Chapter 6
The Basic Interactions between Photons and Charged Particles with Matter
Radiation Dosimetry I

http://www.utoledo.edu/med/depts/radther

Photon interactions

• With energy deposition
  – Photoelectric effect
  – Compton scattering
  – Pair production (above the threshold of 1.02 MeV)
  – Photo-nuclear interactions for higher energies (above 10 MeV)

• Without energy deposition
  – Coherent scattering

Photoelectric effect

• Collision between a photon and an atom results in ejection of a bound electron
• The photon disappears and is replaced by an electron ejected from the atom with kinetic energy \( KE = h\nu - E_b \)
• Highest probability if the photon energy is just above the binding energy of the electron (absorption edge)
• Additional energy may be deposited locally by Auger electrons and/or fluorescence photons

• Interaction probability \( \sim Z^3/\left(h\nu\right)^3 \)
• No universal analytical expression for cross-section

Thomson scattering (classical treatment)

• Elastic scattering of photon on free electron
• Electron is accelerated by EM wave and radiates a wave
• No energy is given to the electron; wavelength of the scattered photon does not change
• Classical scattering coefficient per electron per unit solid angle:
  \[
  \frac{d\sigma_0}{d\Omega} = \frac{\sigma_0}{2} \left(1 + \cos^2 \theta\right)
  \]
  \( d\Omega = 2\pi \sin \theta d\theta \)
  \( \frac{d\sigma}{d\Omega} = 2\pi \sin \theta \)

  \( \sigma_0 = e^2/m_e^2 \) - classical radius of electron

Figures:
- Photon interactions
- Photoelectric effect
- Thomson scattering (classical treatment)
Coherent scattering

- Photon is scattered by combined action of the whole atom
- Photons do not lose energy, just get redirected through a small angle
- No charged particles receive energy, no excitation produced
- Scattering coefficient ($F$ - atomic form factor, tabulated):
  \[
  \frac{d\sigma_{coh}}{d\theta} = \frac{\sigma^2}{2} \left[ 1 + \cos^2 \theta \right] F(x, Z) \frac{2}{\lambda} \sin \theta
  \]
  \[
  x = \frac{\sin \theta/2}{\lambda}
  \]

Example 1

- A 5-keV photon undergoing coherent or classical scatter would be most likely to lose ___ % of its energy in the process.
  
  A. 0  
  B. 10  
  C. 50  
  D. 90  
  E. 100

Compton scattering kinematics

- Incoherent scattering – energy is transferred to electron (inelastic scattering)
- Energy and momentum are conserved

\[
\begin{align*}
E_{\text{photon}} &= h\nu + E \\
E_{\text{electron}} &= h\nu + m_e c^2 \left[ -1 - \sqrt{1 - \left( \frac{m_e c^2}{E_{\text{photon}}} \right)^2} \right] \\
p_x &= \frac{h\nu}{c} \cos \theta + \frac{m_e c^2}{\sqrt{1 - \left( \frac{m_e c^2}{E_{\text{photon}}} \right)^2}} \cos \phi \\
p_y &= 0 = \frac{h\nu}{c} \sin \theta - \frac{m_e c^2}{\sqrt{1 - \left( \frac{m_e c^2}{E_{\text{photon}}} \right)^2}} \sin \phi
\end{align*}
\]

Compton scattering probability

- Klein-Nishina coefficient: Compton scattering on free electron, but includes Dirac’s quantum relativistic theory
  \[
  \frac{d\sigma}{d\Omega} = \frac{d\sigma_0}{d\Omega} \left[ 1 + \alpha(1 - \cos \theta) \right] F_{KN}, \quad \text{where}
  \]
  \[
  F_{KN} = \left[ \frac{1}{1 + \alpha(1 - \cos \theta)} \right] \left[ 1 + \alpha^2 (1 - \cos \theta)^2 \right] \left[ \left[ 1 + \alpha(1 - \cos \theta) \right] \left[ 1 + \cos^2 \theta \right] \right]
  \]

Energy distribution of Compton electrons

- Lower energy and high energy electrons are more likely to be produced
- Higher energy photons are more likely to transfer more energy to Compton electrons
Effect of binding energy

- In real situation electrons are bound and in motion
- Additional factor $S(x, Z)$ is introduced
- Makes the scattering coefficient $Z$-dependent

Example 2

- Compton scattered electrons can be emitted at:
  A. Any angle.
  B. $0^\circ$-$90^\circ$ with respect to the direction of the incident photon
  C. $30^\circ$-$120^\circ$ with respect to the direction of the incident photon.
  D. $90^\circ$-$180^\circ$ with respect to the direction of the incident photon.

Example 3

- In Compton scattering original photon has energy 1.25 MeV, scattered at $30^\circ$.
  Calculate the energy of scattered photon.

A. 1.2 MeV  
B. 1.1 MeV  
C. 0.94 MeV  
D. 0.83 MeV

\[ h\nu = \frac{h\nu}{1 - \cos \theta} = \frac{1.25}{1 + 0.511(1 - \cos 30^\circ)} = 0.941 \text{ MeV} \]

Pair production

- Photon is absorbed giving rise to electron and positron
- Occurs in Coulomb force field - usually near atomic nucleus; threshold energy 1.022 MeV
- Sometimes occurs in a field of atomic electron (triplet production); threshold energy 2.044 MeV
  - For 10 MeV photons in soft tissue, for example, about 10%
  - The ratio of triplet to pair production increases with $E$ of incident photons and decreases with $Z$

Pair production

- Probability of pair production increases with energy (this trend is opposite those of both Compton and photoelectric interactions)
- High Z dependence: increases as $Z^2$

Pair production

- Energy in excess of 1.02 MeV is released as kinetic energy of the electron and positron:
  \[ h\nu = 2m_e c^2 + T^- + T^+ = 1.022 \text{ MeV} + T^- + T^+ \]

- Differential cross section:
  \[ \frac{d\sigma}{d\Omega} = \frac{Z^2 \epsilon^2}{137 \cdot 2\pi} m_0^2 c^4 F_{pair} \]
  \[ F_{pair} \] is a function of momentum, energy and angle of positron projection.
Charged particle interactions

• All charged particles lose kinetic energy through Coulomb field interactions with charged particles (electrons or nuclei) of the medium.
• In each interaction typically only a small amount of particle’s kinetic energy is lost (“continuous slowing-down approximation” – CSDA).
• Typically undergo very large number of interactions, therefore can be roughly characterized by a common path length in a specific medium (range).

Heavy charged particle interactions

• Energy transferred to the electron of the medium
  \[ \Delta E = \frac{z^2 e^4 m_e c^2 M}{b^2 E} \]
• Parameters:
  – E – kinetic energy of the particle; ze – its charge; M – mass
  – b is the impact parameter
• Slower moving particle (lower KE) transfers more energy.

Stopping power

• Mass stopping power – energy loss per unit track length in producing ionization in the absorbing medium
  \[ S_m = \frac{1}{\rho} \left( \frac{dE}{dx} \right) = 4m_e^2 N_e z^2 m_e c^2 \frac{1}{b^2} [\ldots] \]
• Parameters: \( N_e \) – number of electrons per gram; \([\ldots]\) is a slowly increasing function of the particle energy
• For electrons \([\ldots]\) term is more complex
• This is mass collision stopping power.

Stopping power

• When a charged particle moves close to a nucleus – it experiences electrostatic force
• Radiation stopping power – energy loss due to bremsstrahlung (breaking radiation)
• Important for electrons (low mass)
  \[ S_{\text{rad}} = \frac{1}{\rho} \left( \frac{dE}{dx} \right) = 4m_e^2 N_e Z E \frac{137}{157} [\ldots] \]

Stopping power

• Energy loss by radiation increases with atomic number and energy

Range of electrons

• Charge particles are characterized by a range – a finite distance
• Can be calculated from stopping power in continuous slowing down approximation (CSDA)
  \[ R = \frac{\int_0^E \frac{dE}{S(E)}}{S(E)} \]
  Rule of thumb: for electron energies > 0.5 MeV, the range in unit density material (\( \rho \) in g/cm\(^2\)) is, in cm, about half the energy in MeV.
**Example 4**

- A 6 MeV electron beam passes through 2 cm of tissue, overlying lung (density 0.25 g/cm³). The approximate range in the patient is ___ cm.

  A. 1  
  B. 3  
 **C. 6**  
  D. 9  
  E. 12

\[ R_{\text{tissue}} = 3 \text{ cm} \]

\[ R_{\text{tissue+lung}} = 2 \text{ cm} (\rho=1) + 1 \text{ cm} (\rho=0.25) = 3 \text{ cm} + 4 \text{ cm} = 6 \text{ cm} \]

**Example 5**

- A Cerrobend cutout for a 12 MeV electron beam should be approximately ___ cm thick, to stop all the electrons and transmit only the bremsstrahlung. (Hint: Cerrobend requires 20% more thickness than lead.)

  A. 7  
  B. 4  
  C. 1.5  
 **D. 0.7**  
  E. 0.4

\[ \rho_{\text{lead}} = 11 \text{ g/cm}^3 \]

\[ R_{\text{lead}} = 0.5 \times 12/11 = 0.55 \text{ cm} \]

\[ R_{\text{Cerrobend}} = 0.55 \times (1 + 0.2) = 0.66 \text{ cm} \]

**Mean stopping power**

- If electron beam is not monoenergetic need to average over all electron energies

\[ S(E) = \frac{\int d\Phi(E) S_{\text{ion}}(E) dE}{\int d\Phi(E)} \]

- Similarly, if electrons are produced by polyenergetic photon beam with fluence \( \Phi(E) \)

**Restricted stopping power**

- In slowing down an electron may suffer a large energy loss in producing delta-ray: introduce a cutoff energy \( \Delta \) allowing to not account for escaping delta-rays

\[ \text{Restricted stopping power} (\text{or LET – linear energy transfer}) \]

\[ L_{\Delta} = \text{only energy exchanges less than } \Delta \text{ are accounted for} \]

<table>
<thead>
<tr>
<th>( E ) (MeV)</th>
<th>0.01</th>
<th>0.02</th>
<th>0.03</th>
<th>0.04</th>
<th>0.05</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
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<th>2.0</th>
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<td>190.0</td>
<td>230.0</td>
<td>270.0</td>
<td>85.0</td>
<td>95.0</td>
<td>105.0</td>
<td>115.0</td>
<td>125.0</td>
<td>250.0</td>
<td>340.0</td>
<td>430.0</td>
<td>520.0</td>
<td>600.0</td>
<td>800.0</td>
<td></td>
</tr>
</tbody>
</table>

**Example 6**

- A 6 MeV electron travels through 3 m of air. By how much is its average energy reduced?

  A. 0.6 MeV  
  B. 1.0 MeV  
  C. 2.0 MeV  
  D. 3.0 MeV  
 **E. 10.0 MeV**

\[ \rho_{\text{air}} = 0.001 \text{ g/cm}^3 \]

\[ R_{\text{air}} = 0.5 \times 6/10^{-3} \times 10 = 30 \text{ m} \]

Assuming linear decrease in energy:

\[ \text{LET} = 6 \text{ MeV} / 3 \text{ m} = 2 \text{ MeV/m} \]

\[ 0.2 \text{ MeV/m} \times 3 \text{ m} = 0.6 \text{ MeV} \]

**Example 7**

- A 6 MeV alpha particle produces 20,000 ion pairs per cm. What is the range of the alpha particle?

  A. 0.01 mm  
  B. 0.1 mm  
  C. 1.0 mm  
  D. 10.0 mm  
  **E. 100.0 mm**

In air the energy to produce one ion pair is \( W = 34 \text{ eV/pair} \)

\[ \text{LET} = 2 \times 10^4 \text{ (MeV/cm)} \times 34 \text{ eV/pair} = 6 \times 10^6 \text{ eV/cm} \]

Since heavy particles move along a straight-line path

\[ R = \frac{E}{\text{LET}} = \frac{6\text{ MeV}}{6 \times 10^6 \text{ eV/cm}} = 10 \text{ cm} = 100 \text{ mm} \]
Bragg peak

- Slower moving charged particle (lower KE) transfers more energy, resulting in Bragg peak at the end of its track
- Never observed for electrons due to their low mass; electrons constantly change direction as they slow down