Chapter 6
The Basic Interactions between Photons and Charged Particles with Matter

Radiation Dosimetry I

Text: H.E Johns and J.R. Cunningham, The physics of radiology, 4th ed.

http://www.utoledo.edu/med/depts/radther

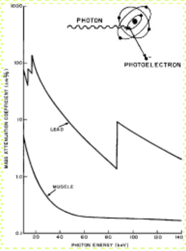
Outline

- Photon interactions
  - Photoelectric effect
  - Compton scattering
  - Pair productions
  - Coherent scattering
- Charged particle interactions
  - Stopping power and range
  - Bremsstrahlung interaction
  - Bragg peak

Photon interactions

- With energy deposition
  - Photoelectric effect
  - Compton scattering
    - No energy deposition in classical Thomson treatment
  - Pair production (above the threshold of 1.02 MeV)
  - Photo-nuclear interactions for higher energies (above 10 MeV)
- Without energy deposition
  - Coherent scattering

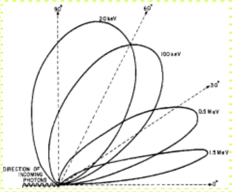
Photoelectric effect



Photoelectric mass attenuation coefficients of lead and soft tissue as a function of photon energy. K and L-absorption edges are shown for lead

- Collision between a photon and an atom results in ejection of a bound electron
- The photon disappears and is replaced by an electron ejected from the atom with kinetic energy $KE = h\nu - E_b$
- Highest probability if the photon energy is just above the binding energy of the electron (absorption edge)
- Additional energy may be deposited locally by Auger electrons and/or fluorescence photons

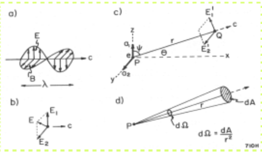
Photoelectric effect



- Electron tends to be ejected at 90° for low energy photons, and approaching 0° with increase in energy
- Kinetic energy given to electron $KE=h\nu\text{-}E_b$, independent of scattering angle
- Interaction probability ~$Z^3/(h\nu)^3$
- No universal analytical expression for cross-section

Thomson scattering (classical treatment)

- **Elastic** scattering of photon (EM wave) on free electron
- Electron is accelerated by EM wave and radiates a wave
- No energy is given to the electron; wavelength of the scattered photon does not change
- Classical scattering coefficient per electron per unit solid angle:



$$\frac{d\sigma_0}{d\Omega} = \frac{r_0^2}{2}\left(1 + \cos^2\theta\right)$$

max at $\theta = 0, 180^o$

½ max at $\theta = 90^o$

$r_0 = \frac{e^2}{m_0 c^2}$ - classical radius of electron

Thomson scattering (classical treatment)

- No energy dependence, total $\sigma_0 = 66.5 \times 10^{-30}\ \mathrm{m}^2 / \mathrm{electron}$
- Works for low photon energies, << m_0c^2
- Overestimates for photon energies > 0.01MeV (factor of 2 for 0.4MeV)

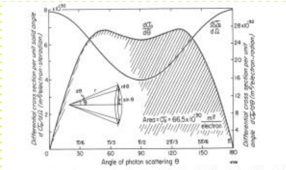


Figure 6-2. Differential cross section per unit solid angle, $d\sigma_0/d\Omega$ and the differential cross section per unit angle, $d\sigma_0/d\Theta$, as a function of the angle of photon scattering, according to Thomson's classical theory.

$$d\Omega = 2\pi \sin\theta d\theta$$

$$\frac{d\sigma_0}{d\theta} = \frac{r_0^2}{2}\left(1 + \cos^2\theta\right) \cdot 2\pi \sin\theta$$

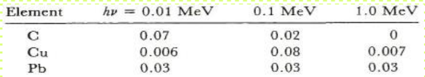
Coherent scattering

- Photon is scattered by combined action of the whole atom
- Photons do not lose energy, just get redirected through a small angle
- No charged particles receive energy, no excitation produced
- Scattering coefficient (F - atomic form factor, tabulated):

$$\frac{d\sigma_{coh}}{d\theta} = \frac{r_0^2}{2}\left(1 + \cos^2\theta\right) \cdot \left[F(x, Z)\right]^2 2\pi \sin\theta$$

$$x = (\sin\theta/2)/\lambda$$

Typical ratios of coherent to total attenuation coefficient



Element	$h\nu$ = 0.01 MeV	0.1 MeV	1.0 MeV
C	0.07	0.02	0
Cu	0.006	0.08	0.007
Pb	0.03	0.03	0.03

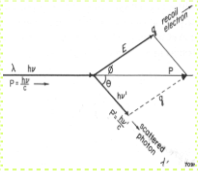
Example 1

- A 5-keV photon undergoing coherent or classical scatter would be most likely to lose ___% of its energy in the process.

[✓ A. 0]
B. 10
C. 50
D. 90
E. 100

Compton scattering kinematics

- Incoherent scattering – energy is transferred to electron (inelastic scattering)
- Energy and momentum are conserved



$$h\nu = h\nu' + E = h\nu' + m_0c^2\left\{\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right\}$$

$$p_{II} = \frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + \frac{m_0 v}{\sqrt{1 - v^2/c^2}} \times \cos\varphi$$

$$p_{\perp} = 0 = \frac{h\nu'}{c}\sin\theta - \frac{m_0 v}{\sqrt{1 - v^2/c^2}} \times \sin\varphi$$

Compton scattering kinematics

- Introducing parameter $\alpha = h\nu / m_0c^2$

$$E = h\nu \frac{\alpha(1 - \cos\theta)}{1 + \alpha(1 - \cos\theta)} \qquad \text{Energy of electron}$$

$$h\nu' = h\nu \frac{1}{1 + \alpha(1 - \cos\theta)} \qquad \text{Energy of scattered photon}$$

- The maximum energy that electron can acquire in a direct hit ($\theta = 180^o$):

$$E_{\max} = h\nu \frac{2\alpha}{1 + 2\alpha}; \quad \text{photon energy}: h\nu'_{\min} = h\nu \frac{1}{1 + 2\alpha} \approx 0.255\ MeV$$

For high-energy photon

- In a grazing hit ($\theta \sim 0^o$) E=0, and for the scattered photon

$$h\nu' = h\nu$$

Compton scattering probability

- Klein-Nishina coefficient: Compton scattering on free electron, but includes Dirac's quantum relativistic theory

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_0}{d\Omega}F_{KN} = \frac{r_0^2}{2}\left(1 + \cos^2\theta\right) \cdot F_{KN}, \quad \text{where}$$

$$F_{KN} = \left\{\frac{1}{1 + \alpha(1 - \cos\theta)}\right\}^2 \left\{1 + \frac{\alpha^2(1 - \cos\theta)^2}{\left[1 + \alpha(1 - \cos\theta)\right](1 + \cos^2\theta)}\right\}$$

- Factor F_{KN} describes the deviation from classical scattering
  - F_{KN} <1 for higher energies;
  - F_{KN} =1 for small α (low energy) or θ=0

Energy distribution of Compton electrons

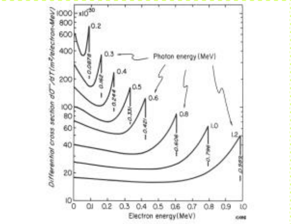


Figure 6-5. Differential cross section per unit kinetic energy interval, dσ/dE, as calculated by the Klein-Nishina coefficient for a free electron (N2). The areas under the curves give the total cross section for that photon energy. The curves give the distribution of electron energies produced by monoenergetic photons of energies 0.2 to 1.2 MeV. The numbers appearing along vertical lines at the ends of each curve are the maximum energy the recoil electron may acquire.

- Lower energy and high energy electrons are more likely to be produced
- Higher energy photons are more likely to transfer more energy to Compton electrons

Effect of binding energy

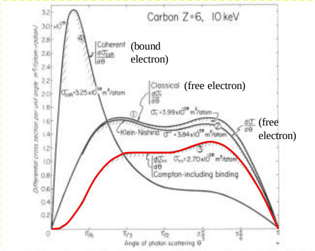


Figure 6-6. Scattering coefficients for carbon at 10 keV. Curve 1 is calculated by classical theory, as given by equation 6.5. Curve 2 is calculated by the Klein-Nishina equation, as given by equation 6-11. Curve 3 uses the Klein-Nishina derivation but takes into account the momentum and binding energy of the electrons within the atom, according to equation 6-17. Curve 4 is the coherent coefficient as given by equation 6-7, taking into account the constructive interference between the electrons of the atom.

- In real situation electrons are bound and in motion
- Additional factor S(x, Z) is introduced

$$\frac{d\sigma_{inc}}{d\theta} = \frac{d\sigma}{d\theta} \cdot S(x, Z)$$

x=sin(θ/2)/λ

- Makes the scattering coefficient Z-dependent

Example 2

- Compton scattered electrons can be emitted at:

A. Any angle.
[✓ B.] 0^o-90^o with respect to the direction of the incident photon
C. 30^o-120^o with respect to the direction of the incident photon.
D. 90^o-180^o with respect to the direction of the incident photon.

Example 3

- In Compton scattering original photon has energy 1.25 MeV, scattered at 30°. Calculate the energy of scattered photon.

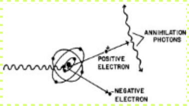
A. 1.2 MeV
B. 1.1 MeV
[✓ C.] 0.94 MeV
D. 0.83 MeV

$$\alpha = \frac{hv}{m_0 c^2} = \frac{1.25}{0.511} = 2.45$$

$$hv' = hv \frac{1}{1 + \alpha(1 - \cos\theta)} =$$

$$1.25 \frac{1}{1 + 2.45(1 - \cos 30)} = 0.941 \text{ MeV}$$

Pair production



- Photon is absorbed giving rise to electron and positron
- Occurs in Coulomb force field - usually near atomic nucleus; threshold energy 1.022 MeV
- Sometimes occurs in a field of atomic electron (triplet production); threshold energy 2.044 MeV
  - For 10-MeV photons in soft tissue, for example, about 10%
- The ratio of triplet to pair production increases with E of incident photons and decreases with Z

Pair production

- Energy in excess of 1.02 MeV is released as kinetic energy of the electron and positron

$$hv = 2m_0c^2 + T^- + T^+ = 1.022 \text{ MeV} + T^- + T^+$$

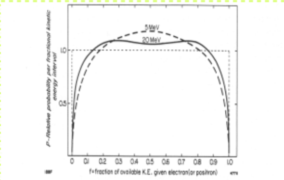


Figure 6-7. Graphs adapted from Evans (E1) to allow one to determine the number of electrons (or positrons) with a given energy produced in pair production.

- Differential cross section:

$$\frac{d\kappa}{d\Omega} = \frac{Z^2}{137} \frac{r_0^2}{2\pi} m_0^2 c^4 F_{pair}$$

- F_{pair} is a function of momentum, energy and angle of positron projection

Charged particle interactions

- All charged particles lose kinetic energy through Coulomb field interactions with charged particles (electrons or nuclei) of the medium
- In each interaction typically only a small amount of particle's kinetic energy is lost ("continuous slowing-down approximation" – CSDA)
- Typically undergo very large number of interactions, therefore can be roughly characterized by a common path length in a specific medium (***range***)

Heavy charged particle interactions

- Energy transferred to the electron of the medium

$$\Delta E = \frac{z^2 r_0^2 m_0 c^4}{b^2} \frac{M}{E}$$

- Parameters:
  - E – kinetic energy of the particle; ze- its charge; M – mass
  - b is the impact parameter
- Slower moving particle (lower KE) transfers more energy

Interactions of electrons

- In any given collision with another electron, one emerging with higher energy is assumed to be primary (max energy exchange is limited to half of its original energy)
- Due to small mass
  - Relativistic effects are important
  - Interactions with nucleus: rapid deceleration results in bremsstrahlung (breaking) radiation

Bremsstrahlung interactions

- Fast moving charged particle of mass *M*, and charge *ze*, passing close to a nucleus of mass $M_N >> M$ and charge Ze will experience electric force, corresponding to accelerations:

$$a = \frac{F}{M} = \frac{kzZe^2}{r^2 M}$$

- Accelerated charge radiates energy at a rate ~a^2
- The rate of energy loss is negligible for particles other than electrons (even protons) due to $1/M^2$

Stopping power

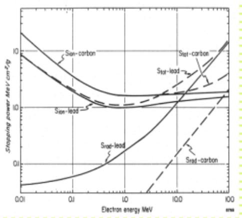
- Mass stopping power – energy loss per unit track length in producing ionization in the absorbing medium

$$S_{ion} = \frac{1}{\rho}\left(\frac{dE}{dx}\right) = 4\pi r_0^2 N_e \frac{z^2 m_0 c^2}{\beta^2}[...]$$

- Parameters: N_e – number of electrons per gram; […] is a slowly increasing function of the particle energy
- For electrons […] term is more complex
- Radiation stopping power – energy loss due to bremsstrahlung

for electrons $$S_{rad} = \frac{1}{\rho}\left(\frac{dE}{dx}\right) = 4 r_0^2 \frac{N_e Z E}{137}[...]$$

Stopping power



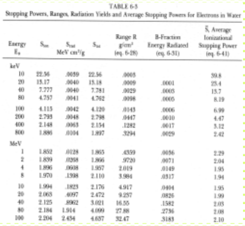
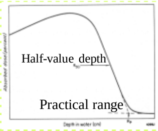


TABLE 6-3
Stopping Powers, Ranges, Radiation Yields and Average Stopping Powers for Electrons in Water

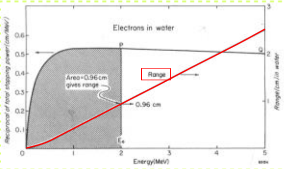
Energy E_0	S_{ion}	S_{rad} MeV cm²/g	S_{tot}	Range R g/cm² (eq. 6-28)	B-Fraction Energy Radiated (eq. 6-31)	$\bar{S}$, Average Ionizational Stopping Power (eq. 6-41)
keV						
10	22.56	.0059	22.56	.0005		39.8
20	13.17	.0040	13.18	.0009	.0001	23.4
40	7.777	.0040	7.781	.0029	.0003	13.7
80	4.757	.0041	4.762	.0098	.0005	8.19
100	4.115	.0042	4.120	.0143	.0006	6.99
200	2.793	.0048	2.798	.0447	.0010	4.47
400	2.148	.0063	2.154	.1282	.0017	3.12
800	1.886	.0104	1.897	.3294	.0029	2.42
MeV						
1	1.852	.0128	1.865	.4359	.0036	2.29
2	1.839	.0268	1.866	.9720	.0071	2.04
4	1.896	.0608	1.957	2.019	.0149	1.95
8	1.970	.1398	2.110	3.984	.0317	1.94
10	1.994	.1823	2.176	4.917	.0404	1.95
20	2.063	.4097	2.472	9.237	.0826	1.99
40	2.125	.8962	3.021	16.55	.1582	2.03
80	2.184	1.914	4.099	27.88	.2736	2.08
100	2.204	2.434	4.637	32.47	.3183	2.10

- Energy loss by radiation increases with atomic number of medium and electron energy

Range of electrons



- Charge particles are characterized by a range – a finite distance
- Can be calculated from stopping power in continuous slowing down approximation (CSDA)



$$R=\int_0^{E_0}\frac{dE}{S(E)}$$

Rule of thumb: for electron energies > 0.5 MeV, the range in unit density material (ρ in g/cm^3) is, in cm, about half the energy in MeV

Example 4

- A 6 MeV electron beam passes through 2 cm of tissue, overlying lung (density 0.25 g/cm3). The approximate range in the patient is ___cm.

A. 1
B. 3
[✓ C. 6]
D. 9
E. 12

R_{tissue} ~3 cm
$R_{tissue+lung}$ ~2 cm (ρ=1) + 1 cm (ρ=0.25)
=2 cm + 4 cm = 6 cm

Example 5

- A Cerrobend cutout for a 12 MeV electron beam should be approximately ___cm thick, to stop all the electrons and transmit only the bremsstrahlung. (Hint: Cerrobend requires 20% more thickness than lead.)

A. 7
B. 4
C. 1.5
[✓ D. 0.7]
E. 0.4

ρ_{lead}=11 g/cm^3
R_{lead} ~0.5x12/11=0.55 cm
$R_{Cerrobend}$~0.55x(1+0.2)= 0.66 cm

Mean stopping power

- If electron beam is not monoenergetic need to average over all electron energies

$$\bar{S}(E_i)=\frac{\int_0^{E_1}\frac{d\Phi(E)}{dE}S_{ion}(E)\,dE}{\int_0^{E_1}\frac{d\Phi(E)}{dE}\,dE}$$

- Similarly, if electrons are produced by polyenergetic photon beam with fluence Φ(E)

Restricted stopping power

- In slowing down an electron may suffer a large energy loss in producing delta-ray: introduce a cutoff energy Δ allowing to not account for escaping delta-rays
- Restricted stopping power (or LET – linear energy transfer) L_Δ - only energy exchanges less than Δ are accounted for

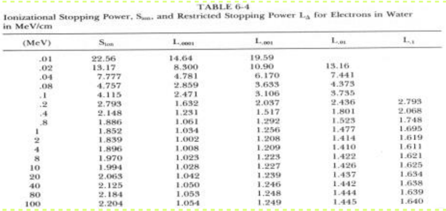


TABLE 6-4

Ionizational Stopping Power, S_{ion}, and Restricted Stopping Power L_Δ for Electrons in Water in MeV/cm

(MeV)	S_{ion}	$L_{.0001}$	$L_{.001}$	$L_{.01}$	$L_{.1}$
.01	22.56	14.64	19.59		
.02	13.17	8.300	10.90	13.16	
.04	7.777	4.781	6.170	7.441	
.08	4.757	2.859	3.633	4.373	
.1	4.115	2.471	3.106	3.735	
.2	2.793	1.632	2.037	2.436	2.793
.4	2.148	1.231	1.517	1.801	2.068
.8	1.886	1.061	1.292	1.523	1.748
1	1.852	1.034	1.256	1.477	1.695
2	1.839	1.002	1.208	1.414	1.619
4	1.896	1.008	1.209	1.410	1.611
8	1.970	1.023	1.223	1.422	1.621
10	1.994	1.028	1.227	1.426	1.625
20	2.063	1.042	1.239	1.437	1.634
40	2.125	1.050	1.246	1.442	1.638
80	2.184	1.053	1.248	1.444	1.639
100	2.204	1.054	1.249	1.445	1.640

Example 6

- A 6 MeV electron travels through 3 m of air. By how much is its average energy reduced?

[✓ A. 0.6 MeV]
B. 1 MeV
C. 2 MeV
D. 3 MeV

Need to remember ρ_{air}~0.001 g/cm^3
R_{air}~0.5xE/10^{-3}=3x10^3 cm=30 m
Assuming linear decrease in energy:
LET=6MeV/30 m=0.2 MeV/m
0.2MeV/m x 3m=0.6 MeV

Example 7

- A 6 MeV alpha particle produces 20,000 ion pairs per cm. What is the range of the alpha particle?

A. 0.01 mm
B. 0.1 mm
C. 1.0 mm
D. 10.0 mm
[✓ E.] 100.0 mm

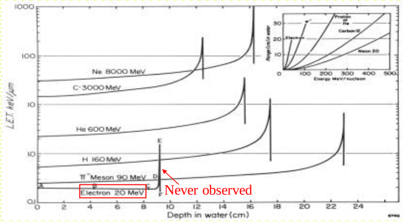
In air the energy to produce one ion pair is W~34 eV/pair

$$LET = 2 \times 10^{4} \frac{pair}{cm} 34 \frac{eV}{pair} \approx 6 \times 10^{5} \frac{eV}{cm}$$

Since heavy particles move along a straight-line path

$$R = \frac{E}{LET} = \frac{6\,MeV}{0.6 \frac{MeV}{cm}} = 10 \text{ cm} = 100 \text{ mm}$$

Bragg peak



Never observed

- Slower moving charged particle (lower KE) transfers more energy, resulting in Bragg peak at the end of its track
- **Never** observed for electrons: due to their low mass, electrons constantly change direction as they slow down

Summary

- Photon interactions
  - Photoelectric effect
  - Compton scattering
  - Pair productions
  - Coherent scattering
- Charged particle interactions
  - Stopping power and range
  - Bremsstrahlung interaction
  - Bragg peak