before target starts melting ($T_{\text{melting}}=3400^\circ$C for tungsten)
- Anode cooling and housing cooling rates determine the number of exposures that may be given in a sequence

Rating of diagnostic tubes

- The combination of current and voltage must lie to the left of the appropriate curve
- The maximum duration of a single exposure depends on spot size, anode rotation speed, current, voltage, power supply type

X ray tubes for radiotherapy

- Mostly for superficial treatments
- No need for a small spot source
- The instantaneous energy input is small (about 1/10) but the average energy input is ~ 10 times greater compared with a diagnostic tube
- Due to much higher energy (>200keV) of electrons bombarding the target, there is a problem of secondary electrons emerging from the target
  - Solution: the target is placed in a “hood” - hollow tube with copper shielding intercepting the secondary electrons

X-ray spectra

- Characteristic radiation from the tungsten target
- Continuous white radiation (Compton scattering)
- End point energy (Rest energy)
Interactions of electrons with the target to give x rays

Most probable: no x-rays produced

Breaking radiation

Figure 2.14. Typical electron interactions with a target. (a) Electron suffers inelastic losses, giving rise to delta rays and Compton scatter. (b) The electron spits a K electron, giving rise to characteristic radiation. (c) Collision between an electron of energy E and a nucleus, leading to bremsstrahlung of energy E. The electron emerges from the "table" with energy E' < E. (d) Rare collision when the electron is completely stopped in one collision, giving rise to a photon of energy E = hν.

Breaking radiation

Bremsstrahlung interaction

Figure 2.15. Melting source of x-rays. In each collision energy is produced only a small amount of energy is converted into x-rays. This electron is scattered at a large angle, giving rise to a light spectrum of characteristic x-rays. The electron has a kinetic energy in the range of 0.1 to 1 MeV. The upper limit is set by the maximum energy of photon and the lower limit by the maximum energy that can be absorbed by the target.

• Thin target approximation: one collision per electron
• Thick target approximations: I(E) = CZ(E_{max} - E)

Example 4

• The energy levels of K, L, and M shells in tungsten are -69.5, -11.0, and -2.5 keV. What photon energies will be present in its characteristic X-ray spectrum?

A. 67.0, 58.5, 8.5 keV
B. 80.5, 72.0, 13.5 keV
C. 69.5, 11.0, 2.5 keV
D. Continuous spectrum from 2.5 to 69.5 keV
E. Continuous spectrum below 2.5 keV

Photon energies are equal to the differences between corresponding energy levels

Example 5

• A target material has the following binding energies: K=30 keV, L=4 keV, M=0.7 keV. If 40.0 keV electrons are fired at the target, what kind of x-rays can have the following energies?
  6-1: 34 keV
  6-2: 26 keV
  6-3: 40.7 keV

A. Characteristic only
B. Bremsstrahlung only
C. Both A and B
D. Neither A nor B

Answers:
• 6-1: B
• 6-2: C
• 6-3: D

Summary

• X-rays are produced via bremsstrahlung interactions of high-energy electrons within a target
  - Efficiency is low, most energy goes into target heating
  - Continuous spectrum includes characteristic x-rays due to target material
• Required high voltage (≈50-100 kV) to accelerate electrons
  - Power source: ac to dc conversion

Characteristic radiation

TABLE 2-3
Principal Emission Lines in keV for Tungsten and Molybdenum

<table>
<thead>
<tr>
<th>Transition</th>
<th>Symbol</th>
<th>Energy (keV)</th>
<th>Relative Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-(\gamma)</td>
<td>K(_\gamma)</td>
<td>69.081</td>
<td>7</td>
</tr>
<tr>
<td>K-(\delta)</td>
<td>K(_\delta)</td>
<td>67.244</td>
<td>21</td>
</tr>
<tr>
<td>K-(\alpha)</td>
<td>K(_\alpha)</td>
<td>66.050</td>
<td>11</td>
</tr>
<tr>
<td>K-(\beta)</td>
<td>K(_\beta)</td>
<td>59.321</td>
<td>100</td>
</tr>
<tr>
<td>K-(\alpha)</td>
<td>K(_\alpha)</td>
<td>57.984</td>
<td>58</td>
</tr>
<tr>
<td>K-(\delta)</td>
<td>K(_\delta)</td>
<td>19.602</td>
<td>24</td>
</tr>
<tr>
<td>K-(\beta)</td>
<td>K(_\beta)</td>
<td>17.479</td>
<td>100</td>
</tr>
<tr>
<td>K-(\alpha)</td>
<td>K(_\alpha)</td>
<td>17.375</td>
<td>32</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Transition</th>
<th>Symbol</th>
<th>Energy (keV)</th>
<th>Relative Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>L-(\gamma)</td>
<td>L(_\gamma)</td>
<td>11.137</td>
<td>10</td>
</tr>
<tr>
<td>L-(\delta)</td>
<td>L(_\delta)</td>
<td>9.253</td>
<td>127</td>
</tr>
<tr>
<td>K-(\gamma)</td>
<td>K(_\gamma)</td>
<td>8.395</td>
<td>100</td>
</tr>
<tr>
<td>K-(\delta)</td>
<td>K(_\delta)</td>
<td>8.355</td>
<td>11</td>
</tr>
</tbody>
</table>

From Storm and Israel (SI)

• Different transitions have different probabilities, according to quantum mechanics selection rules (some transitions are forbidden)