Absorbed Dose in Radioactive Media

Chapter 5

F.A. Attix, Introduction to Radiological Physics and Radiation Dosimetry

Outline

• General dose calculation considerations, absorbed fraction
• Radioactive disintegration processes and associated dose deposition
  – Alpha disintegration
  – Beta disintegration
  – Electron-capture transitions
  – Internal conversion
• Summary

Introduction

• We are interested in calculating the absorbed dose in radioactive media, applicable to cases of
  – Dose within a radioactive organ
  – Dose in one organ due to radioactive source in another organ
• If conditions of CPE or RE are satisfied, dose calculation is straightforward
• Intermediate situation is more difficult but can be handled at least in approximations

Radiation equilibrium

a. The atomic composition of the medium is homogeneous
b. The density of the medium is homogeneous
c. The radioactive source is uniformly distributed
d. No external electric or magnetic fields are present

Charged-particle equilibrium

• Each charged particle of a given type and energy leaving the volume is replaced by an identical particle of the same energy entering the volume
• Existence of RE is sufficient condition for CPE
• Even if RE does not exist CPE may still exist (for a very large or a very small volume)

Limiting cases

• Emitted radiation typically includes both photons (longer range) and charged particles (shorter range)
• Assume the conditions for RE are satisfied
• Consider two limited cases based on the size of the radioactive object
Limiting cases: small object

- A radioactive object V having a mean radius not much greater than the maximum charged-particle range \( d \)
- CPE is well approximated at any internal point \( P \) that is at least a distance \( d \) from the boundary of \( V \)
- If \( d \approx l/\mu \) for the \( \gamma \)-rays, the absorbed dose \( D \) at \( P \) approximately equals to the energy per unit mass of medium that is given to the charged particles in radioactive decay (less their radiative losses)
- The photons escape from the object and are assumed not to be scattered back by its surroundings

Limiting cases: large object

- A radioactive object \( V \) with mean radius \( \gg l/\mu \) for the most penetrating \( \gamma \)-rays
- RE is well approximated at any internal point \( P \) that is far enough from the boundary of \( V \) so \( \gamma \)-ray penetration through that distance is negligible
- The dose at \( P \) will then equal the sum of the energy per unit mass of medium that is given to charged particles plus \( \gamma \)-rays in radioactive decay

Absorbed fraction

- An intermediate-size radioactive object \( V \)
- Dose at \( P \) will then equal the sum of the energy per unit mass of medium that is given to charged particles plus dose from some \( \gamma \)-rays
- To estimate the dose from \( \gamma \)-rays define absorbed fraction:

\[
AF = \frac{\gamma \text{-ray radiant energy absorbed in target volume}}{\gamma \text{-ray radiant energy emitted by source}}
\]

Intermediate-size radioactive object

- Consider volume \( V \) filled by a homogeneous medium and a uniformly distributed \( \gamma \)-source
- The volume may be surrounded by
  - Case 1: infinite homogeneous medium identical to \( V \), but non-radioactive (an organ in the body)
  - Case 2: infinite vacuum (object in air)

Case 1

- Reciprocity theorem: energy spent in \( dv \) due to the source in \( dv' \):
  \[
  \epsilon_{dv',dv} = \epsilon_{dv,dv'} \quad \text{and} \quad \epsilon_{dv,V} = \epsilon_{V,dv}
  \]
- No source in \( dv'' \)
- Define \( \bar{R}_d \), as the expectation value of the \( \gamma \)-ray radiant energy emitted by the source in \( dv \), and \( \bar{R}_{dv,V} \), the part of that energy that is spent in \( V \) (source \( dv \) and target \( V \)
Case 1 continued

- The absorbed fraction with respect to source $dv$ and target $V$ is:
  $$ AF_{dv,V} = \frac{\bar{e}_{dv,V}}{\bar{R}_{dv}} $$

- Estimates reduction in absorbed dose relative to RE condition
- For very small radioactive objects ($V \rightarrow dv$) this absorbed fraction approaches zero; for an infinite radioactive medium it equals unity

Case 1 example

- Dose calculations published in MIRD reports
- The larger the radius, the lower the $\gamma$-ray energy – the closer to RE condition ($AF\approx 1$)

Case 2

- To obtain a crude estimate of the dose at some point $P$ within a uniformly $\gamma$-active homogeneous object, it may suffice to obtain the average distance $\bar{r}$ from the point to the surface of the object by
  $$ \bar{r} = \frac{1}{4\pi} \int_0^\pi \int_0^{2\pi} r \sin \theta d\theta d\beta $$

- Then one may employ $\mu_{\text{en}} = \bar{\mu}$ in the straight-ahead approximation to obtain
  $$ AF_{dv,V} \approx 1 - e^{-\mu_{\text{en}}\bar{r}} $$

Case 1 continued

- Reduction in the absorbed dose due to $\gamma$-rays energy escaping from $V$
- Can estimate $AF$ using mean effective attenuation coefficient $\bar{\mu}$ for $\gamma$-rays energy fluence through a distance $r$ in the medium
  $$ AF_{dv,V} = \frac{1}{4\pi} \int_0^\pi \int_0^{2\pi} (1 - e^{-\bar{\mu}r}) \sin \theta d\theta d\beta $$

- For poly-energetic sources have to find an average value of the absorbed fraction

Case 2

- More difficult to calculate
- The reciprocity theorem is only approximate due to the lack of backscattering
- Dose is lower than in Case 1

MIRD tables

- AF is incorporated in “S-value” tabulated for each radionuclide and source-target configuration used in nuclear medicine
- Dose absorbed in any organ (target) due to a source in some other organ is calculated based on cumulative activity, $D=AxS$
- S values are typically calculated by Monte Carlo technique
- Straightforward approach, but accuracy is limited
MIRD tables

- The accuracy is continuously improved through revisions and updates
- Example: kidneys represent a frequent source of radiopharmaceutical uptake in both diagnostic and therapeutic nuclear medicine

Radioactive disintegration processes

- Radioactive nuclei undergo transformations to a more stable state through expulsion of energetic particles
- Example: decay of radium to radon, presented by the mass-energy balance equation
- Each of the elemental terms represents the rest mass of a neutral atom of that element
- Energy term represents kinetic energy released

Alpha disintegration

- Occurs mainly in heavy nuclei
- Example: decay of radium to radon, presented by the mass-energy balance equation
- Each of the elemental terms represents the rest mass of a neutral atom of that element
- Energy term represents kinetic energy released

Absorbed dose from $\alpha$-disintegration

- Take into account both branches through the average branching ratios
- For radium average kinetic energy given to charged particles per disintegration
- For CPE condition in a small (1 cm) radium-activated object: $D=4.77\times n\text{ MeV/g}$, where $n$ = number of disintegrations per gram of the matter
- For RE condition (large object): $D=4.78\times n\text{ MeV/g}$, includes $\gamma$-ray energy
Beta disintegration

- Nuclei having excess of neutrons typically emit an electron, $\beta^-$ particle; atomic number $Z$ is increased by 1
- Nuclei having excess of protons typically emit positron, $\beta^+$ particle; atomic number $Z$ is decreased by 1
- Nucleus is left in an exited state, emitting one or more $\gamma$-rays
- Example:

$$^{32}_{15}P \rightarrow ^{32}_{16}S + e^- + 0\nu + 1.71\text{MeV}$$

• Kinetic energy $1.71\text{MeV}$ is shared between $\beta^-$ and neutrino
• Charge balance is realized through initially ion of $^{32}_{16}S^+$ capturing an electron

Absorbed dose from $\beta$- disintegration

- Under CPE condition $D = nE_{\text{avg}} \text{MeV/g}$ for $n$ disintegrations per gram of medium
- Any additional contributions to energy deposition due to $\gamma$-rays must be included for RE condition
- Radiative losses by $\beta$-rays, such as bremsstrahlung and in-flight annihilation, are ignored

Example 5.1

• Uniformly distributed $\beta$- and $\gamma$-ray source
• The rest-mass loss is spent
  – half in 1-MeV $\gamma$ -ray production and
  – half in $\beta^-$-decay, for which $E_{\text{max}}=5$ MeV and $E_{\text{avg}}=2$ MeV
• The point of interest $P$ is located $>5$ cm inside the boundary of the object, at an average distance $r \approx 20$ cm from the boundary.
• $\mu_{\text{in}}=0.0306 \text{ cm}^{-1}$ and $\mu_{\gamma}=0.0699 \text{ cm}^{-1}$ for the $\gamma$ -rays
• A total energy of $10^2 \text{ J}$ converted from rest mass in each kg of the object
• Estimate the absorbed dose at $P$
Example 5.1

- For β-ray $E_{\text{max}}=5 \text{ MeV}$ corresponds to maximum particle range (Appendix E) $d \approx 2.6 \text{ cm} \ll 1/\mu = 14.3 \text{ cm}$
- CPE exists at P, therefore dose due to β-rays:
  
  \[ D_\beta = E_{\text{avg}} \times \frac{1}{2} \times 10^{-3} \text{J/kg} = 10^{-3} \text{J/kg} \]
  
  - For γ-ray 20 cm is not $\gg 1/\mu = 14.3 \text{ cm}$
  - RE does not exist at P, therefore have to use absorbed fraction:

Example 5.2

- $E_{\text{max}} = 1.71 \text{ MeV}$ corresponds to maximum particle ($\beta^-$) range of ~0.8 cm $< 1 \text{ cm}$
- CPE condition
- Absorbed dose rate:
  
  \[ D_\beta = 0.46 \times (1/2) \times 10^{-3} \text{J/kg} = 2.3 \times 10^{-3} \text{J/kg} \]
  
  \[ D_{\text{eff}} = (2.3 + 10) \times 10^{-3} \text{J/kg} = 1.23 \times 10^{-2} \text{J/kg} \]

Electron-capture transitions

- Example: $^{22}_{11} \text{Na} \rightarrow ^{22}_{10} \text{Ne}$ with half-life for both branches of 2.60 years
  - $\beta^+$ branch
    
    \[ ^{22}_{11} \text{Na} \rightarrow ^{22}_{10} \text{Ne} + e^+ + \beta^+ + \gamma + 0.546 \text{ MeV (k.e.)} \]
    
    $2m_0 = 1.022 \text{ MeV}$
  
  - EC branch
    
    \[ ^{22}_{11} \text{Na} \rightarrow ^{22}_{10} \text{Ne} + 0^+ \nu + E_\beta + 1.275 \text{ MeV (E}\gamma) \]
    
    $1.568 \text{ MeV}$

Electron-capture transitions

- Binding energy for K-shell $E_b \approx 1 \text{ keV}$
- For $\beta^+$ to occur the minimum atomic mass decrease of $2m_0$ between the parent and daughter nuclei is required to supply $\beta^+$ with kinetic energy; EC does not have this requirement
Absorbed dose for EC process

- Most of the energy is carried away by neutrino
- The only available energy for dose deposition comes from electron binding term $E_b$, which is very small compared to that of neutrino

Internal conversion

- An excited nucleus can impart its energy directly to its own atomic electron, which then escapes with the net kinetic energy of $h\nu - E_0$ ($h\nu$ is the excitation energy)
- No photon is emitted in this case
- Process competing with $\gamma$-ray emission
- Internal conversion coefficient is the ratio of $N_e/N_\gamma$

Internal conversion

- Example: $^{137}\text{Cs} \rightarrow ^{137}\text{Ba}$

Absorbed dose for internal conversion

- If IC occurs in competition with $\gamma$-ray emission, it results in increase in absorbed dose in small objects (CPE condition) due to release of electron locally depositing the energy

$$E_{KC} = h\nu - E_b$$
- In addition electron binding energy is contributed to the dose unless it escapes as a fluorescence x-ray

Absorbed dose for internal conversion

- If the fraction $p = 1 - AF$ of these fluorescence x-rays escape, then the energy contributed to dose per IC event under CPE condition

$$f_w = h\nu - p_k Y_k h\nu_K - p_L Y_L h\nu_L$$
- Using straight-ahead approximation $p \approx e^{-\mu\rho}$
- Values of fluorescence yield $Y_{K,L}$ and the mean emitted x-ray energies $h\nu_{K,L}$ are tabulated

Fluorescence data

- Fluorescence yield for K and L shells
- Electron binding energies and mean fluorescence x-ray energies, K and L shells
Example 5.5

- A sphere of water 10 cm in diameter contains a uniform source of $^{137}$Cs undergoing $10^3$ dis/(g s).
- What is the absorbed dose at the center, in grays, for a 10-day period, due only to the decay of $^{137m}$Ba?
- Use the mean-radius straight-ahead approximation.

For $\gamma$-ray of 0.662 MeV in water $\mu_w=0.0327$ cm$^{-1}$.

Example 5.5

- First check RE condition: $1/\mu=30.6$ cm $>> r=5$
- Find absorbed fraction $AF \approx 1-e^{-0.0327 \times 5} \approx 0.151$
- Dose in 10 days $= 8.64 \times 10^5$ s is $151.01\times 0.0327 \approx 4.92$ Gy

Example 5.5

- Similarly, for the L+M+…-shell conversion process we need $Y_k=0.90$, $h\nu_k=0.032$ MeV, and $\mu_w=24$ cm$^{-1}$ for 6 keV. Then $p_k \equiv e^{-2\nu_k} = 0$ and the corresponding dose contribution

$$D_L = 10^5 \frac{\text{dis}}{\text{g s}} \times 0.078 \frac{\text{IC}(L)}{\text{dis}} \times (h/\nu_k Y_k h\nu_k) \frac{\text{MeV}}{\text{IC}(L)} \times 1.602 \times 10^{-10} \frac{\text{Gy}}{\text{MeV} / \text{g}} \times 8.64 \times 10^5 s \times AF$$

$$= 1.65 \times 10^{-7} \text{Gy}$$

- The total absorbed dose is $D_{\text{tot}} = D_\gamma + D_x + D_L + D_K = 2.03 \times 10^{-7} \text{Gy}$

Summary

- General approach to dose calculation within and outside of distributed radioactive source
  - Absorbed fraction
  - Radioactive disintegration processes and calculation of absorbed dose
    - Alpha disintegration
    - Beta disintegration
    - Electron-capture transitions
    - Internal conversion 
    - Less important for dose deposition