Chapter 4
High Energy Machines

Radiation Dosimetry I

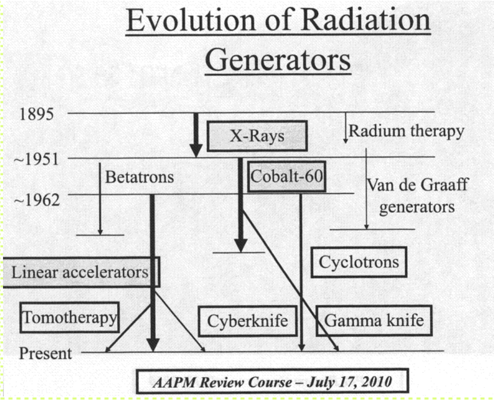
Text: H.E Johns and J.R. Cunningham, The physics of radiology, 4th ed.
http://www.utoledo.edu/med/depts/radther

Outline

- Brief history of radiation generators
- Betatrons
- Linear accelerator: main components, components of the accelerator head
  - – Thomotherapy
- Cyclotrons

General considerations

- Depth-dose properties:
  - – High penetration
  - – Delivery of maximum dose at a depth
  - – Minimal low-energy electron contamination
  - – Skin sparing
- Field flatness (not a requirement anymore)
- Constant output and monitoring
- Sharp field edges (penumbra)



Evolution of Radiation Generators

AAPM Review Course – July 17, 2010

Evolution of radiation generators

1895 - Roentogen discovers X-rays
1913 W.E.Coolidge develops vacuum X-ray tube
1931 E.O.Lawrence develops a cyclotron
1932 1MV Van de Graaff accelerator installed, Boston, MA (USA)
1939 First medical cyclotron for neutron therapy, Crocker, CA (USA)
1946 20MeV electron beam therapy with a Betatron, Urbana, IL (USA)
1952 First Co-60 teletherapy units, Saskatoon (Canada)
1956 First 6MeV linear accelerator, Stanford, CA (USA)
1958 First proton beam therapy (Sweden)
1959 First scanning electron beam therapy, Chicago, IL (USA)
1976 First pion beam therapy, LAMPF, NM (USA)
1990 First hospital based proton therapy, Loma Linda, CA (USA)

Yoichi Watanabe, MPHY5170/TRAD7170, http://www.tc.umn.edu/~watan016/Teaching.htm

Betatrons

- Device for accelerating electrons
- Electrons from a filament are injected
- AC magnet performs two functions:
  - – Electrons are bent into a circular path
  - – Changing magnetic flux creates E-field accelerating electrons
- Electrons spiral inward until reach equilibrium orbit
- They are accelerated as they travel around at equilibrium orbit

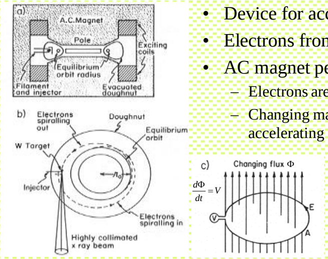


Figure 4-3. Diagrams illustrating the construction and operation of the betatron. (a) Cross-sectional diagram showing the AC magnet, the poles, the doughnut, and injector. (b) The paths of the electrons within the doughnut and the method of production of the x rays. (c) How an electric field is produced by a changing magnetic flux. (d) The cycle

Betatrons

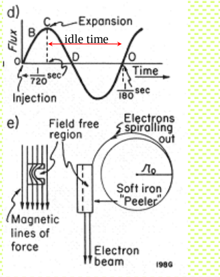


Figure 4-3 (d) The cycle of operation of the betatron showing the time of injection and expansion. (e) The operation of the electron "peeler" for obtaining an electron beam. The sketch showing the magnetic lines of force is a cross-sectional view of the "peeler" device taken at right angles to the diagram through the center of the "peeler."

- As the electrons get faster they need a larger magnetic field to keep moving at a constant radius orbit, which is provided by the increasing magnetic flux
- After electron attains its maximum energy (pt. C) the field at the orbit is decreased, and electron spirals outward to strike the target
- Betatron can be a source of electrons or x-rays (with a target)
- Radiation is produced in pulses
- Efficiency is very high, but the dose rate could be low due to inefficient injection of electrons

Example 1

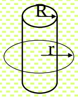
- A long solenoid has a diameter of 2R=12 cm. When a current I exists in its windings, a uniform magnetic field of magnitude B=30 mT is produced in its interior. By decreasing I, the field is caused to decrease at the rate of 6.5 mT/s. Calculate the magnitude of the induced electric field E at r=8.2 cm from the axis of the solenoid.

A. 10.0 μV/m
B. 31 μV/m
C. 75 μV/m
[✓ D.] 143 μV/m
E. 200 μV/m

$$\frac{d\Phi}{dt} = V = E \times 2\pi r; \qquad B = \Phi / \left(\pi R^2\right)$$

$$\frac{d\Phi}{dt} = \frac{d(B\pi R^2)}{dt} = \pi R^2 \frac{dB}{dt}$$

$$E = \frac{R^2}{2r}\frac{dB}{dt} = \frac{(6\times10^{-2})^2}{2\times8.2\times10^{-2}} 6.5\times10^{-3} = 143\mu\text{V/m}$$



Linear Accelerators

- A charged particle can be accelerated while moving through a potential difference between two electrodes
- Arranging electrodes in a certain configuration and using electromagnetic wave rather than a static potential difference allows for higher beam energies and intensities
- Two main components: power source and resonant cavity

Linear Accelerators

- I. POWER SOURCE
  1. Why not DC: Problems of electrical breakdown, physical size of electrical equipment
  2. Apply technique of repeated pulses, V = nv
     - Need oscillating form of power supply
  3. Leads to principle of cyclic and linear accelerators
  4. Wavelength has to be short enough to accelerate electrons in a reasonable distance
  5. S-band microwave technology, developed for radar in WWII, has a frequency of ~ 3 GHz or λ= 10 cm
  6. High power is also needed to ensure sufficient energy gain per cycle

P.J. Biggs, AAPM Review course 2010

Linear Accelerators

- II. RESONANT CAVITY FOR GENERATION OF HIGH POWER MICROWAVE PULSES:
  1. At high frequencies, ordinary resonant circuits become impractical. Also problems of radiation loss
  2. Hollow cavities as resonant circuits (skin effect)
  3. The quality factor, or Q value, of a resonant circuit or cavity is defined as

     Q = energy stored in cycle/energy lost in cycle

  For a circuit, Q ~ 10^2, whereas for a cavity, Q ~ 10^4
  4. Achieved through devices called magnetrons and klystrons
  5. Cavity resonators feature in **both** power sources and accelerating structures

  ROLE OF RESONANT CAVITIES IN LINEAR ACCELERATORS
  1. Cavity acts as an acceleration module
  2. Multiple cavity arrangement can act as an RF amplifier - klystron
  3. Multiple cavity arrangement can act as a high power oscillator - magnetron

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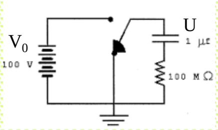
RC circuit



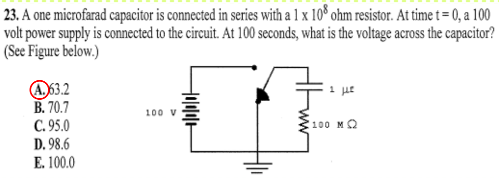
$$V_0 + U - IR = 0; \qquad I = dq/dt = d(CU)/dt$$

$$V_0 + U - RC\frac{dU}{dt} = 0$$

$$\frac{dU}{dt} - \frac{U}{RC} = \frac{V_0}{RC}$$

$$U = V_0\left(1 - e^{-t/RC}\right)$$

Example 2



23. A one microfarad capacitor is connected in series with a 1 x 10^8 ohm resistor. At time t = 0, a 100 volt power supply is connected to the circuit. At 100 seconds, what is the voltage across the capacitor? (See Figure below.)

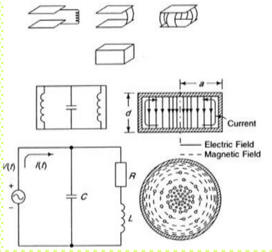
[✓ **A.** 63.2]
B. 70.7
C. 95.0
D. 98.6
E. 100.0

$$RC = 100 \times 10^{6} \times 10^{-6} = 100$$

$$U = V_0\left(1 - e^{-t/RC}\right) =$$

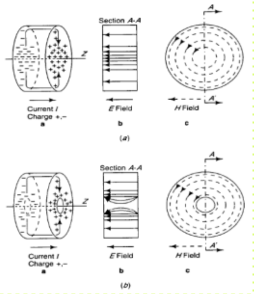
$$100\left(1 - e^{-100/100}\right) = 100 \times 0.632 = 63.2$$

Cavity Principle



- Microwave cavity design is derived from a simple resonant circuit. The components of such a circuit need to become smaller as the resonant frequency increases
- The capacitor, resistance and inductance become the faces and sides of a pill box such that the entire surface is made of conducting material
- When this cavity is excited, strong electric fields are generated perpendicular to the faces that alternate in sign with the frequency of the oscillation

Cavity Principle



Electric, magnetic field configurations for lowest resonant mode

- In general, ω=$(LC)^{-1/2}$ for a resonant circuit, so one needs very small L and C to obtain high frequencies: use of cavities as a form of resonant circuit since they have low L and C
- Only the electric field plays a role in electron acceleration (along the axis of a cylindrical cavity)
- Electric field configuration in cylindrical microwave cavity with hole on axis for electrons to pass through is only slightly modified
- Efficient transfer of energy to electron beam, i.e., low energy losses, since for a resonant circuit Q ~ 10^2 (where Q=$f_0/2\Delta f$ and 2Δf is FWHM), whereas for a cavity, Q ~10^4

Example 3

- Determine the lowest frequency allowed in a cubic microwave oven of linear dimension of a=30 cm.

A. 0.1 GHz
B. 0.2 GHz
[✓ C.] 0.5 GHz
D. 1 GHz
E. 5 GHz

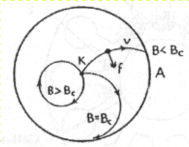
$$2a = n\lambda$$

$$n = 1$$

$$\lambda = 2x30 = 60cm$$

$$\nu = c/\lambda = 3\times10^{10}/60 = 5\times10^{8} = 0.5\,\text{GHz}$$

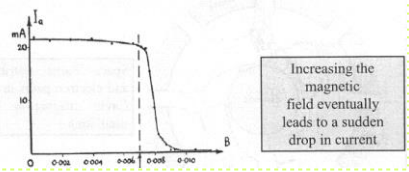
Magnetron



At low magnetic fields, the electron paths' are bent, but the electrons still reach the anode. At a field Bc, the electron paths' just graze the anode. For higher fields, individual electrons are unable to reach the anode.

- Generator of microwaves
- A cylindrical anode (A) has the cathode (K) along its axis.
- Magnetic field is directed along the axis of the cylinder
- Magnetic field bends the electron trajectory, and accelerating electron emits energy in microwave range
- The magnetic field has to be high enough to provide electron collection, but low enough to avoid electron accumulation at K

Magnetron



Increasing the magnetic field eventually leads to a sudden drop in current

- Current between anode and cathode is not used in a power source
- If it stops the magnetron is blocked due to electron accumulation around the cathode

Magnetron

Resonant cavities

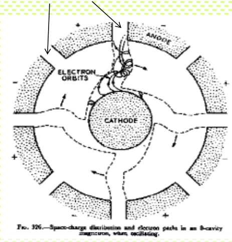


FIG. 326.—Space-charge distribution and electron paths in an 8-cavity magnetron, when oscillating.

- The circulating electrons induce a charge distribution between adjacent segments of the anode
- This, in turn, perturbs the electron orbits so that the space charge distribution into a form resembling the spokes of a wheel
- The resonant cavities gain energy from the orbiting electrons which are slowed and eventually spiral into the anode, leading to anode current

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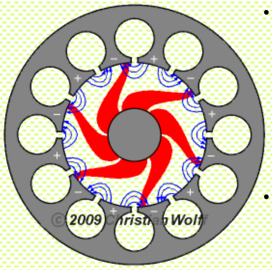
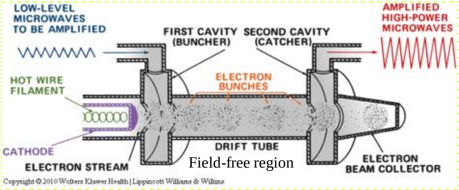
Magnetron



Twelve-cavity magnetron
http://www.radartutorial.eu/08.transmitters/pic/mag06.big.gif

- The dc field extends radially from adjacent anode segments to the cathode (shown in red); the ac fields extend between adjacent segments (blue lines), their magnitude changes with the RF oscillations occurring in the cavities
- Rotating space-charge wheel results in continuously delivered energy to sustain the RF filed

Klystron



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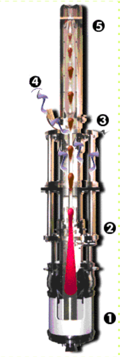
- Klystron is a microwave amplifier
- Low-power signal is supplied to the "buncher" cavity, its E-field modulates velocities of electrons, which then merge into bunches going through the drift tube
- Amplified signal is collected from "catcher" cavity, where electrons decelerate emitting at the same *resonant* frequency

Klystron



http://www2.slac.stanford.edu/vvc/accelerators/klystron.html

- The electron gun 1 produces electrons
- The bunching cavities 2 regulate the speed of the electrons (ac field accelerates slow electrons and slows down fast ones) so that they arrive in bunches at the output cavity
- The bunches of electrons decelerate and excite microwaves in the output cavity 3 (E-filed of the same frequency but polarity opposite to that applied in bunching cavities)
- The amplified microwaves flow into waveguide 4, to the accelerating structure
- The electrons are absorbed in the beam stop 5
- Usually klystrons have with multiple amplifying cavities

Waveguides

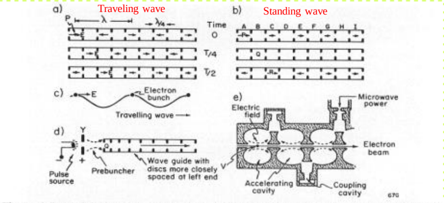
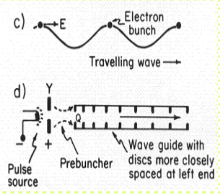


Figure 4-5. (a) Schematic representation of wave guide with a series of equally spaced discs λ/4 apart. The electric field E at various positions in the wave guide is shown at times of 0, T/4, T/2, etc. when a *traveling* wave is passing down the guide from left to right. (b) Same as a except the electric field configuration is now shown for a *standing* wave. (c) Schematic representation of electron bunch being carried on the crest of a traveling wave. (d) Diagram of buncher showing how electrons are captured into the wave guide. (e) Details of one type of wave guide used to create standing waves.

Waveguides

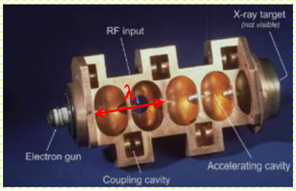


- The velocity of the wave is matched with the velocity of the electron at time of injection
- Disk spacing is gradually increased to correspond to velocity c down the waveguide

Waveguides

- Cavities in a waveguide serve dual purpose: accelerate electrons (those that are at the accelerating polarity) and transfer microwave power to the next cavity (coupling)
- In standing wave configuration half of cavities have essentially zero E-field (no acceleration); those are moved off-axis thus making the waveguide shorter
- Traveling waveguides (E~80kV/cm) are longer than standing waveguides (E~150kV/cm)

Accelerating waveguides



Cut-away of standing wave accelerating structure: 30cm long with 5 accelerating cavities (from Karzmark/ Podgorzak)

- Most linacs use waveguides operating at 3000 MHz (S-band)
- Compact waveguides (used in CyberKnife, Tomotherapy) operate at 9000 MHz (X-band)
- Traveling waveguides (E~80kV/cm) are longer than standing waveguides (E~150kV/cm)

Example 4

- What is the minimum size of a cubical cavity in a standing wave waveguide for 3000 MHz microwaves?

A. 2 cm
B. 3 cm
[✓ C] 5 cm
D. 10 cm

$$d=\frac{\lambda}{2}=\frac{c}{2\nu}=\frac{3\times10^{10}\,\text{cm/s}}{2\times3000\times10^{6}/\text{s}}=5\,\text{cm}$$

Medical linacs

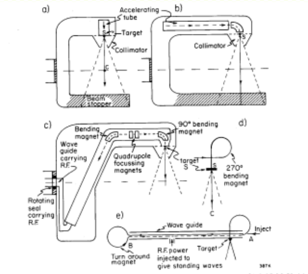
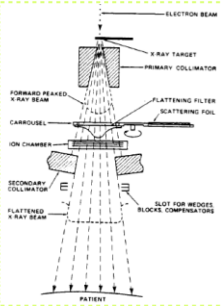


Figure 4-6. Medical Linacs: Arrangements suitable for (a) 4 to 6 MeV, (b) 8-12 MeV, (c) 30 to 35 MeV. (d) A method of bending the electron beam through 90° by using a 270° bending magnet. (e) The "turn around," or "two pass," linac.

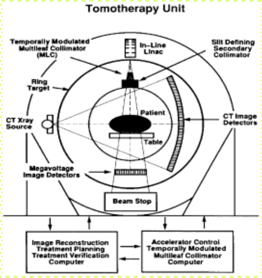
- Most use isocentric mounts with the distance of 100 (80) cm to the axis of rotation
- The center of the tumor is positioned at the axis

Medical linacs



- Various components serve a purpose of delivering flat beam with pre-defined field sizes and adequate shielding beyond the target
- Ion chamber is used to monitor the output

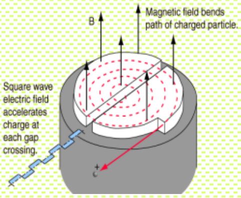
Tomotherapy



Conceptual drawing of a helical tomotherapy unit in the first tomotherapy paper (Mackie *et al 1993)*.

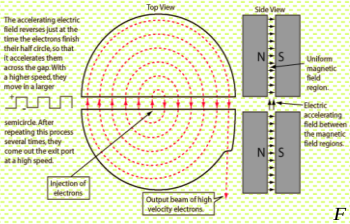
- A linac is mounted on a CT-like ring gantry
  - Conformal treatment (MLC)
  - Image guidance (MV imaging)
- The source can move either through a set of discrete angular positions or helically
- First patient treated in 2002

Cyclotron



- For sources of protons and other heavy particles
- Massive particle moves through relatively small potential difference many times
- Magnetic field is used to bring the particle to an accelerating region
- High-frequency oscillator creates potential difference between D’s and is used for acceleration

Cyclotron



$$F = Bq\mathrm{v}, \frac{m\mathrm{v}^2}{r} = Bq\mathrm{v}$$

- Due to increase in mass particle may get out of step with the E field

$$t_t = f_{osc}^{-1} = \frac{2\pi r}{\mathrm{v}} = \frac{2\pi m}{Bq}$$

Example 5

- A deutron (m=3.34x10^{-27} kg) circulates in a cyclotron of radius 53 cm and operating frequency 12 MHz, beginning approximately at rest at the center. The electric potential between D’s is 80 kV. How many passes will it complete before reaching the edge of the cyclotron?

A. 80
[✓ B.] 105
C. 130
D. 150
E. 200

$$\mathrm{v}_{final} = 2\pi rf$$

$$K_{final} = \frac{m\mathrm{v}^2}{2} = \frac{m(2\pi rf)^2}{2} =$$

$$\frac{3.34\times10^{-27}(2\pi\times0.53\times12\times10^6)}{2} =$$

$$2.7\times10^{-12} J\times6.24\times10^{18} J/eV \approx 16.8 MeV$$

$$K_{pass} = 2\times80\times10^3 = 1.6\times10^5 eV = 0.16 MeV$$

$$n = K_{final}/K_{pass} = 16.8/0.16 \approx 105$$

Summary

- Particle accelerators: circular and linear
- High energy x-ray machines use electron accelerator and a target
- Using electromagnetic wave rather than a static potential difference allows for higher beam energies and intensities

Additional reading

- P.J. Biggs, AAPM Review course 2010, available at https://www.aapm.org/meetings/2010AM/documents/biggs2.pdf
- C.J. Karzmak, R.J. Morton, A primer on theory and operation of linear accelerators in radiation therapy, Med. Phys. Publishing corp., 1989
- E.B.Podgorsak, Radiation Physics for Medical Physicists, 2nd edition, Springer 2010, Chapters 13-14