Chapter 5
The Interaction of Ionizing Radiation with Matter

Radiation Dosimetry I

Text: H.E Johns and J.R. Cunningham, The physics of radiology, 4th ed.
http://www.utoledo.edu/med/depts/radther

Outline

- Absorption of energy
- Quantitative characterization
  - Attenuation, HVL, TVL
  - Absorption coefficients
- Basic interactions of radiation with matter
- Photon interactions

Absorption of energy

- When an x-ray beam passes into an absorbing medium such as body tissues, some of the energy carried by the beam is transferred to the medium where it may produce biological damage
- The energy deposited per unit mass of the medium is known as the **absorbed dose** and is a very useful quantity for the prediction of biological effects
- The events that result in this absorbed dose and subsequent biological damage are quite complicated

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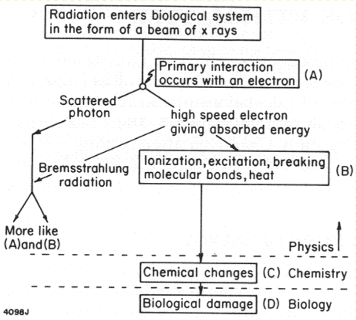
Absorption of energy



Beam attenuation

- As beam passes through a medium its attenuation (change in the number of photons) can be described by

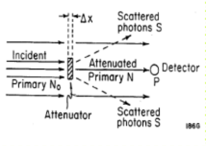
$$N = N_0 e^{-\mu x}$$

- μ is the linear attenuation coefficient - the fraction of photons that interact per unit thickness of attenuator
- The special thickness that attenuates the beam to 50% is called the half-value layer or HVL:

$$HVL = x_h = \ln 2 / \mu = 0.693 / \mu$$

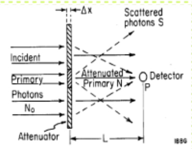
$$N = N_0 e^{-\mu x} = N_0 2^{-x/x_h}$$

Beam attenuation



$$N = N_0 e^{-\mu x}$$

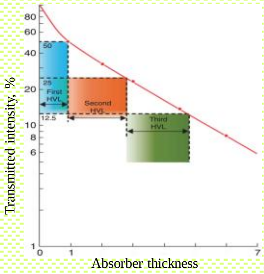
Narrow beam attenuation



$$N = N_0 e^{-\mu x} B(x, hv, A, L)$$

Broad beam attenuation

Beam attenuation



- Only mono-energetic beams can be characterized by a single value of attenuation coefficient and HVL
- For poly-energetic (realistic) beams: concept of first, second, etc. HVL
  - First HVL is always smaller due to preferential absorption of low energy photons

Example 1

- For a beam of monoenergetic photons (such as gamma rays or characteristic X-rays), what is the relationship between the first and the second HVL?

A. The first HVL is thicker than the second.
B. The second HVL is thicker than the first.
[✓ C.] Both HVLs are equal.
D. It depends upon the beam energy.
E. It depends upon filtration.

$$\frac{N}{N_0} = e^{-\mu x_h} = 0.5$$

$$x_h = -\frac{\ln 0.5}{\mu} = \frac{\ln 2}{\mu}$$

Beam attenuation

- Another special thickness that attenuates the beam to 10% is called the tenth-value layer or TVL:

$$TVL = x_t = \ln 10 / \mu \approx 2.3 / \mu$$

$$N = N_0 10^{-x/x_t}$$

- Used mostly in shielding calculations

Example 2

- Approximately how many HVLs are in 6 TVL? (TVL – tenth value layer)

A. 1
B. 10
[✓ C.] 18
D. 28
E. 31

$$HVL: \frac{N}{N_0} = 0.5 = e^{-\mu x_h} \Rightarrow x_h = -\frac{\ln 0.5}{\mu} = \frac{\ln 2}{\mu}$$

$$TVL: \frac{N}{N_0} = 0.1 = e^{-\mu x_t} \Rightarrow x_t = -\frac{\ln 0.1}{\mu} = \frac{\ln 10}{\mu}$$

$$x_t = \frac{\ln 10}{\mu} = \frac{\ln 10}{\ln 2} \times x_h \approx 3x_t \Rightarrow 6x_t \approx 18x_h$$

Attenuation coefficients

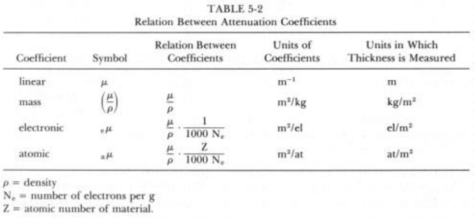


TABLE 5-2
Relation Between Attenuation Coefficients

Coefficient	Symbol	Relation Between Coefficients	Units of Coefficients	Units in Which Thickness is Measured
linear	μ		m^{-1}	m
mass	$\left(\frac{\mu}{\rho}\right)$	$\frac{\mu}{\rho}$	m^2/kg	kg/m^2
electronic	${}_e\mu$	$\frac{\mu}{\rho} \cdot \frac{1}{1000\ N_e}$	m^2/el	el/m^2
atomic	${}_a\mu$	$\frac{\mu}{\rho} \cdot \frac{Z}{1000\ N_e}$	m^2/at	at/m^2

ρ = density
N_e = number of electrons per g
Z = atomic number of material.

- Introduction of different attenuation coefficient allows for more general description and comparison of absorbers
  - Number of atoms per gram = N_A/A
  - Number of electrons per gram = $N_AZ/A = N_e$

Example 3

- For 100 keV photons mass attenuation coefficients of aluminum (ρ= 2.7g/cm^3) and lead (ρ=11.4 g/cm^3) are 0.17 and 5.46 cm^2/g. What is the thickness of aluminum in mm equivalent to 0.2 mm of lead?

A. 8
B. 10
C. 16
D. 19
[✓ E.] 27

$$I = I_0 e^{-\frac{\mu_1}{\rho_1}\rho_1 d_1} = I_0 e^{-\frac{\mu_2}{\rho_2}\rho_2 d_2}$$

$$d_2 = \frac{\frac{\mu_1}{\rho_1}\rho_1}{\frac{\mu_2}{\rho_2}\rho_2} d_1 = \frac{5.46 \cdot 11.4}{0.17 \cdot 2.7} 0.2 = 27\ \text{mm}$$

Example 4

- The mass attenuation coefficient of bone with a density of 1.8 g/cm^3, is 0.2 cm^2/g for an 80-keV gamma ray. The percentage of 80-keV photons attenuated by a slab of bone 4 cm thick is _____%.

A.36
B.45
C.55
D.64
[✓ E.]76

Fractionof transmitted:

$$N/N_0 = e^{-\mu x} = e^{-0.2\times1.8\times4} = 0.24$$

$$N_{atten}/N_0 = 1-0.24 = 0.76$$

Types of ionizing radiations

- Electromagnetic radiations
  - – X-rays and gamma-rays
- Particulate radiations
  - – Electrons, protons, α-particles, heavy charged particles
  - – Neutrons
- All charged particles: directly ionizing radiation
- X and γ-rays, as well as neutrons – indirectly ionizing radiation

Energy transfer and energy absorption

- Photons transfer their energy to a medium as they interact
- Generally, only a portion of that energy can be converted into kinetic energy of electrons, and eventually get absorbed
- For average energy transferred (absorbed) can introduce corresponding attenuation coefficients:

$$\mu_{tr} = \mu(\overline{E}_{tr}/h\nu)$$

$$\mu_{ab} = \mu(\overline{E}_{ab}/h\nu)$$

Energy transfer and energy absorption

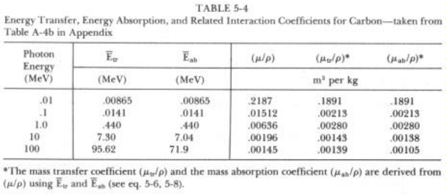


TABLE 5-4

Energy Transfer, Energy Absorption, and Related Interaction Coefficients for Carbon—taken from Table A-4b in Appendix

Photon Energy (MeV)	$\overline{E}_{tr}$ (MeV)	$\overline{E}_{ab}$ (MeV)	(μ/ρ)	$(\mu_{tr}/\rho)^*$	$(\mu_{ab}/\rho)^*$
			m^2 per kg		
.01	.00865	.00865	.2187	.1891	.1891
.1	.0141	.0141	.01512	.00213	.00213
1.0	.440	.440	.00636	.00280	.00280
10	7.30	7.04	.00196	.00143	.00138
100	95.62	71.9	.00145	.00139	.00105

*The mass transfer coefficient (μ_{tr}/ρ) and the mass absorption coefficient (μ_{ab}/ρ) are derived from (μ/ρ) using $\overline{E}_{tr}$ and $\overline{E}_{ab}$ (see eq. 5-6, 5-8).

- Especially in MeV range, part of the energy is lost (scattering, bremsstrahlung)

Example 5

- An x-ray tube emits 10^{12} photons per second in a highly collimated mono-energetic (40-keV) beam that strikes 0.1 mm thick radiographic screen. The attenuation coefficient of the screen is 23 m^{-1}, and the absorption coefficient is 5 m^{-1}. Find the total energy absorbed by the screen during 0.5 sec exposure.

A. 0.6x10^{10} keV
[✓ B.] 1.0x10^{10} keV
C. 1.5x10^{10} keV
D. 1.8x10^{10} keV
E. 2.3x10^{10} keV

$$N_{\text{interactions}} = N_0(1-e^{-\mu x}) = 10^{12}\,\frac{ph}{s}\times 0.5s\times(1-e^{-23\times0.1/10^3}) =$$

$$5\times10^{11}(1-e^{-23\times10^{-4}}) = 1.15\times10^9$$

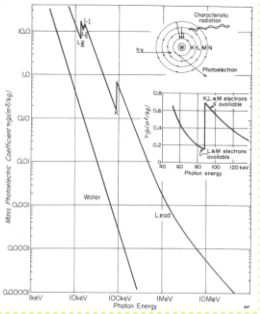
$$E_{ab} = N_{\text{interactions}}\frac{\mu_{ab}}{\mu}E_{ph} =$$

$$1.15\times10^9\,\frac{5\text{m}^{-1}}{23\text{m}^{-1}}\times 40\text{keV} = 1\times10^{10}\text{ keV}$$

Absorption of radiation

- Photons (x and γ-rays) are absorbed by
  - – Photoelectric effect - dominant interaction at lower energies (diagnostic range)
  - – Compton (incoherent) scattering is the dominant mechanism of interaction for higher energy (therapy range)
  - – Pair production for higher energy (above 1.02 MeV)
  - – Photo-nuclear interactions for higher energy (above 10 MeV)
  - – There is also a photon interaction with *no* energy absorption: coherent (Rayleigh) scattering
- Charged particles interact through Coulomb-force field of atom or nucleus
  - – Soft and hard collisions; nuclear interactions of heavy charged particles
- Neutrons interact with nuclei of atoms

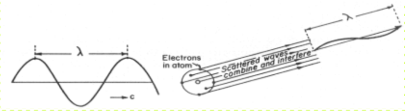
Photoelectric effect



Involves bound electrons

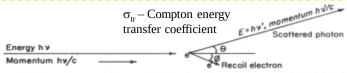
- The probability of ejection is maximum if the photon has just enough energy to knock the electron from its shell
- The photoelectric cross section varies with photon energy approximately as $1/(hv)^3$
- The coefficient per electron or per gram varies with atomic number - as $\sim Z^3$ for high Z materials and $\sim Z^{3.8}$ for low Z materials
- The coefficient per atom for low Z materials varies at $Z^{4.8}$
- In tissue $E_{tr} \approx E_{ab} \approx hv$ and the transfer, absorption, and the attenuation coefficients are nearly equal

Coherent (Rayleigh) scattering



- *No* energy is converted into kinetic energy and all is scattered
- The scattered waves from electrons within the atom combine with each other to form the scattered wave
- The effect of the process is to broaden the angular width of a beam slightly
- Negligibly small for energies greater than about 100 keV in low atomic number materials

Incoherent (Compton) scattering



σ_{tr} – Compton energy transfer coefficient

- It involves an interaction between a photon and an electron
- It is almost independent of Z; decreases with increase in E
- In each collision some energy is scattered and some transferred to an electron, the amount depending on the angle of emission of the scattered photon and the energy of the photon.
- On the average, the fraction of the energy transferred to K.E. per collision increases with increase in photon energy. For low energy photons $\sigma_{tr} \ll \sigma$, for high energy photons $\sigma_{tr} \sim \sigma$
- In soft tissue the Compton process is the most important for photons in the range 100 keV to 10 MeV

Pair production

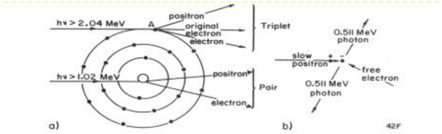


Figure 5-9. (a) The absorption of photons by pair and triplet production. (b) The annihilation of a positron and an electron to form two photons of radiation, each with an energy of 0.511 MeV.

- A photon is absorbed in the Coulomb field of a nucleus (electron in triplet production), producing an electron and a positron
- The threshold for the process is 1.022 MeV; it increases rapidly with energy above this threshold
- The coefficient per atom varies approximately as Z^2
- The coefficient per unit mass depends on $\sim Z^1$
- The energy transferred to K.E. is *hv* - 1.022 MeV.
- Two annihilation photons, each of 0.511 MeV, are produced per interaction and radiated from the absorber

Total attenuation coefficient

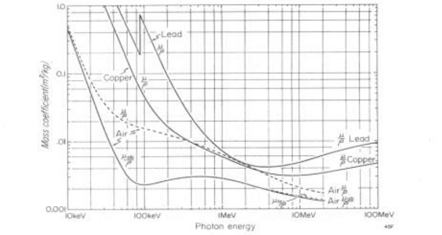
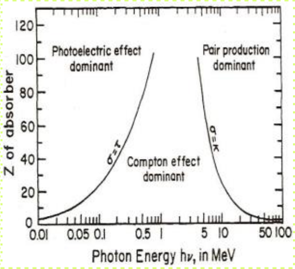


Figure 5-11. Graphs showing the total mass attenuation coefficient (μ/ρ) for lead, copper, and air. Also shown are the mass energy absorption coefficient (μ_{ab}/ρ) and the mass energy transfer coefficient (μ_{tr}/ρ) for air.

$$\underset{\text{total}}{\mu} = \underset{\text{photoelectric}}{\tau} + \underset{\text{coherent}}{\sigma_{\text{coh}}} + \underset{\text{Compton}}{\sigma_{\text{inc}}} + \underset{\text{pair}}{\kappa}$$

The relative importance of different interactions



For water (*Z*=7.4):

- Up to 50 keV: Photoelectric absorption is important
- 60 keV to 90 keV: Photoelectric and Compton are both important
- 200 keV to 2 MeV: Compton absorption alone is present
- 5 MeV to 10 MeV: Pair production begins to be important
- 50 MeV to 100 MeV: Pair production is most important

Example 6

- A 9.5 MeV photon passes by the nucleus of a lead atom. How many pair productions can take place?

A. 0
[✓ B. 1]
C. 3
D. 5
E. 9

During pair production, energy in excess of 1.022 MeV is released as kinetic energy of the two particles, electron and positron

Example 7

- The loss of contrast in a therapy verification image compared with a simulator radiographic image is mostly a result of _____.

A. an increased number of pair productions
B. an increased number of Compton interactions
C. an increased number of photoelectric interactions
[✓ D. a decreased number of photoelectric interactions]
E. a decreased number of Compton interactions

Summary

- Absorption of energy
- Quantitative characterization
  - Attenuation, HVL, TVL
  - Absorption coefficients
- Basic interactions of radiation with matter
- Photon interactions