Ionizing Radiation

Chapter 1

F.A. Attix, Introduction to Radiological Physics and Radiation Dosimetry

Outline

- Radiological physics and radiation dosimetry
- Types and sources of ionizing radiation
- Description of ionizing radiation fields
  - Random nature of radiation
  - Non-stochastic quantities
- Summary

Introduction

- Radiological physics studies ionizing radiation and its interaction with matter
- Began with discovery of x-rays, radioactivity and radium in 1890s
- Special interest is in the energy absorbed in matter
- Radiation dosimetry deals with quantitative determination of the energy absorbed in matter

Ionizing radiation

- By general definition ionizing radiation is characterized by its ability to excite and ionize atoms of matter
- Lowest atomic ionization energy is ~ eV, with very little penetration
- Energies relevant to radiological physics and radiation therapy are in keV – MeV range

Types and sources of ionizing radiation

- γ-rays: electromagnetic radiation (photons) emitted from a nucleus or in annihilation reaction
  - Practical energy range from 2.6 keV (K_α from electron capture in ${}^{37}_{18}Ar$) to 6.1 and 7.1 MeV (γ-rays from ${}^{16}_{7}N$)
- x-rays: electromagnetic radiation (photons) emitted by charged particles (characteristic or bremsstrahlung processes). Energies:
  - 0.1-20 kV "soft" x-rays
  - 20-120 kV diagnostic range
  - 120-300 kV orthovoltage x-rays
  - 300 kV-1 MV intermediate energy x-rays
  - 1 MV and up megavoltage x-rays

Types and sources of ionizing radiation

- Fast electrons (positrons) emitted from nuclei (β-rays) or in charged-particle collisions (δ-rays). Other sources: Van de Graaf generators, linacs, betatrons, and microtrons
- Heavy charged particles emitted by some radioactive nuclei (α-particles), cyclotrons, heavy particle linacs (protons, deuterons, ions of heavier elements, etc.)
- Neutrons produced by nuclear reactions (cannot be accelerated electrostatically)

Types of interaction

- ICRU (The *International Commission on Radiation Units* and Measurements; established in 1925) terminology
- *Directly ionizing radiation*: by charged particles, delivering their energy to the matter directly through multiple Coulomb interactions along the track
- *Indirectly ionizing radiation*: by photons (x-rays or γ-rays) and neutrons, which transfer their energy to charged particles (two-step process)

Description of ionizing radiation fields

- To describe radiation field at a point P need to define non-zero volume around it
- Can use *stochastic* or *non-stochastic* physical quantities

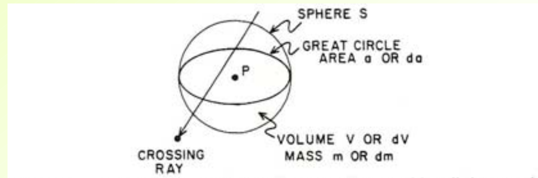


FIGURE 1.1. Characterizing the radiation field at a point *P* in terms of the radiation traversing the spherical surface *S*.

Stochastic quantities

- Values occur randomly, cannot be predicted
- Radiation is random in nature, associated physical quantities are described by probability distributions
- Defined for finite domains (non-zero volumes)
- The expectation value of a stochastic quantity (e.g. number of x-rays detected per measurement) is the mean of its measured value for infinite number of measurements

$$\bar{N} \to N_e \text{ for } n \to \infty$$

Stochastic quantities

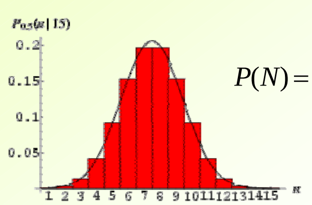
- For a "constant" radiation field a number of x-rays observed at point P per unit area and time interval follows Poisson distribution
- For large number of events it may be approximated by normal (Gaussian) distribution, characterized by standard deviation σ (or corresponding percentage standard deviation *S*) for a single measurement

$$\sigma = \sqrt{N_e} \cong \sqrt{\bar{N}}$$

$$S = \frac{100\sigma}{N_e} = \frac{100}{\sqrt{N_e}} \cong \frac{100}{\sqrt{\bar{N}}}$$

Stochastic quantities

- Normal (Gaussian) distribution is described by probability density function $P(x)$
- Mean $\bar{N}$ determines position of the maximum, standard deviation σ defines the width of the distribution



$$P(N) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(N-\bar{N}\right)^2/\left(2\sigma^2\right)}$$

Stochastic quantities

- For a given number of measurements *n* standard deviation is defined as

$$\sigma' = \frac{\sigma}{\sqrt{n}} = \sqrt{\frac{N_e}{n}} \cong \sqrt{\frac{\bar{N}}{n}}$$

$$S' = \frac{100\sigma'}{N_e} = \frac{100}{\sqrt{nN_e}} \cong \frac{100}{\sqrt{n\bar{N}}}$$

- $\bar{N}$ will have a 68.3% chance of lying within interval $\pm\sigma'$ of N_e, 95.5% to be within $\pm 2\sigma'$, and 99.7% to be within interval $\pm 3\sigma'$. No experiment-related fluctuations

Stochastic quantities

- In practice one always uses a detector. An estimated precision (proximity to N_e) of any single random measurement N_i

$$\sigma \cong \left[\frac{1}{n-1} \sum_{i=1}^{n} \left(N_i - \overline{N} \right)^2 \right]^{1/2}$$

$$\overline{N} = \left(\sum N_i \right) / n$$

- Determined from the data of n such measurements

Stochastic quantities

- An estimate of the precision (proximity to N_e) of the mean value $\overline{N}$ measured with a detector n times

$$\sigma' = \sigma / \sqrt{n}$$

$$\sigma' \cong \left[\frac{1}{n(n-1)} \sum_{i=1}^{n} \left(N_i - \overline{N} \right)^2 \right]^{1/2}$$

- N_e is as correct as your experimental setup

Stochastic quantities: Example

- A γ-ray detector having 100% counting efficiency is positioned in a constant field, making 10 measurements of equal duration, Δt=100s (exactly). The average number of rays detected ("counts") per measurement is $1.00\text{x}10^5$. What is the mean value of the count rate C, including a statement of its precision (i.e., standard deviation)?

$$\overline{C} = \frac{\overline{N}}{\Delta t} = \frac{1.00 \times 10^5}{100} = 1.00 \times 10^3 \text{ c/s}$$

$$\sigma'_C \cong \sqrt{\frac{\overline{C}}{n}} = \sqrt{\frac{1.00 \times 10^3}{10}} = 1 \text{ c/s}$$

$$\overline{C} = 1.00 \times 10^3 \pm 1 \text{ c/s}$$

- Here the standard deviation is due entirely to the stochastic nature of the field, since detector is 100% efficient

Non-stochastic quantities

- For given conditions the value of non-stochastic quantity can, in principle, be calculated
- In general, it is a "point function" defined for infinitesimal volumes
  - It is a continuous and differentiable function of space and time; with defined spatial gradient and time rate of change
- Its value is equal to, or based upon, the *expectation value* of a related stochastic quantity, if one exists
  - In general does not need to be related to stochastic quantities, they are related in description of ionizing radiation

Description of radiation fields by non-stochastic quantities

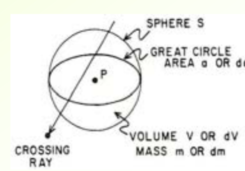
- Fluence
- Flux Density (or Fluence Rate)
- Energy Fluence
- Energy Flux Density (or Energy Fluence Rate)

Non-stochastic quantities: Fluence

- A number of rays crossing an infinitesimal area surrounding point P, define fluence as

$$\Phi = \frac{dN_e}{da}$$

- Units of m^{-2} or cm^{-2}



Non-stochastic quantities: Flux density (Fluence rate)

- An increment in fluence over an infinitesimally small time interval

$$\varphi = \frac{d\Phi}{dt} = \frac{d}{dt}\left(\frac{dN_e}{da}\right)$$

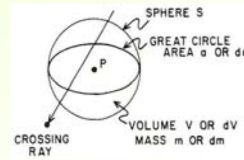
- Units of m^{-2} s^{-1} or cm^{-2} s^{-1}
- Fluence can be found through integration:

$$\Phi(t_0, t_1) = \int_{t_0}^{t_1} \varphi(t)dt$$

Non-stochastic quantities: Energy fluence

- For an expectation value R of the energy carried by all the N_e rays crossing an infinitesimal area surrounding point P, define energy fluence as

$$\Psi = \frac{dR}{da}$$



- Units of J m^{-2} or erg cm^{-2}
- If all rays have energy E

$$R = EN_e$$

$$\Psi = E\Phi$$

Non-stochastic quantities: Energy flux density (Energy fluence rate)

- An increment in energy fluence over an infinitesimally small time interval

$$\psi = \frac{d\Psi}{dt} = \frac{d}{dt}\left(\frac{dR}{da}\right)$$

- Units of J m^{-2} s^{-1} or erg cm^{-2} s^{-1}
- Energy fluence can be found by integration:

$$\Psi(t_0, t_1) = \int_{t_0}^{t_1} \psi(t)dt$$

Differential distributions

- More complete description of radiation field is often needed
- Generally, flux density, fluence, energy flux density, or energy fluence depend on all variables: θ, β, or E
- Simpler, more useful differential distributions are those which are functions of only one of the variables

Differential distributions by energy and angle of incidence

- Differential flux density as a function of energy and angles of incidence: distribution

$$\varphi'(\theta, \beta, E)$$

- Typical units are m^{-2} s^{-1}sr^{-1}keV^{-1}
- Integration over all variables gives the flux density:

$$\varphi = \int_{\theta=0}^{\pi} \int_{\beta=0}^{2\pi} \int_{E=0}^{E_{max}} \varphi'(\theta, \beta, E) \sin\theta d\theta d\beta dE$$

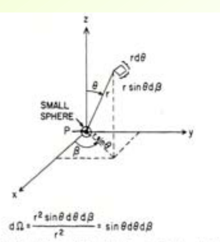


FIGURE 1.2. Polar coordinates. The element of solid angle is dΩ

(x,y,z) –> (r,θ,β)

Differential distributions: Energy spectra

- If a quantity is a function of energy only, such distribution is called the *energy spectrum* (e.g.$\varphi'(E)$)
- Typical units are m^{-2} s^{-1}keV^{-1} or cm^{-2} s^{-1}keV^{-1}
- Integration over angular variables gives flux density spectrum

$$\varphi'(E) = \int_{\theta=0}^{\pi} \int_{\beta=0}^{2\pi} \varphi'(\theta, \beta, E) \sin\theta d\theta d\beta$$

- Similarly, may define energy flux density $\psi'(E)$

Differential distributions: Energy spectrum example

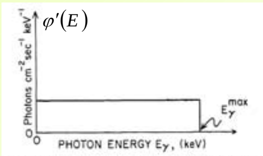


FIGURE 1.3a. A flat spectrum of photon flux density $\varphi'(E)$.

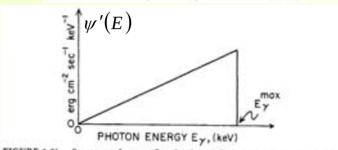


FIGURE 1.3b. Spectrum of energy flux density $\psi'(E)$ corresponding to Fig. 1.3a.

- A "flat" distribution of photon flux density
- Energy flux density spectrum is found by

$$\psi'(E) = E\varphi'(E)$$

Typically units for E are joule or erg, so that [ψ'] = $\mathrm{Jm^{-2}s^{-1}keV^{-1}}$

Example: Problem 1.8

An x-ray field at a point P contains $7.5\mathrm{x}10^8$ photons/($\mathrm{m^2}$-sec-keV), uniformly distributed from 10 to 100 keV.

a) What is the photon flux density at P?
b) What would be the photon fluence in one hour?
c) What is the corresponding energy fluence, in $\mathrm{J/m^2}$ and $\mathrm{erg/cm^2}$?

Example: Problem 1.8

Energy spectrum of a flux density $\varphi'(E)$ =$7.5\mathrm{x}10^8$ photons/$\mathrm{m^2}$-sec-keV

a) Photon flux density

$$\varphi = \varphi'(E)\cdot(E_{\max} - E_{\min}) =$$
$$7.5\times 10^8\cdot 90 = 6.75\times 10^{10}\ \text{photons/m}^2\text{s}$$

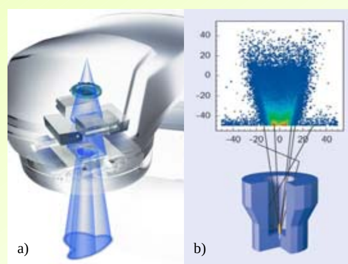
b) The photon fluence in one hour

$$\Phi(t = 1\ \text{hour}) = \varphi\cdot\Delta t =$$
$$6.75\times 10^{10}\cdot 3600 = 2.43\times 10^{14}\ \text{photons/m}^2$$

c) The corresponding energy fluence, in $\mathrm{J/m^2}$ and $\mathrm{erg/cm^2}$

$$\Psi = \Delta t\cdot\int_{E=10}^{E=100}\varphi'(E)\cdot E\,dE = \Delta t\cdot\varphi'\cdot\frac{E^2}{2}\Big|_{10}^{100} =$$
$$3600\cdot 7.5\times 10^8\cdot\frac{1}{2}\left(100^2 - 10^2\right) = 1.336\times 10^{16}\ \text{keV/m}^2 =$$
$$1.336\times 10^{16}\cdot 1.602\times 10^{-16} = 2.14\ \text{J/m}^2 = 2.14\times 10^3\ \text{erg/cm}^2$$

Differential distributions: Angular distributions



Azimuthal symmetry: a) accelerator beam after primary collimator; b) brachytherapy surface applicator

- Full differential distribution integrated over energy leaves only angular dependence
- Often the field is symmetrical with respect to a certain direction, then only dependence on polar angle θ or azimuthal angle β

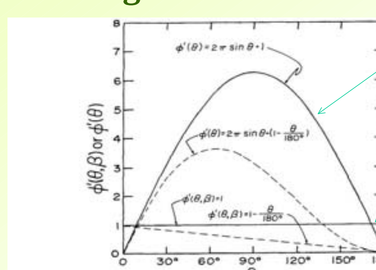
Differential distributions: Angular distributions

- If the field is symmetrical with respect to the vertical (z) axis, it is independent of azimuthal angle β
- This results in distribution per unit polar angle

$$\varphi'(\theta) = \int_{\beta=0}^{2\pi}\int_{E=0}^{E_{\max}}\varphi'(\theta,\beta,E)\sin\theta\,d\beta\,dE$$

- Alternatively, can obtain distribution per unit solid angle for particles of all energies

Differential distributions: Angular distributions



Distribution per unit polar angle, azimuthal symmetry

Field isotropic per unit solid angle, $d\Omega = \sin\theta d\theta d\beta$ spherical symmetry

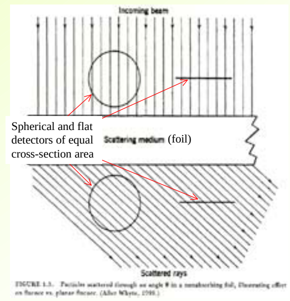
FIGURE 1.4. Isotropic radiation field expressed in terms of its flux-density distribution per unit solid angle, $\varphi'(\theta, \beta)$ = constant = 1 $\mathrm{m^{-2}\,s^{-1}\,sr^{-1}}$ (lower solid curve). The same field is also shown in terms of its distribution per unit polar angle, $\varphi'(\theta)$, in $\mathrm{m^{-2}\,s^{-1}\,radian^{-1}}$ (upper solid curve). These two curves are related by the factor $2\pi\sin\theta$, which is also true if $\varphi'(\theta, \beta)$ is a function of θ only [e.g., see dashed curves for $\varphi'(\theta, \beta) = 1 - (\theta/180°)$].

Planar fluence

- Planar fluence: number of particles crossing a fixed plane in either direction (i.e., summed by scalar addition) per unit area of the plane
- Vector-sum quantity gives *net flow* – number of particles crossing a fixed plane in one direction minus those crossing in the opposite direction (used in MC calculations)
- Fluence vs. planar fluence – definition matters

Planar fluence

- Assume that the energy imparted is ~ proportional to the total track length of the rays crossing the detector
- For *penetrating* radiation both detectors will read more below the foil, E~|1/cosθ| times the number striking it above foil
- For *non-penetrating* rays the flat detector responds the same above and below the foil (~to only the number of x-rays striking it; track length is irrelevant)
  - The energy deposited in this case is related to the *planar* fluence



Summary

- Types and sources of ionizing radiation
  - γ-rays, x-rays, fast electrons, heavy charged particles, neutrons
- Description of ionizing radiation fields
  - Standard deviation due random nature of radiation; accuracy of a measurement
  - Non-stochastic quantities: fluence, flux density, energy fluence, energy flux density, differential distributions