Charged-Particle and Radiation Equilibria

Chapter 4

F.A. Attix, Introduction to Radiological Physics and Radiation Dosimetry

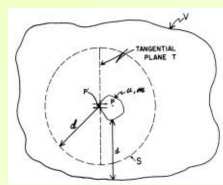
Outline

- Radiation equilibrium
- Charged particle equilibrium
- Causes of CPE failure
- Transient CPE
- Summary

Introduction

- The concepts of radiation equilibrium (RE) and charged-particle equilibrium (CPE) are useful in relating certain basic quantities
- CPE allows the equating of the absorbed dose D to the collision kerma K_c and exposure X
- Radiation equilibrium makes dose D equal to the net rest mass converted to energy per unit mass at the point of interest

Radiation equilibrium

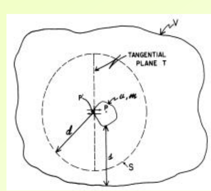


- Consider an extended volume V containing a distributed radioactive source with a smaller internal volume v about a point of interest, P
- Radioactivity is emitted isotropically on average
- V is required to be large enough so that the maximum distance of penetration d of any emitted ray and its progeny (i.e., scattered and secondary rays) is less than the minimum separation s of the boundaries of V and v

Radiation equilibrium

- *Radiation equilibrium* (RE) exists for the volume v if the following four conditions exist throughout V (in the nonstochastic limit):
  a. The atomic composition of the medium is homogeneous
  b. The density of the medium is homogeneous
  c. The radioactive source is uniformly distributed
  d. There are no electric or magnetic fields present to perturb the charged-particle paths, except the fields associated with the randomly oriented individual atoms

Radiation equilibrium



- Consider a plane T that is tangent to the volume v at a point P', and the rays crossing the plane per unit area
- In the nonstochastic limit there will be perfect reciprocity of rays of each type and energy crossing both ways, due to uniform distribution of the radioactive source within the sphere S
- This will be true for all possible orientations of tangent planes around the volume v; therefore, *in the nonstochastic limit, for each type and energy of ray entering v, another identical ray leaves*
- This condition is called *radiation equilibrium* (RE) with respect to v

Radiation equilibrium

- As a consequence of radiation equilibrium the energy carried in and that carried out of *v* are balanced for both indirectly and directly ionizing radiation:

$$(\overline{R}_{\text{in}})_u = (\overline{R}_{\text{out}})_u \quad \text{and} \quad (\overline{R}_{\text{in}})_c = (\overline{R}_{\text{out}})_c$$

- The energy imparted can then be simplified to

$$\bar{\varepsilon} = \overline{\sum Q}$$

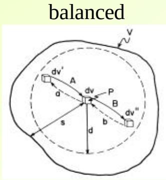
- Therefore under RE conditions the expectation value of the energy imparted to the matter in the volume *v* is equal to that emitted by the radioactive material in *v*

Radiation equilibrium

- In non-stochastic consideration the volume *v* can be reduced to infinitesimal *dv*, then RE exists at the point P
- Since $D = d\varepsilon / dm$, under condition of radiation equilibrium at a point in a medium, the absorbed dose is equal to the expectation value of the energy released by the radioactive material per unit mass at that point, $\frac{d}{dm}\overline{\sum Q}$

Radiation equilibrium

- Presence of homogeneous constant magnetic and/or electric field throughout V can make radiation field anisotropic
- RE will still be satisfied if the flow of radiation is balanced



- Consider flows of particles between dv`, dv, and dv``: a+B=A+b
- If anisotropy is homogeneous everywhere in V, the same balance of flow is held

Radiation equilibrium

- The concept of RE has practical importance in the fields of nuclear medicine and radiobiology, where distributed radioactive sources may be introduced into the human body or other biological systems for diagnostic, therapeutic, or analytical purposes
- The resulting absorbed dose at any given point depends on the size of the object relative to the radiation range and on the location of the point within the object

Charged-particle equilibrium

- Charged particle equilibrium (CPE) exists for the volume *v* if each charged particle of a given type and energy leaving *v* is replaced by an identical particle of the same energy entering (in terms of expectation values)
- If CPE exists,

$$(\overline{R}_{\text{in}})_c = (\overline{R}_{\text{out}})_c$$

- RE condition is sufficient for CPE to exist

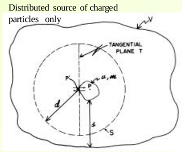
Charged-particle equilibrium

- In many practical cases RE condition is not satisfied, but cab be adequately approximated if CPE condition exists
- Consider two general situations:
  1. distributed radioactive sources
  2. indirectly ionizing radiation from external sources

CPE for distributed sources – case 1

- For the trivial case of a distributed source emitting only charged particles, in a system where radiative losses are negligible
- The dimension *s* (the minimum separation of *v* from the boundary) is taken to be greater than the maximum range *d* of the particles

Distributed source of charged particles only

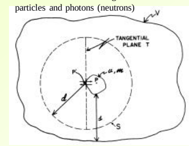


- If all of the four conditions *a* - *d* are satisfied, both RE and CPE exist (they are identical for this case)

CPE for distributed sources – case 2

- Consider now the case where both charged particles and relatively more penetrating indirectly ionizing radiation are emitted
- Let the distance *d* be the maximum range of the *charged particles only*, and distance $s > d$

Distributed source of charged particles and photons (neutrons)



- Conditions a through d are satisfied
- Only CPE exists in this case
- RE is not attained since rays escaping from the volume *v* are not replaced, $(\overline{R}_{\text{out}})_u > (\overline{R}_{\text{in}})_u$

CPE for distributed sources – case 2

- The equation for the expectation value of the energy imparted $\bar{\varepsilon} = (\overline{R}_{\text{in}})_u - (\overline{R}_{\text{out}})_u + \overline{\sum Q}$
- Since the indirectly ionizing rays are so penetrating that they do not interact significantly in *v*, $\bar{\varepsilon}$ is equal to the kinetic energy given to charged-particles by the radioactive source in *v*, less any radiative losses by those particles while in *v*
- The average absorbed dose in *v* is thus $\overline{D} = \bar{\varepsilon} / m$ for CPE condition

CPE for distributed sources – case 2

- Now assume that the size of the volume *V* occupied by the source is expanded so that distance *s* increases to being greater than the effective range of the indirectly ionizing rays and their secondaries
- This transition will cause the $(\overline{R}_{\text{in}})_u$ term to increase until it equals $(\overline{R}_{\text{out}})_u$ in value
- RE will be restored
- The energy imparted would be transformed into that for RE: $\bar{\varepsilon} = \overline{\sum Q}$

CPE for distributed sources – case 3

- A distributed source emitting penetrating indirectly ionizing radiation
- Achieving CPE will also require that RE is attained
- The following equations are applicable

$$(\overline{R}_{\text{in}})_u = (\overline{R}_{\text{out}})_u \quad \text{and} \quad (\overline{R}_{\text{in}})_c = (\overline{R}_{\text{out}})_c$$

$$\bar{\varepsilon} = \overline{\sum Q}$$

CPE for distributed sources

- The calculation of the absorbed dose is straightforward for either of these limiting cases (CPE or RE)
- Intermediate situations are more difficult to deal with, i.e., when the volume *V* is larger than necessary to achieve CPE in *v*, but not large enough for RE
- In that case some fraction of the energy of the indirectly ionizing radiation component will be absorbed, and it is relatively difficult to determine what that fraction is

CPE for indirectly ionizing radiation from external sources

- A volume *V* contains a smaller volume *v*
- The boundaries of *v* and *V* are required to be separated by at least the maximum distance of penetration of any secondary charged particle present
- If the following conditions are satisfied throughout *V*, CPE will exist for the volume *v:*

a. The atomic composition of the medium is homogeneous

b. The density of the medium is homogeneous

c. There exists a uniform *field* of indirectly ionizing radiation (rays are only negligibly attenuated passing through the medium)

d. No inhomogeneous electric or magnetic fields are present

CPE for indirectly ionizing radiation from external sources

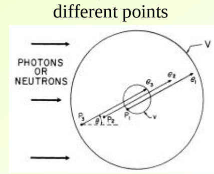
- These conditions are similar to those of RE, except for:
  - Condition *c*: uniform *field* of radiation replaces the uniform radioactive *source*
  - The separation of boundaries of *v* and *V* are required to be at least the maximum distance of penetration of any secondary charged particle, rather than that of the most penetrating radiation (indirectly ionizing)
  - The last condition *d* has been shown to be a sufficient substitute for the requirement of a complete absence of electric or magnetic fields

CPE for indirectly ionizing radiation from external sources

- Analogy with the distributed source case: since the radiation field is uniform, the number of charged particles produced per unit volume in each energy interval and element of solid angle will be uniform
- However, since neutron and photon interactions generally result in anisotropic angular distributions of secondary and scattered radiations, these particles are not emitted isotropically as in the case of radioactive point sources
- This anisotropy will be *homogeneous* throughout *V*

CPE for indirectly ionizing radiation from external sources

- Homogeneous anisotropy, together with a uniform medium in which the charged particles can slow down throughout *V* (as guaranteed by the first two conditions) is sufficient to produce CPE for the volume *v*
- Example: a simplified case of straight charged-particle tracks, all emitted at angle θ through identical interactions at different points



- Three charged particles e_1-e_3 will contribute the same kinetic energy as e_1 alone would, if its entire track remained inside of *v*
- Thus CPE exists inside of *v*

$$\left(\overline{R}_{\text{in}}\right)_c = \left(\overline{R}_{\text{out}}\right)_c$$

CPE for indirectly ionizing radiation from external sources

- Relating $\overline{\varepsilon}_{tr}^{\,n}$ and $\overline{\varepsilon}$ for CPE conditions:

$$\overline{\varepsilon}_{tr}^{\,n} = \overline{\varepsilon} + \left(\overline{R}_{\text{out}}\right)_u - \left(\overline{R}_{\text{out}}\right)_u^{\text{non-}r} - \overline{R}_u^{\,r}$$

- However, under those same conditions we may also assume that any radiative interaction by a charged particle after it leaves *v* will be replaced by an identical interaction inside of *v*, resulting in

$$\left(\overline{R}_{\text{out}}\right)_u = \left(\overline{R}_{\text{out}}\right)_u^{\text{non-}r} + \overline{R}_u^{\,r}$$

CPE for indirectly ionizing radiation from external sources

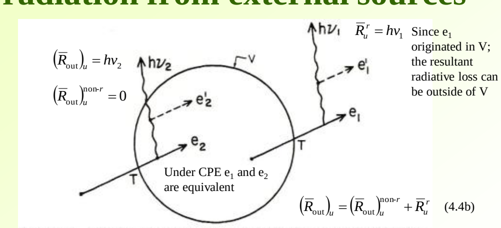


FIGURE 4.4. Illustrating Eqs. (4.4b) and (4.4c). CPE exists (in the nonstochastic limit) because electron e_2 enters the volume *v* with a kinetic energy *T* equal to that carried out by electron e_1. If e_1 then emits an x-ray $h\nu_1$, e_2 will also emit an identical x-ray $h\nu_2$ (on the average). If $h\nu_2$ escapes from *v*, then $(\overline{R}_{\text{out}})_u = h\nu_2 = h\nu_1 = \overline{R}_u^{\,r}$, and since $\overline{R}_{\text{out}})_u^{\text{non-}r} = 0$, Eq. (4.4b) is satisfied. However, if $h\nu_2$ is absorbed within *v*, producing secondary electron e'_2, then $(\overline{R}_{\text{out}})_u = 0$ but $\overline{R}_u^{\,r}$ still equals $h\nu_1$ and $(\overline{R}_{\text{out}})_u^{\text{non-}r} = 0$ as before, so Eq. (4.4b) is no longer satisfied. Therefore Eq. (4.4c) also is only valid for small enough volumes to allow the escape of radiative losses. Equations (4.5) and (4.6) are surely valid because the relevant volume is infinitesimal.

CPE for indirectly ionizing radiation from external sources

- Provided that the volume v is small enough to allow radiative-loss photons to escape

$$\bar{\varepsilon} = \bar{\varepsilon}_{\mathrm{tr}}^{\mathrm{n}}$$

- Reducing v to the infinitesimal volume dv, containing mass dm about a point of interest P, we can write

$$\frac{d\bar{\varepsilon}}{dm} = \frac{d\bar{\varepsilon}_{\mathrm{tr}}^{n}}{dm}$$

hence under CPE:

$$D \overset{\mathrm{CPE}}{=} K_c$$

CPE for indirectly ionizing radiation from external sources

- The derivation of this equation proves that under CPE conditions at a point in a medium, the absorbed dose is equal to the collision kerma there
- That is true irrespective of radiative losses
- This relationship equates the measurable quantity D with the calculable quantity $K_c = \Psi \bullet \mu_{\mathrm{en}}/\rho$

CPE for indirectly ionizing radiation from external sources

- If the same photon energy fluence Ψ is present in media A and B having two different average energy absorption coefficients, the ratio of absorbed doses under CPE conditions in the two media will be given by

$$\frac{D_A}{D_B} \overset{\mathrm{CPE}}{=} \frac{(K_C)_A}{(K_C)_B} = \frac{\left(\overline{\mu_{\mathrm{en}}/\rho}\right)_A}{\left(\overline{\mu_{\mathrm{en}}/\rho}\right)_B}$$

- Doses D_A and D_B can differ due to either different atomic compositions A and B, or radiation spectra

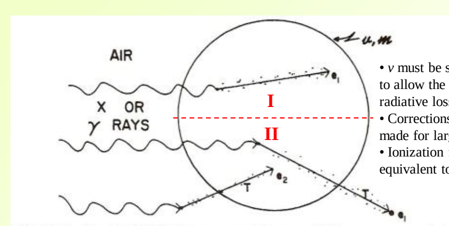
CPE for indirectly ionizing radiation from external sources

- It is possible for CPE to exist in a volume without satisfying all the requirements, *a* through *d*, under certain geometrical conditions
- The ion-collecting region of a free-air chamber represents such a situation (Chapter 12)
- Another example is the trivial case of a point source within a volume large enough so the radiation cannot reach the boundary surface, hence no replacement particles are required

CPE in measurement of exposure

- Exposure is only defined for x and γ rays, $X = K_c(e/\overline{W})$ for air
- This poses a practical difficulty in the measurement of X, since collision kerma (K_c) cannot be readily measured by any direct means
- The attainment of CPE in an ionization chamber, however, allows the measurement of the ionization collected within a defined volume and mass of air (in place of the ionization produced everywhere by all the secondary electrons that start within the defined volume, as called for by the exposure definition)

Ion chamber operation



- v must be small enough to allow the escape of radiative losses
- Corrections must be made for larger volumes
- Ionization from I is equivalent to II

FIGURE 4.5. The role of CPE in the measurement of exposure X. The average exposure in the finite air volume v equals the total charge of either sign released in air by all electrons (e_1) that originate in v, divided by the air mass m in v. If CPE exists, each electron carrying an energy (say, T) out of v is compensated by another electron (e_2) carrying the same energy in. Thus the same ionization occurs in v as if all electrons e_1 remained there. The measurement of that charge divided by m is thus equivalent to a measurement of the average exposure in v. Radiative losses are assumed to escape from v, and any ionization they produce is not to be included in X.

Relating absorbed dose to exposure

- It is sometimes useful to know how much absorbed dose would be deposited at some point in air as a result of an exposure X
- The relationship is indeterminate in the absence of CPE, since

$$D_{\text{air}} \overset{\text{CPE}}{=} (K_c)_{\text{air}} = X \cdot \left(\frac{\overline{W}}{e}\right)_{\text{air}}$$

where the first equality is valid only if CPE exists at the point in question

Relating absorbed dose to exposure

- If D_{air} is expressed in rads and X in roentgens,

$$D_{\text{air}} \overset{\text{CPE}}{=} (K_c)_{\text{air}} = 0.876X$$

- This equation is valid only where X is the exposure at the point of interest in air, under CPE conditions

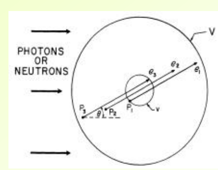
Causes of CPE failure in a field of indirectly ionizing radiation

There are four basic causes for CPE failure in an indirectly ionizing field:

a. Inhomogeneity of atomic composition within volume V
b. Inhomogeneity of density in V
c. Non-uniformity of the field of indirectly ionizing radiation in V
d. Presence of a non-homogeneous electric or magnetic field in V

CPE failure: Proximity to the source

- If the volume V is too close to the source of the indirectly ionizing radiation, then the energy fluence will be significantly nonuniform within V, being larger on the side nearest to the source



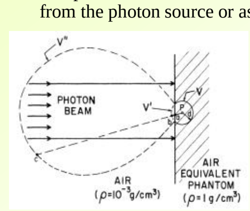
- Thus there will be more particles (e_3) produced at points like P_3 than particles e_1 at P_1 => more particles will enter v than leave it
- CPE consequently fails for v

CPE failure: proximity to a boundary of inhomogeneity in the medium

- If the volume V is divided by a boundary between dissimilar media, loss of CPE may result at v, since the number of charged particles arriving at v will generally be different than would be the case for a homogeneous medium
- This difference may be due to a change in charged-particle production, or a change in the range or geometry for scattering of those particles, or a combination of these effects

CPE failure: proximity to a boundary of inhomogeneity

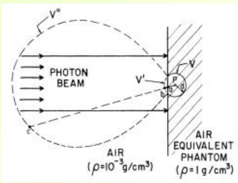
- Consider a case of a beam of MV photons incident on a solid unit-density air-equivalent phantom
- The photon beam is not contaminated by secondary electrons from the photon source or associated hardware



- The absorbed dose D in the phantom first increases steeply up to a maximum, then decreases more gradually in a condition called *transient charged particle equilibrium* (TCPE)
- We temporarily consider TCPE as being ~ the same as CPE

CPE failure: proximity to a boundary of inhomogeneity

- The spherical volume *V*, having radius *d* equal to the maximum range of secondary electrons, must contain a uniformly irradiated homogeneous medium throughout if TCPE is to be produced at point *P*



- If *P* is too close to the surface, the portion *V′* of *V* will project out of the phantom
- To replace the solid missing from that volume, a 1000-fold larger volume *V″* of air is required (distance ac=1000ab)
- But *V″* is not homogeneously (nor even completely) irradiated

CPE failure: High energy radiation

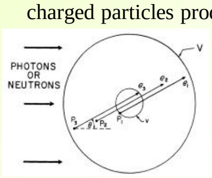
- As the energy of indirectly ionizing radiation increases, the penetrating power of the secondary charged particles increases faster than the penetrating power of the primary radiation
- For example, a 7% attenuation of γ-rays would occur in a water layer equal in thickness (≈ 5 cm) to the maximum range of secondary electrons produced by 10-MeV γ-rays
- The neutron effect is much smaller (1%) at that energy, assuming hydrogen-recoil proton secondaries

TABLE 4.1. Approximate Attenuation[a] of Gamma Rays and Neutrons within a Layer of Water Equal to the Maximum Range of Secondary Charged Particles

Primary Radiation Energy (MeV)	Gamma-Ray Attenuation (%) in Maximum Electron Range	Neutron Attenuation (%) in Maximum Proton Range
0.1	0	0
1.0	1	0
10	7	1
30	15	4

CPE failure: High energy radiation

- As a result, there is a significant attenuation of primary radiation over the distance *d*
- The radiation field is not uniform anymore (failure of condition *c* in CPE definition)
- CPE fails due to the difference in the number of charged particles produced



- Due to attenuation fewer photons reach P_1 than P_3
- The number of charged particles generated at P_3 is greater than at P_1

CPE failure: High energy radiation

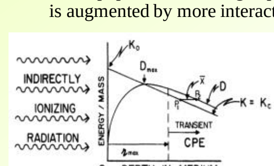
- Since the measurements of exposure from *x*- and γ-rays relies on the existence of CPE, exposure measurements have been conventionally assumed to be infeasible for photon energies > 3 MeV
- However, if some known relationship between D_{air} and $(K_c)_{air}$ can be attained under achievable conditions, and substituted for the simple equality that exists for CPE, exposure can still be measured, at least in principle
- Such a relationship does exist for a situation known as TCPE

Transient charged-particle equilibrium

- TCPE is said to exist at all points within a region in which $D \sim K_c$, with the constant of proportionality >1
- Consider two cases of a broad "clean" beam of indirectly ionizing radiation (no contamination by charged particles) falling perpendicularly on a slab of material:
  1. Negligible radiative losses from secondary charged particles
  2. Significant radiative losses; photons are allowed to escape from the phantom

Transient charged-particle equilibrium

- Case 1: Assume negligible radiative losses from secondary charged particles ($K_r \approx 0$)
  - Strictly true only for incident neutrons
  - In low-Z media (carbon, water, air) $K_r = K - K_c$ <1% of *K* for photons up to 3 MeV
- The absorbed-dose curve is rising with depth near the surface as the population of charged particles flowing toward the right is augmented by more interactions of indirectly ionizing rays



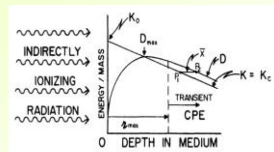
- The dose curve reaches a maximum (D_{max}) at the depth where the rising slope due to buildup of charged particles is balanced by the descending slope due to attenuation of the indirectly ionizing radiation

Transient charged-particle equilibrium

- For a “clean” beam of indirectly ionizing radiation D_{max} occurs at approximately the same depth as where the *D*-curve crosses the K_c-curve
- The presence of charged-particle “contamination” in the beam is often observed to shift the depth of D_{max} closer to the surface, where it no longer approximates the depth at which $D = K_c$
- Thus one should not assume that $D = K_c$ at D_{max}

Transient charged-particle equilibrium

- At a somewhat greater depth r_{max}, equal to the maximum distance the secondary charged particles starting at the surface can penetrate in the direction of the incident rays, the *D*-curve becomes parallel to the K_c– and *K*–curves, although all may gradually change slope together with depth



- *D* therefore becomes proportional to K_c, and we say that TCPE exists

Transient charged-particle equilibrium

- Roesch suggest a relationship between the *D*- and *K*-curves for TCPE conditions, in assumption of no radiative interactions, and ignoring scattered photons:

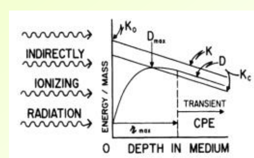
$$D \overset{\text{TCPE}}{=} K_c e^{\mu'\bar{x}} \overset{\text{TCPE}}{=} K_c\left(1+\mu'\bar{x}+\frac{(\mu'\bar{x})^2}{2!}+\cdots\right)$$

$$\overset{\text{TCPE}}{=} K_c\left(1+\mu'\bar{x}\right)$$

- Here *D* and K_c are for the same given depth, at which TCPE is required, μ′ is the common slope of the *D*, *K*, and K_c curves at that depth; and $\bar{x}$ is the mean distance the secondary charged particles carry their kinetic energy in the direction of the primary rays while depositing it as dose

Transient charged-particle equilibrium

- Case 2: The case of significant radiative losses from secondary charged particles (such as bremsstrahlung interactions) is similar to the previous one
- The D-curve bears the same relationship to the K_c-curve, which is moved down by the amount K_r



$$K = K_c + K_r$$

$$K_r = \left[\left(\mu_{tr} - \mu_{en}\right)/\,\mu_{tr}\right]K$$

Transient charged-particle equilibrium

- Existence of a relationship between *D* and K_c under transient CPE conditions is valuable for high-energy indirectly ionizing radiations, where CPE may fail
- However, a knowledge of $\bar{x}$ and the effective attenuation coefficient μ′ is required for each case, so the equation is not as readily applicable as the simple equality of *D* and K_c that exists under CPE conditions

Summary

- Radiation equilibrium
- Charged particle equilibrium
- Causes of CPE failure
- Transient CPE