Chapter 3
The Fundamentals of Nuclear Physics
Radiation Dosimetry I

http://www.utoledo.edu/med/depts/radther

Outline
• Terms: activity, half life, average life
• Nuclear disintegration schemes
• Parent-daughter relationships
• Activation of isotopes

Natural radioactivity
• Particles inside a nucleus are in constant motion; can escape if acquire enough energy
• Most lighter atoms with Z<82 (lead) have at least one stable isotope
• All atoms with Z > 82 are radioactive and disintegrate until a stable isotope is formed
• Artificial radioactivity: nucleus can be made unstable upon bombardment with neutrons, high energy protons, etc.

Activity
• Activity – number of disintegrations per unit time; directly proportional to the number of atoms present
  \[ A = \frac{\Delta N}{\Delta t} = \lambda N = N_0 e^{-\lambda t} \]  
  Average lifetime
  \[ t_a = 1.44 t_b \]
  \[ A = N_0 e^{-\lambda t_{1/2}} = A_0 \cdot 2^{-t/t_{1/2}} \]  
  Half-life
• Units: Bq = 1/s, Ci=3.7x10^10 Bq
Example 1

- A prostate implant has a half-life of 17 days. What percent of the dose is delivered in the first day?
  
  \[ \frac{N}{N_0} = 2^{-t/t_{1/2}} \]

A. 0.5 
B. 2 
C. 4  
D. 15 
E. 30

Delivered : 4%

Example 1A

- A prostate implant has a half-life of 17 days. If the initial dose rate is 10 cGy/h, what is the total dose delivered?

\[ D_{\text{total}} = D_{\text{water}} \]

A. 9 
B. 29 
C. 59 
D. 75 
E. 300

\[ 10 \times 10^2 \times 17 \frac{cGy}{h} 	imes 0.693 \times \frac{24}{\lambda} = 39 \text{ Gy} \]

Example 2

- Initial activity of an \(^{123}\)I sample (\(t_0 = 13\) h) injected in the blood is 480 MBq. After 12 h measured activity is 20 MBq/l. Assuming the volume of the blood is 6 l find biological half-life, \(h\).

A. 4 
B. 6 
C. 9 
D. 11 
E. 13

\[ \lambda = \frac{0.693}{h}, \, A_t = A_0 \times 2^{-t/t_{1/2}} \]

\[ t_{\text{eff}} = \frac{1}{1/6 - 1/13} = 11 \text{ h} \]

\[ 1/h_{\text{eff}} = 1/t_0 + 1/t_b \]

Decay schemes

- A decay (disintegration) scheme depicts possible routes of radioactive decay for a nuclide:
  1) \(\alpha\) decay
  2) \(\beta^+\) (positron) decay or electron capture
  3) \(\beta^-\) (negatron) decay
  4) Isomeric transition

Alpha disintegrations

- Emission of helium nucleus, mainly from heavy nuclei

Beta disintegrations

- Ejection of positive or negative electron
- Positron (\(\beta^+\)) or negatron (\(\beta^-\)) emitters
- Proton-rich nucleus – emits positron, neutron-rich emits electron

\[ \beta^- \text{ emission: } n \rightarrow p + \beta^- + \bar{\nu} \quad Z \text{ increases by 1} \]

\[ \beta^+ \text{ emission: } p \rightarrow n + \beta^+ + \nu \quad Z \text{ decreases by 1} \]

In beta-minus emission an antineutrino is emitted instead of neutrino. They have opposite helicity (projection of spin on direction of momentum)
Beta disintegrations

- Co-60 decay scheme:
  - Energy is released in transitions from excited levels of Ni-60 to the ground state via emission of gamma rays (average $\gamma$-ray energy 1.25MeV)

Beta disintegrations

- The ejected beta particle shares its energy with neutrino (anti-neutrino), and therefore may have energy in a continuous spectrum up to $E_{\text{max}}$.

- In the production of isotopes in a nuclear reactor, a neutron is usually added to a stable nucleus, resulting in $\beta^-$ emitters.
- Example: to produce $^{60}\text{Co}$ a neutron is added to $^{59}\text{Co}$.
- Particle accelerators (more expensive) produce $\beta^+$ emitters.
- Since $2m_e c^2 = 1.022$ MeV energy is required to produce electron-positron pair – this is the minimum difference between the initial and final energy (parent-daughter) for $\beta^+$ emitters.

Electron capture

- $\Delta Z X \rightarrow Z\Delta Q$:
  - Orbital electron can be captured by nucleus
  - $p + e\text{ (usually K electron)} \rightarrow n + \nu$.
  - This process is competing with positron emission in proton-rich nuclei.
  - The only process when the energy difference between the initial and final state < 1.022 MeV.

Isomeric transitions

- Isomeric transitions: no change in $A$ or $Z$.
- Always preceded by another transition, leaving nucleus in excited state.
- Energy is released by:
  - Emission of a photon
  - Internal conversion
- Metastable state: an excited state with the lifetime $>10^6$ s.

Isomeric transitions: Photon emission

- Technetium is predominantly an artificially produced radioactive metal. Example: $^{99m}\text{Mo}$ ($t_\nu = 66.7$ hours) produced in nuclear reactors by $\beta^-$ decay to $^{99}\text{Tc}$.
- All isotopes are radioactive, most common: $^{99}\text{Tc}$ ($t_\nu = 210,000$ years) and $^{99m}\text{Tc}$ ($t_\nu = 6$ hours).
- $^{99}\text{Tc}$ is widely used:
  - as a tracer for medical diagnosis ($\gamma$ of 140 keV is detected with gamma-camera)
  - functional scans of brain, bone, liver, spleen, kidney, thyroid
  - for blood flow studies.

Image from https://www.boundless.com
Isomeric transitions:
Internal conversion

- Nucleus relaxes to the ground state
- Alternatively to a $\gamma$-ray emission an orbital electron may be ejected with the energy $E = \gamma - E_b$

Auger electron production

- A hole in K, L, or M shell has to be filled
- The relative probability of the emission of characteristic radiation to the emission of the Auger electron is called fluorescence yield
- Fluorescence yield is higher for larger $Z$

Charts of isotopes

- Almost all the radioactive isotopes of the lighter elements achieve stability by keeping their mass number constant, through beta decay or K capture
- Decay is usually along an isobaric line (constant mass number) and often involves a number of successive transitions before stability is reached

Decay series

- The mass number ($A$) changes only for ___ decay:
  
  A. alpha  
  B. beta minus  
  C. beta plus  
  D. electron capture  
  E. isomeric

Example 3

- The mass number ($A$) changes only for ___ decay:
  
  A. alpha  
  B. beta minus  
  C. beta plus  
  D. electron capture  
  E. isomeric

Nuclear transformations

<table>
<thead>
<tr>
<th>$\Delta A$</th>
<th>$\Delta Z$</th>
<th>Energy deposition</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>-2</td>
<td>Total energy of a particle</td>
</tr>
<tr>
<td>0</td>
<td>+1</td>
<td>$E_{\text{kinetic}} - \frac{1}{3} E_{\gamma}$</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
<td>$E_{\text{kinetic}} - \frac{1}{3} E_{\gamma}$ and $E_{\gamma} = 0.02 \text{ MeV}$</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
<td>Most energy carried away by $\gamma$; $E_{\gamma}$, available and/or $E_{\text{kinetic}}$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>$E_{\gamma}$, available</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>$E_{\text{kinetic}} - E_{\gamma}$, and/or $E_{\text{kinetic}}$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>$E_{\text{kinetic}} - E_{\gamma}$, and/or $E_{\text{kinetic}}$</td>
</tr>
</tbody>
</table>
Example 4

\[ ^{99m}_{43}\text{Mo} \rightarrow ^{99m}_{43}\text{Tc} \rightarrow ^{99}_{43}\text{Tc} \]

- In the above decay of molybdenum, the modes of decay labeled (I) and (II) are, respectively:
  A. beta minus, isomeric transition
  B. isomeric transition, beta minus
  C. beta plus, isomeric transition
  D. beta minus, beta minus
  E. electron capture, beta minus

ilit

Growth of radioactive daughter

- Radioactive daughter is formed at the rate at which the parent decays (\( \lambda_1 \)), and itself decays with a different rate (\( \lambda_2 \)).

\[
dN_2 = \lambda_1 N_1 - \lambda_2 N_2 \\

\text{Solution: } N_2 = \frac{\lambda_1}{\lambda_2 - \lambda_1} (N_1) e^{\lambda_1 t} - e^{\lambda_2 t}
\]

Growth of radioactive daughter

- Express the same solution for activity

\[
A_t = A_0 \frac{\lambda_2}{\lambda_2 - \lambda_1} \left[ 1 - e^{(\lambda_1 - \lambda_2)t} \right]
\]

- For a special case of \( \lambda_2 > \lambda_1 \), can simplify to

\[
A_t \approx A_0 \left[ 1 - e^{-\lambda_1 t} \right]
\]

- After some time – reach a state of parent-daughter equilibrium, where \( A_1 = A_2 \)

Example 5

- After ____ hours, \(^{99m}\text{Tc}\) will be approximately in radioactive equilibrium with its parent element \(^{99}\text{Mo}\). (Half-lives: 67 hours for \(^{99}\text{Mo}\) and 6 hours for \(^{99m}\text{Tc}\).)

A. 6
B. 24
C. 48
D. 54
E. 268

Activation of isotopes

- Atomic nuclei can be activated by neutron bombardment with the probability dependent on the interaction cross section \( \sigma \)

\[
\Delta N = N_0 \sigma \phi \Delta t
\]

- Number of activated atoms:

\[ \sigma \approx 10^{-24} \text{ cm}^2 \]
Activation of isotopes

• More practically relevant quantity is the activity of the irradiated sample 
  \[ \Delta A = \lambda \Delta N = \lambda N_0 \phi \Delta t \]

• If the bombardment time \( \Delta t >> t_h \), activity reaches its maximum
  \[ A = N_0 \phi \]

• Accounting for decay during irradiation:
  \[ A = A_0 \left(1 - e^{-\lambda_0 \Delta t} \right) \]

Example 6

What is the activity of a 20 g sample of \(^{59}\)Co
irradiated in a neutron flux of \(10^{14}\) neutrons/(cm\(^2\)s)
for 6 years? Cross section \(\sigma=36\) barns, \(t_h=5.3\) years.

First find the number of atoms in the target

\[ N_i = \frac{20}{59/693.01} \times 5.3 \times 10^{14} = 2.04 \times 10^{21} \]

The activity at a time \(t\):

\[ A = \phi \sigma N_0 \left(1 - e^{-\lambda_0 t} \right) = 10^{14} \times 2.04 \times 10^{21} \times 36 \times 10^{-26} \left(1 - e^{-0.0001135} \right) = 4.0 \times 10^{14} \text{Bq} \]

Nuclear fission and fusion

• Nuclear fission – a nucleus is split into lighter fragments with release of very large energy. Example:
  \[ ^{235}_{92} \text{U} + ^{1}_0 \text{n} \rightarrow ^{144}_{56} \text{Ba} + ^{89}_{36} \text{Kr} + 200\ \text{MeV} \]

• Nuclear fusion – two lighter atoms are fused into a heavier one also with release of energy. Example:
  \[ ^{1}_1 \text{H} + ^{1}_1 \text{H} \rightarrow ^{3}_{2} \text{He} + n + \text{Energy} \sim 3.3\ \text{MeV} \]

Summary

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