Quantities for Describing the Interaction of Ionizing Radiation with Matter

Chapter 2

F.A. Attix, Introduction to Radiological
Physics and Radiation Dosimetry

Outline

- Kerma, components of kerma
- Absorbed dose
- Exposure
- Quantities for use in radiation protection
- Summary

Introduction

- Need to describe interactions of ionizing radiation with matter
- Special interest is in the energy absorbed in matter, absorbed dose – delivered by directly ionizing radiation
- Two-step process for indirectly ionizing radiation involves kerma and absorbed dose

Definitions

- Most of the definitions are by ICRU
- Energy transferred by indirectly ionizing radiation leads to the definition of kerma
- Energy imparted by ionizing radiation leads to the definition of absorbed dose
- Energy carried by neutrinos is ignored
  - Very small mass, no electric charge => negligibly small cross section for interactions with matter

Energy transferred

- ε_{tr} - *energy transferred* in a volume V to charged particles by indirectly ionizing radiation (photons and neutrons)
- *Radiant energy R* – the energy of particles emitted, transferred, or received, excluding rest mass energy
- Q - energy delivered from *rest mass* in V (positive if m →E, negative for E →m)

Energy transferred

- The energy transferred in a volume V

$$\varepsilon_{tr} = \left(R_{in}\right)_u - \left(R_{out}\right)_u^{nonr} + \sum Q$$

uncharged

- $\left(R_{out}\right)_u^{nonr}$ does not include radiative losses of kinetic energy by charged particles (bremsstrahlung or in-flight annihilation)
- Energy transferred is only the kinetic energy received by charged particles

Kerma

- Kerma K is the energy transferred to charged particles per unit mass

$$K = \frac{d\left(\varepsilon_{tr}\right)_e}{dm} \equiv \frac{d\varepsilon_{tr}}{dm}$$

- Includes radiative losses by charged particles (bremsstrahlung or in-flight annihilation of positron)
- Excludes energy passed from one charged particle to another
- Units: 1 Gy = 1 J/kg = 10^2 rad = 10^4 erg/g

Relation of Kerma to Energy fluence for photons

- For mono-energetic photon of energy E and medium of atomic number Z, relation is through the mass energy-transfer coefficient:

$$K = \Psi \cdot \left(\frac{\mu_{tr}}{\rho}\right)_{E,Z}$$

- For a spectrum of energy fluence $\Psi'(E)$

$$K = \int_{E==0}^{E=E\max} \Psi'(E) \cdot \left(\frac{\mu_{tr}}{\rho}\right)_{E,Z} dE$$

Energy-transfer coefficient

- Linear energy-transfer coefficient μ_{tr} , units of m^{-1} or cm^{-1}
- Mass energy-transfer coefficient $\left(\frac{\mu_{tr}}{\rho}\right)_{E,Z}$, units of m^2/kg or cm^2/g
- Set of numerical values, tabulated for a range of photon energies, Appendix D.3

Relation of Kerma to Fluence for neutrons

- Neutron field is usually described in terms of fluence rather than energy fluence
- Kerma factor is tabulated instead of kerma (units are rad cm^2/neutron, Appendix F)

$$\left(F_n\right)_{E,Z} = \left(\frac{\mu_{tr}}{\rho}\right)_{E,Z} \cdot E$$

- For mono-energetic neutrons

$$K = \Phi \cdot \left(F_n\right)_{E,Z}$$

Components of Kerma

- Energy received by charged particles may be spent in two ways
  - *Collision* interactions – local dissipation of energy, ionization and excitation along electron track
  - *Radiative* interactions, such as bremsstrahlung or positron annihilation, carry energy away from the track
- Kerma may be subdivided in two components, collision and radiative:

$$K = K_c + K_r$$

- When kerma is due to neutrons, resulting charged particles are much heavier, K=K_c

Collision Kerma

- Subtracting radiant energy emitted by charged particles R_u^r from energy transferred results in *net* energy transferred locally

$$\varepsilon_{tr}^{net} = \varepsilon_{tr} - R_u^r =$$

$$\left(R_{in}\right)_u - \left(R_{out}\right)_u^{nonr} - R_u^r + \sum Q$$

- Now collision kerma can be defined

$$K_c = \frac{d\varepsilon_{tr}^{net}}{dm}$$

Mass energy-absorption coefficient

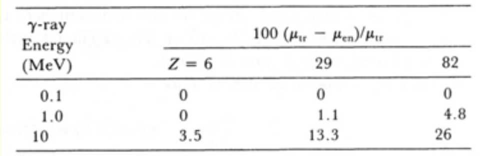
- Since collision kerma represents energy deposited (absorbed) locally, introduce mass energy-absorption coefficient. For mono-energetic photon beam

$$K_c = \Psi \cdot \left(\frac{\mu_{en}}{\rho}\right)_{E,Z}$$

- Depends on materials present along particle track before reaching point P

Mass energy-absorption coefficient

- For low Z materials and low energy radiative losses are small, therefore values of μ_{tr} and μ_{en} are close



γ-ray Energy (MeV)	100 $(\mu_{tr} - \mu_{en})/\mu_{tr}$		
	$Z = 6$	29	82
0.1	0	0	0
1.0	0	1.1	4.8
10	3.5	13.3	26

Kerma rate

- Kerma rate at point P and time *t*

$$\dot{K} = \frac{dK}{dt} = \frac{d}{dt}\left(\frac{d\varepsilon_{tr}}{dm}\right)$$

- Units of J/(kg s), erg/(g s), or rad/s
- Knowing kerma rate, kerma

$$K(t_0, t_1) = \int_{t_0}^{t_1} \dot{K}(t)\,dt$$

Absorbed dose

- Energy imparted by ionizing radiation to matter of mass m in volume V

$$\varepsilon = \underbrace{(R_{in})_u - (R_{out})_u}_{\text{due to uncharged}} + \underbrace{(R_{in})_c - (R_{out})_c}_{\text{due to charged}} + \sum Q$$

- Absorbed dose is defined as

$$D = \frac{d\varepsilon}{dm}$$

- Units: 1 Gy = 1 J/kg = 10^2 rad = 10^4 erg/g

Absorbed dose

- *D* represents the energy per unit mass which remains in the matter at P to produce any effects attributable to radiation
- The most important quantity in radiological physics
- Absorbed dose rate:

$$\dot{D} = \frac{dD}{dt} = \frac{d}{dt}\left(\frac{d\varepsilon}{dm}\right)$$

Example 1

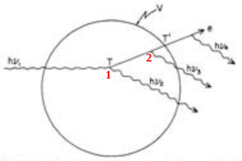


FIGURE 2.1a. Illustration of the concepts of energy imparted, energy transferred, and net energy transferred for the case of a $\underbrace{\text{Compton interaction}}_{1}$ followed by $\underbrace{\text{bremsstrahlung emission}}_{2}$ (Attix, 1983).

$(R_{in})_u = h\nu_1$

$(R_{out})_u = (R_{out})_u^{nonr} = h\nu_2$

$(R_{in})_c = 0, \quad Q = 0$

$(R_{out})_c = h\nu_3 + T'$

$R_u^r = h\nu_3$

Energy imparted $\varepsilon = (R_{in})_u - (R_{out})_u + (R_{in})_c - (R_{out})_c + \sum Q$

Energy transferred $\varepsilon_{tr} = (R_{in})_u - (R_{out})_u^{nonr} + \sum Q$

Net energy transferred $\varepsilon_{tr}^{net} = (R_{in})_u - (R_{out})_u^{nonr} - R_u^r + \sum Q$

Example 1

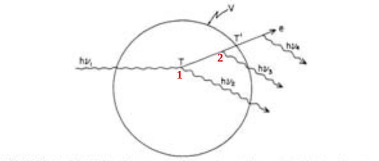


FIGURE 2.1a. Illustration of the concepts of energy imparted, energy transferred, and net energy transferred for the case of a Compton interaction followed by bremsstrahlung emission (Attix, 1983).

$$(R_{in})_u = h\nu_1$$

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$$(R_{in})_c = 0, \quad Q = 0$$

$$(R_{out})_c = h\nu_3 + T'$$

$$R_u^r = h\nu_3$$

$$\varepsilon = h\nu_1 - (h\nu_2 + h\nu_3 + T') + 0$$

$$\varepsilon_{tr} = h\nu_1 - h\nu_2 + 0 = T$$

$$\begin{aligned}\varepsilon_{tr}^{net} &= h\nu_1 - h\nu_2 - h\nu_3 + 0\\ &= T - h\nu_3\end{aligned}$$

Example 2

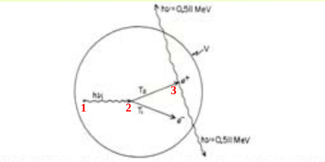


FIGURE 2.1b. Example involving γ-ray emission, pair production, and positron annihilation (Attix, 1983).

Positron has no excess kinetic energy to transfer to photons after annihilation

$$(R_{in})_u = (R_{in})_c = (R_{out})_c = R_u^r = 0$$

$$(R_{out})_u = (R_{out})_u^{nonr} = 2h\nu = 1.022\text{MeV}$$

$$\sum Q = h\nu_1 - \underset{\text{pair production}}{2m_0c^2} + \underset{\text{annihilation}}{2m_0c^2} = h\nu_1$$

$$\varepsilon = \varepsilon_{tr} = \varepsilon_{tr}^{net} = h\nu_1 - 1.022\text{ MeV} = T_1 + T_2$$

Example 3

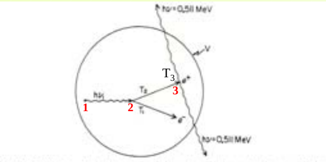


FIGURE 2.1b. Example involving γ-ray emission, pair production, and positron annihilation (Attix, 1983).

- Positron transfers excess kinetic energy T_3 to photons after annihilation.
- It generates radiative loss from charged-particle kinetic energy
- Affects ε and ε_{tr}^n by subtraction of T_3

$$(R_{in})_u = (R_{in})_c = (R_{out})_c = 0$$

$$(R_{out})_u = 2h\nu + \boxed{T_3} = 1.022\text{MeV} + T_3$$

$$(R_{out})_u^{nonr} = 2h\nu = 1.022\text{MeV}$$

$$R_u^r = \boxed{T_3}$$

$$\sum Q = h\nu_1 - 2m_0c^2 + 2m_0c^2 = h\nu_1$$

Exposure

- Historically, was introduced before kerma and dose, measured in roentgen (R)
- Defined as a quotient

$$X = \frac{dQ}{dm}$$

- *dQ* is absolute value of the total charge of the ions of one sign produced in air when all electrons liberated by photons in air of mass *dm* are completely stopped in air
- Ionization from the absorption of radiative loss of kinetic energy by electrons is *not included*

Exposure

- Exposure is the ionization equivalent of the collision kerma in air for x and γ-rays
- Introduce mean energy expended in a gas per ion pair formed, $\overline{W}$, constant for each gas, independent of incoming photon energy
- For dry air

$$\begin{aligned}\frac{\overline{W}_{air}}{e} &= \frac{33.97\,\text{eV/i.p.}}{1.602\times10^{-19}\text{C/electron}}\times 1.602\times10^{-19}\,\text{J/eV}\\ &= 33.97\ \text{J/C}\end{aligned}$$

Relation of exposure to energy fluence

- Exposure at a point due to energy fluence of mono-energetic photons

$$X = \Psi\cdot\left(\frac{\mu_{en}}{\rho}\right)_{E,air}\left(\frac{e}{\overline{W}}\right)_{air} =$$

$$(K_c)_{air}\left(\frac{e}{\overline{W}}\right)_{air} = (K_c)_{air}/33.97$$

- Units of [X]=C/kg in SI

Units of exposure

- The roentgen R is the customary unit
- The roentgen is defined as exposure producing in air one unit of esu of charge per 0.001293 g of air irradiated by the photons. Conversion

$$1R = \frac{1esu}{0.001293g} \times \frac{1C}{2.998 \times 10^9 esu} \times \frac{10^3 g}{1kg}$$

$$= 2.580 \times 10^{-4}\,\mathrm{C/kg}$$

$$1\,\mathrm{C/kg} = 3876\,\mathrm{R}$$

Exposure rate

- Exposure rate at a point P and time t:

$$\dot{X} = \frac{dX}{dt}$$

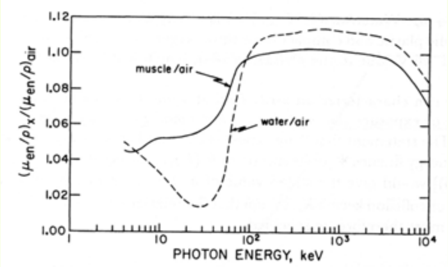
- Units are C/(kg-sec) or R/sec
- Exposure

$$X = \int_{t_0}^{t_1} \dot{X}(t)\,dt$$

Significance of exposure

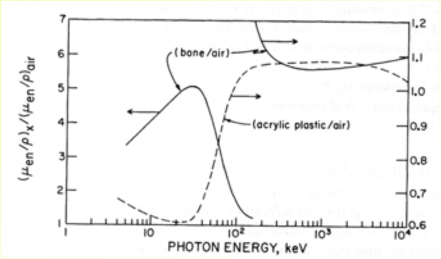
- Energy fluence is proportional to exposure for any given photon energy or spectrum
- Due to similarity in effective atomic number
  - Air can be made a tissue equivalent medium with respect to energy absorption – convenient in measurements
  - Collision kerma in muscle per unit of exposure is nearly independent of photon energy

Significance of exposure



Ratio of mass energy-absorption coefficients for muscle/air and water/air are nearly constant (within <5%) for energies from 4keV to 10 MeV

Significance of exposure



Ratio of mass energy-absorption coefficients for bone/air and acrylic/air are nearly constant for energies above 100keV

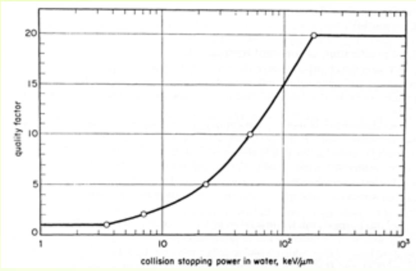
Significance of exposure

- X-ray field at a point can be characterized by means of exposure regardless of whether there is air actually located at this point
- It implies that photon energy fluence at that point is such that it would produce exposure of a stated value
- Same is applicable to kerma or collision kerma, except that reference medium (not necessarily air) has to be specified

Radiation protection quantities

- Quality factor Q – weighting factor to be applied to absorbed dose to provide an estimate of the relative human hazard of ionizing radiation
- It is based on relative biological effectiveness (RBE) of a particular radiation source
- Q is dimensionless

Radiation protection quantities



- Higher-density charged particle tracks (higher collision stopping power) are more damaging per unit dose

Radiation protection quantities

- Dose equivalent *H*, is defined as

$$H \equiv DQN$$

- Here D – dose, Q- quality factor, N-product of modifying factors (currently=1)
- Units of H:
  - severs, *Sv*, if dose is expressed in J/kg
  - *rem*, if dose is in rad (10^{-2} J/kg)

Summary

- Quantities describing the interaction of ionizing radiation with matter
  - Kerma, components of kerma
  - Absorbed dose
  - Exposure
- Relationship with fluence and energy fluence
- Quantities for use in radiation protection
  - Quality factor Q
  - Dose equivalent H