

Absorbed Dose in Radioactive Media

Chapter 5

F.A. Attix, Introduction to Radiological Physics and Radiation Dosimetry

Outline

- General dose calculation considerations, absorbed fraction
- Radioactive disintegration processes and associated dose deposition
 - Alpha disintegration
 - Beta disintegration
 - Electron-capture transitions
 - Internal conversion
- Summary

Introduction

- We are interested in calculating the absorbed dose in radioactive media, applicable to cases of
 - Dose within a radioactive organ
 - Dose in one organ due to radioactive source in another organ
- If conditions of CPE or RE are satisfied, dose calculation is straightforward
- Intermediate situation is more difficult but can be handled at least in approximations

Radiation equilibrium

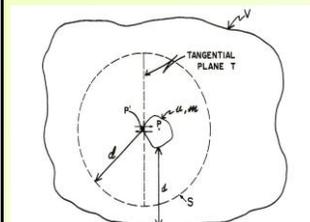


FIGURE 4.1. Radiation equilibrium. Extended volume V contains a homogeneous medium and a homogeneous isotropic source distribution. Radiation equilibrium will exist in the smaller internal volume v if the maximum distance of penetration (d) of primary rays plus their secondaries is less than the minimum separation (j) of v from the boundary of V. Neutrons are ignored. (See text.)

- The atomic composition of the medium is homogeneous
- The density of the medium is homogeneous
- The radioactive source is uniformly distributed
- No external electric or magnetic fields are present

Charged-particle equilibrium

- Each charged particle of a given type and energy leaving the volume is replaced by an identical particle of the same energy entering the volume
- Existence of RE is sufficient condition for CPE
- Even if RE does not exist CPE may still exist (for a very large or a very small volume)

Limiting cases

- Emitted radiation typically includes both photons (longer range) and charged particles (shorter range)
- Assume the conditions for RE are satisfied
 - Consider two limited cases based on the size of the radioactive object

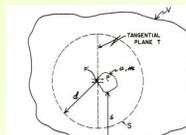


FIGURE 4.1. Radiation equilibrium. Extended volume V contains a homogeneous medium and a homogeneous isotropic source distribution. Radiation equilibrium will exist in the smaller internal volume v if the maximum distance of penetration (d) of primary rays plus their secondaries is less than the minimum separation (j) of v from the boundary of V. Neutrons are ignored. (See text.)

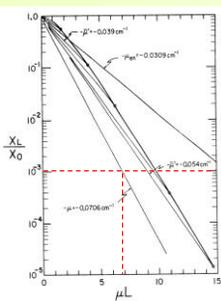
Limiting cases: small object

- A radioactive object V having a mean radius not much greater than the maximum charged-particle range d
- CPE is well approximated at any internal point P that is at least a distance d from the boundary of V
- If $d \ll 1/\mu$ for the γ -rays, the absorbed dose D at P approximately equals to the energy per unit mass of medium that is given to the charged particles in radioactive decay (less their radiative losses)
- The photons escape from the object and are assumed not to be scattered back by its surroundings

Limiting cases: large object

- A radioactive object V with mean radius $\gg 1/\mu$ for the most penetrating γ -rays
- RE is well approximated at any internal point P that is far enough from the boundary of V so γ -ray penetration through that distance is negligible
- The dose at P will then equal the sum of the energy per unit mass of medium that is given to charged particles plus γ -rays in radioactive decay

Limiting cases: large object



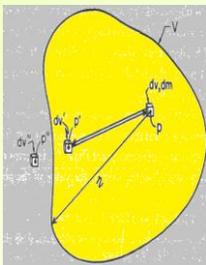
- Deciding upon a maximum γ -ray “range” L for this case requires quantitative criterion
 - For primary beam only (μ) the penetration is $< 0.1\%$ through $L \sim 7$ mean free paths
 - Taking into account scatter, (broad beam geometry, $\bar{\mu}'$) will increase the required object size to satisfy the attenuation objectives

Absorbed fraction

- An *intermediate*-size radioactive object V
- Dose at P will then equal the sum of the energy per unit mass of medium that is given to charged particles plus dose from some γ -rays
- To estimate the dose from γ -rays define absorbed fraction:

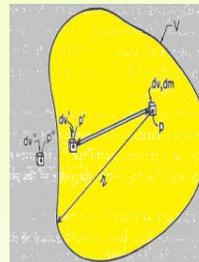
$$AF = \frac{\gamma\text{-ray radiant energy absorbed in target volume}}{\gamma\text{-ray radiant energy emitted by source}}$$

Intermediate-size radioactive object



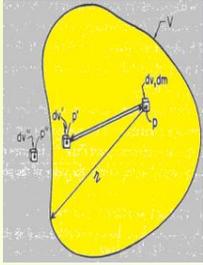
- Consider volume V filled by a homogeneous medium and a uniformly distributed γ -source
- The volume may be surrounded by
 - Case 1: infinite homogeneous medium identical to V , but non-radioactive (an organ in the body)
 - Case 2: infinite vacuum (object in air)

Case 1



- Reciprocity theorem: energy spent in dv due to the source in dv' :
 $\epsilon_{dv',dv} = \epsilon_{dv,dv'}$ and $\epsilon_{dv,V} = \epsilon_{V,dv}$
- No source in dv''
- Define \bar{R}_{dv} as the expectation value of the γ -ray radiant energy emitted by the source in dv , and $\bar{\epsilon}_{dv,V}$ the part of that energy that is spent in V (source dv and target V)

Case 1 continued



- The absorbed fraction with respect to source dv and target V is :

$$AF_{dv,V} = \frac{\bar{\epsilon}_{dv,V}}{R_{dv}} = \frac{\bar{\epsilon}_{V,dv}}{R_{dv}}$$

source, target

- Estimates reduction in absorbed dose relative to RE condition
- For very small radioactive objects ($V \rightarrow dv$) this absorbed fraction approaches zero; for an infinite radioactive medium it equals unity

Case 1 continued

- Reduction in the absorbed dose due to γ -rays energy escaping from V
- Can estimate AF using mean effective attenuation coefficient $\bar{\mu}'$ for γ -rays energy fluence through a distance r in the medium

$$AF_{dv,V} = \frac{1}{4\pi} \int_{\theta=0}^{\pi} \int_{\beta=0}^{2\pi} (1 - e^{-\bar{\mu}'r}) \sin \theta d\theta d\beta$$

- For poly-energetic sources have to find an average value of the absorbed fraction

Case 1 example

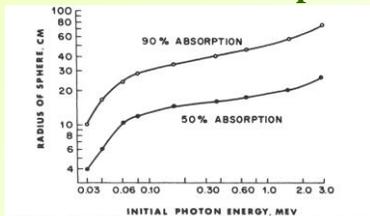
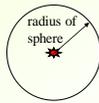


FIGURE 5.2. Radius of unit-density tissue sphere needed to absorb 50% and 90% of the emitted photon energy from a central point source in an infinite homogeneous medium. (After Brownell et al., 1968.) Reproduced with permission of the authors and The Society of Nuclear Medicine.



- Dose calculations published in MIRD reports
- The larger the radius, the lower the γ -ray energy – the closer to RE condition ($AF=1$)

Case 2

Radioactive object V surrounded by vacuum

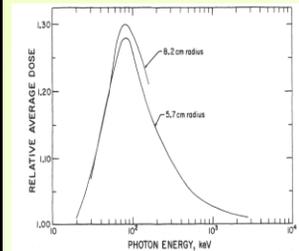


FIGURE 5.3. Radius of average absorbed doses in uniformly radioactive tissue spheres with/without surrounding non-radioactive tissue medium. Lower curve: 780-g (5.7-cm-radius) sphere; upper curve: 2.5-kg (8.2-cm-radius) sphere. (Data from Elliot, 1968.)

- More difficult to calculate
- The reciprocity theorem is only approximate due to the lack of backscattering
- Dose is lower than in Case 1

Case 2

- To obtain a crude estimate of the dose at some point P within a uniformly γ -active homogeneous object, it may suffice to obtain the average distance \bar{r} from the point to the surface of the object by

$$\bar{r} = \frac{1}{4\pi} \int_{\theta=0}^{\pi} \int_{\beta=0}^{2\pi} r \sin \theta d\theta d\beta$$

- Then one may employ $\mu_{en} = \bar{\mu}'$ in the straight-ahead approximation to obtain

$$AF_{dv,V} \cong 1 - e^{-\mu_{en}\bar{r}}$$

MIRD tables

- AF is incorporated in “S-value” tabulated for each radionuclide and source-target configuration used in nuclear medicine
- Dose absorbed in any organ (target) due to a source in some other organ is calculated based on cumulative activity, $D=A \times S$
- S values are typically calculated by Monte Carlo technique
- Straightforward approach, but accuracy is limited

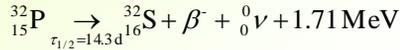
Pamphlet 11, 1975

Target organ	Source organs			
	Colon	Transverse colon	Bladder contents	Total
Red marrow	4.1E-06	9.1E-06	2.3E-06	9.9E-06
Bladder wall	2.1E-07	9.1E-07	1.4E-04	2.3E-06
Ovaries	7.1E-07	7.1E-07	2.3E-06	2.4E-06
Testes	6.4E-07	6.4E-07	2.7E-06	1.7E-06

FIG. 8-2. Anterior view of the principal organs in the head and neck of the phantom.

Beta disintegration

- Nuclei having excess of neutrons typically emit an electron, β^- particle; atomic number Z is increased by 1
- Nuclei having excess of protons typically emit positron, β^+ particle; atomic number Z is decreased by 1
- Nucleus is left in an excited state, emitting one or more γ -rays
- Example:



Beta disintegration

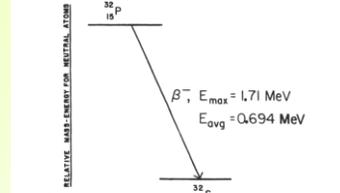


FIGURE 5.6. Atomic energy-level diagram for ${}_{15}^{32}\text{P} \rightarrow {}_{16}^{32}\text{S}$ β^- -disintegration.

- Kinetic energy 1.71 MeV is shared between β^- and neutrino
- Charge balance is realized through initially ion of ${}_{16}^{32}\text{S}^+$ capturing an electron

Beta disintegration

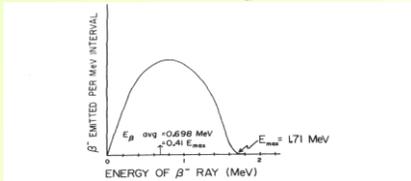


FIGURE 5.5. β^- -ray spectrum emitted from ${}_{15}^{32}\text{P}$. The average energy of the particles is 0.694 MeV. The abscissa may be alternatively labeled " $1.71 - \text{neutrino energy (MeV)}$ " to make the curve indicate the neutrino spectrum.

- Average kinetic energy of the β^+ or β^- particle is typically $0.3-0.4 E_{\text{max}}$; $1/3 E_{\text{max}}$ is often used for purposes of roughly estimating absorbed dose
- Neutrino is not included in dose estimates due to almost zero rest mass energy and no charge

Beta disintegration

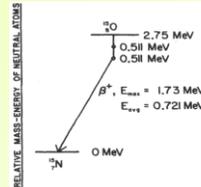
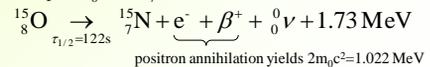


FIGURE 5.7. Atomic energy-level diagram for β^+ -emissions by ${}_{8}^{15}\text{O}$. The vertical line segments below the ${}_{8}^{15}\text{O}$ energy level indicate the loss of the rest-mass energy of the β^+ and the corresponding valence electron released by the atom during disintegration.

- In β^+ disintegration valence electron is emitted simultaneously
- Example: ${}_{8}^{15}\text{O} \rightarrow {}_{7}^{15}\text{N}$



Absorbed dose from β^- disintegration

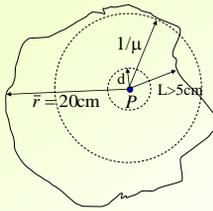
- Under CPE condition $D=nE_{\text{avg}}$ MeV/g for n disintegrations per gram of medium
- Any additional contributions to energy deposition due to γ -rays must be included for RE condition
- Radiative losses by β -rays, such as bremsstrahlung and in-flight annihilation, are ignored

Example 5.1

- Uniformly distributed β^- and γ -ray source
- The rest-mass loss is spent
 - half in 1-MeV γ -ray production and
 - half in β^- -decay, for which $E_{\text{max}}=5\text{MeV}$ and $E_{\text{avg}}=2\text{MeV}$
- The point of interest P is located $>5\text{cm}$ inside the boundary of the object, at an average distance $\bar{r}=20\text{cm}$ from the boundary.
- $\mu_{\text{en}}=0.0306\text{cm}^{-1}$ and $\mu=0.0699\text{cm}^{-1}$ for the γ -rays
- A total energy of 10^{-2}J converted from rest mass in each kg of the object
- Estimate the absorbed dose at P

Example 5.1

- For β -ray $E_{\max}=5$ MeV corresponds to maximum particle range (Appendix E) $d \sim 2.6$ cm $\ll 1/\mu=14.3$ cm
- CPE exists at P, therefore dose due to β -rays:



$$D_{\beta} = E_{\text{avg}} \times \frac{1}{2} \times 10^{-2} \text{ J/kg} = 10^{-2} \text{ J/kg}$$

- For γ -ray 20cm is not $\gg 1/\mu=14.3$ cm
- RE does not exist at P, therefore have to use absorbed fraction:

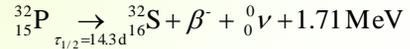
$$AF \cong 1 - e^{-\mu_a r} = 0.46$$

$$D_{\gamma} = 0.46 \times (1/2) \times 10^{-2} \text{ J/kg} = 2.3 \times 10^{-3} \text{ J/kg}$$

$$D_{\text{tot}} = (2.3 + 10) \times 10^{-3} \text{ J/kg} = 1.23 \times 10^{-2} \text{ J/kg}$$

Example 5.2

- What is the absorbed dose rate (Gy/h) at the center of a sphere of water 1 cm in radius, homogeneously radioactivated by $^{32}_{15}\text{P}$, with 6×10^5 disintegrations per second occurring per gram of water? (Assume time constancy)



Example 5.2

- $E_{\max}=1.71$ MeV corresponds to maximum particle (β^{-}) range of ~ 0.8 cm < 1 cm
- CPE condition
- Absorbed dose rate: $\dot{D} = \dot{N} \times E_{\text{avg}}$

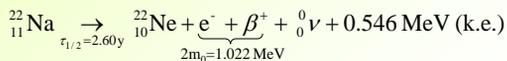
$$\begin{aligned} \dot{D} &= 6 \times 10^5 \frac{\text{dis}}{\text{g sec}} \times 0.694 \frac{\text{MeV}}{\text{dis}} \\ &= 4.164 \times 10^5 \frac{\text{MeV}}{\text{g sec}} \times 3600 \frac{\text{s}}{\text{hr}} \times 1.602 \times 10^{-10} \frac{\text{Gy}}{\text{MeV/g}} \\ &= 0.24 \text{ Gy/h} \end{aligned}$$

Electron-capture transitions

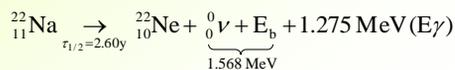
- Parent nucleus captures its own atomic electron from K-shell ($\sim 90\%$ probability) or L-shell ($\sim 10\%$ probability) and emits monoenergetic neutrino
- Resulting shell vacancy is filled with electron from a higher orbit, leading to emission of a fluorescence x-ray
- Process competing with β^{+} disintegrations

Electron-capture transitions

- Example: $^{22}_{11}\text{Na} \rightarrow ^{22}_{10}\text{Ne}$ with half-life for both branches of 2.60 years
- β^{+} branch



- EC branch



Electron-capture transitions

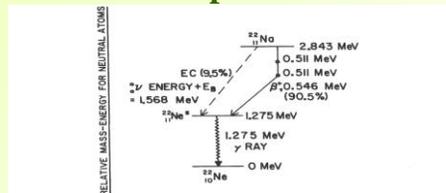


FIGURE 5.8. Atomic energy-level diagram for β^{+} and EC disintegration of $^{22}_{11}\text{Na}$ ($\tau_{1/2}=2.60\text{y}$) \rightarrow $^{22}_{10}\text{Ne}$. ($E_{\beta_{\max}} = 0.546 \text{ MeV}$; $E_{\gamma} = 0.216 \text{ MeV}$.)

- Binding energy for K-shell $E_b \sim 1$ keV
- For β^{+} to occur the minimum atomic mass decrease of $2m_0$ between the parent and daughter nuclei is required to supply β^{+} with kinetic energy; EC does not have this requirement

Absorbed dose for EC process

- Most of the energy is carried away by neutrino
- The only available energy for dose deposition comes from electron binding term E_b , which is very small compare to that of neutrino

Internal conversion

- An excited nucleus can impart its energy directly to its own atomic electron, which then escapes with the net kinetic energy of $h\nu - E_b$ ($h\nu$ is the excitation energy)
- No photon is emitted in this case
- Process competing with γ -ray emission
- Internal conversion coefficient is the ratio of N_e/N_γ

Internal conversion

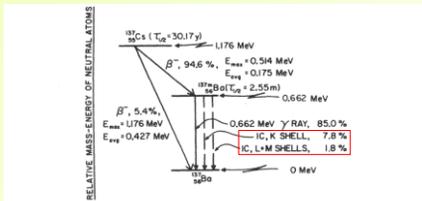


FIGURE 5.9. Atomic energy-level diagram for $^{137}\text{Cs} \rightarrow ^{137}\text{Ba}$, illustrating competition between γ -ray emission and internal conversion. Percentages all refer to disintegrations of parent atoms of ^{137}Cs . ($\alpha/\gamma = 0.0916$, $R/(L + M + \dots) = 4.41$.)

- Example: $^{137}_{55}\text{Cs} \rightarrow ^{137}_{56}\text{Ba}$

Absorbed dose for internal conversion

- If IC occurs in competition with γ -ray emission, it results in increase in absorbed dose in small objects (CPE condition) due to release of electron locally depositing the energy

$$E_{IC} = h\nu - E_b$$

- In addition electron binding energy is contributed to the dose unless it escapes as a fluorescence x-ray

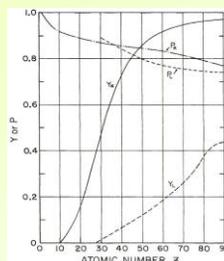
Absorbed dose for internal conversion

- If the fraction $p = 1 - AF$ of these fluorescence x-rays escape, then the energy contributed to dose per IC event under CPE condition

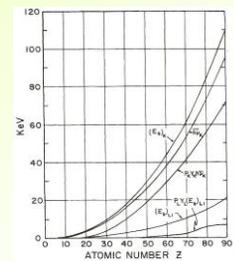
$$f_{IC} = h\nu - p_K Y_K h\nu_K - p_L Y_L h\nu_L$$

- Using straight-ahead approximation $p \cong e^{-\mu_m \bar{r}}$
- Values of fluorescence yield $Y_{K,L}$ and the mean emitted x-ray energies $h\nu_{K,L}$ are tabulated

Fluorescence data



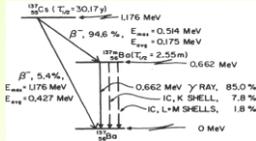
Fluorescence yield for K and L shells



Electron binding energies and mean fluorescence x-ray energies, K and L shells

Example 5.5

- A sphere of water 10 cm in diameter contains a uniform source of ^{137}Cs undergoing 10^3 dis/(g s). What is the absorbed dose at the center, in grays, for a 10-day period, due only to the decay of $^{137\text{m}}_{56}\text{Ba}$? Use the mean-radius straight-ahead approximation.



For γ -ray of 0.662 MeV in water $\mu_{\text{en}}=0.0327 \text{ cm}^{-1}$

Example 5.5

- First check RE condition: $1/\mu=30.6 \text{ cm} \gg r=5$
- Find absorbed fraction $AF \cong 1 - e^{-0.0327 \times 5} = 0.151$
- Dose in 10 days = $8.64 \times 10^5 \text{ s}$ is

$$D_{\gamma} = 10^3 \frac{\text{dis}}{\text{g s}} \times 0.85 \frac{\gamma\text{-rays}}{\text{dis}} \times 0.662 \frac{\text{MeV}}{\gamma\text{-ray}} \\ \times 1.602 \times 10^{-10} \frac{\text{Gy}}{\text{MeV/g}} \times 8.64 \times 10^5 \text{ s} \times AF \\ = 1.17 \times 10^{-2} \text{ Gy}$$

Example 5.5

- For the K-shell conversion process we need $Y_K=0.90$, $h\bar{\nu}_K = 0.032 \text{ MeV}$, and $\mu_{\text{en}}=0.13 \text{ cm}^{-1}$ for 0.032 MeV. Then $p_K \cong e^{-\mu_{\text{en}} r} = e^{-0.13 \times 5} = 0.52$ and the dose contribution is

$$D_{ic}^K = 10^3 \frac{\text{dis}}{\text{g s}} \times 0.078 \frac{IC(K)}{\text{dis}} \times (hv - p_K Y_K h\bar{\nu}_K) \frac{\text{MeV}}{IC(K)} \\ \times 1.602 \times 10^{-10} \frac{\text{Gy}}{\text{MeV/g}} \times 8.64 \times 10^5 \text{ s} \\ = 1.080(0.662 - 0.52 \times 0.90 \times 0.032) \times 10^{-2} \text{ Gy} \\ = 6.99 \times 10^{-3} \text{ Gy}$$

Example 5.5

- Similarly, for the L+M+...-shell conversion process we need $Y_K=0.90$, $h\bar{\nu}_L = E_b^L = 6 \text{ keV}$, and $\mu_{\text{en}}=24 \text{ cm}^{-1}$ for 6 keV. Then $p_L \cong e^{-24 \times 5} = 0$ and the corresponding dose contribution

$$D_{ic}^L = 10^3 \frac{\text{dis}}{\text{g s}} \times 0.078 \frac{IC(L)}{\text{dis}} \times (hv - p_L Y_L h\bar{\nu}_L) \frac{\text{MeV}}{IC(L)} \\ \times 1.602 \times 10^{-10} \frac{\text{Gy}}{\text{MeV/g}} \times 8.64 \times 10^5 \text{ s} \\ = 1.65 \times 10^{-3} \text{ Gy}$$

- The total absorbed dose is $D_{\text{tot}} = D_{\gamma} + D_{ic}^K + D_{ic}^L = 2.03 \times 10^{-2} \text{ Gy}$

Summary

- General approach to dose calculation within and outside of distributed radioactive source
 - Absorbed fraction
 - Radioactive disintegration processes and calculation of absorbed dose
 - Alpha disintegration
 - Beta disintegration
 - Electron-capture transitions
 - Internal conversion
- } Less important for dose deposition