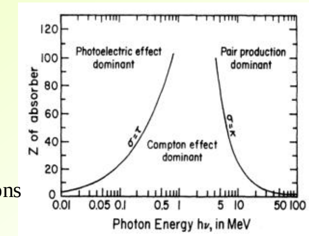
Gamma- and X-Ray Interactions in Matter

Chapter 7

F.A. Attix, Introduction to Radiological Physics and Radiation Dosimetry

Photon interactions in matter

- Compton effect
- Photoelectric effect
- Pair production
- Rayleigh (coherent) scattering
- Photonuclear interactions

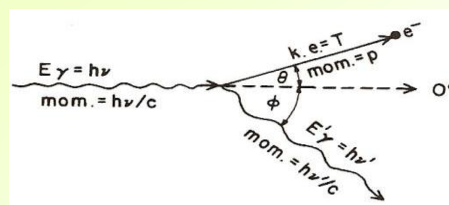


Kinematics
Interaction cross sections
Energy-transfer cross sections
Mass attenuation coefficients

Compton interaction

- Inelastic photon scattering by an electron
- Main assumption: the electron struck by the incoming photon is *unbound* and *stationary*
  - The largest contribution from binding is under condition of high Z, low energy
  - Under these conditions photoelectric effect is dominant
- Consider two aspects: kinematics and cross sections

Compton interaction: Kinematics



- Inelastic collision
- After the collision the electron departs at angle θ, with kinetic energy T and momentum p
- The photon scatters at angle φ with a new, lower quantum energy $h\nu'$ and momentum $h\nu'/c$

Compton interaction: Kinematics

- An earlier theory of γ-ray scattering by Thomson, based on observations only at low energies, predicted that the scattered photon should always have the same energy as the incident one, regardless of $h\nu$ or φ
- The failure of the Thomson theory to describe high-energy photon scattering necessitated the development of Compton's theory

Compton interaction: Kinematics

- The collision kinetics is based upon conservation of both energy and momentum
- Energy conservation requires

$$T = h\upsilon - h\upsilon'$$

- Conservation of momentum along the (0°) direction

$$h\upsilon = h\upsilon' \cos\varphi + pc\cos\theta$$

- Conservation of momentum perpendicular to the direction of incidence:

$$h\upsilon' \sin\varphi = pc\sin\theta$$

Compton interaction: Kinematics

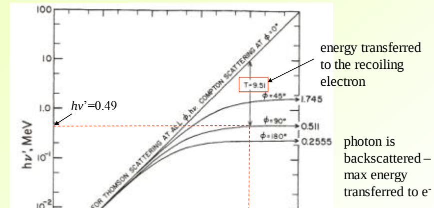
- pc can be written in terms of T : $pc = \sqrt{T(T + 2\, m_0 c^2)}$ where m_0 is the electron's rest mass
- We get a set of three simultaneous equations in these five parameters: $h\nu$, $h\nu'$, T, θ, and φ :

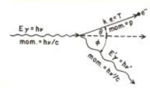
$$h\upsilon' = \frac{h\upsilon}{1 + \left(h\upsilon / m_0 c^2\right)\left(1 - \cos\varphi\right)}$$

$$T = h\upsilon - h\upsilon'$$

$$\cot\theta = \left(1 + \frac{h\upsilon}{m_0 c^2}\right)\tan\left(\frac{\varphi}{2}\right)$$

Compton interaction: Kinematics





When photon energy is lower than electron rest mass energy m_0c^2, relativistic effects do not contribute => pure elastic scattering

Compton interaction: Kinematics

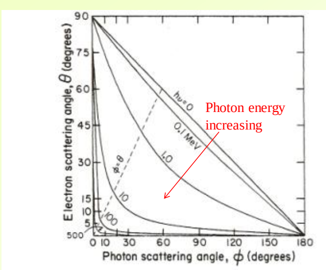
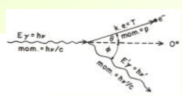


FIGURE 7.4. Relationship of the electron scattering angle θ to the photon scattering angle φ in the Compton effect, from Eq. (7.10). Curves are shown for the incident photon energies 0, 0.1, 1.0, 10, 100, and 500 MeV. The dashed line is the locus where θ = φ, when the electron and photon are scattered at equal angles on opposite sides of the incident photon's direction.



Dependence of θ on φ is a strong function of photon energy:

- Low photon energies $\theta \cong 90° - \varphi / 2$
- High photon energies: electrons are mostly forward scattered

Compton interaction: Cross sections
Interaction cross section

Cross section describes the probability of interaction

- Thomson: *elastic* scattering on a free electron, no energy is transferred to electron
- Differential cross section (per electron for a photon scattered at angle φ, per unit solid angle)

$$\frac{d_e\sigma_T}{d\Omega_\varphi} = \frac{r_0^2}{2}\left(1 + \cos^2\varphi\right)$$

max at $\varphi = 0, 180^o$
½ max at $\varphi = 90^o$

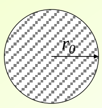
$r_0 = \dfrac{e^2}{m_0 c^2}$ - classical radius of electron

Compton interaction: Cross sections
Interaction cross section

Thomson: elastic scattering on free electron

- total cross section (integrated over all directions)

$$_e\sigma_T = \frac{8\pi r_0^2}{3} = 6.65 \cdot 10^{-25}\ \text{cm}^2/\text{electron}$$



$r_0 = \dfrac{e^2}{m_0 c^2}$ - classical radius of electron

Works well for low photon energies, << m_0c^2
Overestimates for photon energies > 0.01MeV (factor of 2 for 0.4MeV)

Compton interaction: Cross sections
Interaction cross section

- This cross section (can be thought of as an effective target area) is equal to the probability of a Thomson-scattering event occurring when a single photon passes through a layer containing one electron per cm^2
- It is also the fraction of a large number of incident photons that scatter in passing through the same layer, e.g., approximately 665 events for 10^{27} photons
- As long as the fraction of photons interacting in a layer of matter by *all processes combined* remains less than about 0.05, the fraction may be assumed to be proportional to absorber thickness; for greater thicknesses the exponential relation must be used

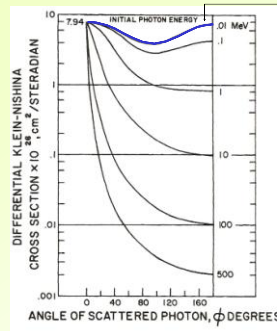
Compton interaction: Cross sections
Interaction cross section

- Klein-Nishina: Compton scattering on free electron but includes Dirac's quantum relativistic theory
- Differential cross section:

$$\frac{d_e\sigma_{K-N}}{d\Omega_\varphi} = \frac{r_0^2}{2}\left(\frac{h\nu'}{h\nu}\right)\left(\frac{h\nu}{h\nu'} + \frac{h\nu'}{h\nu} - \sin^2\varphi\right)$$

- For elastic scattering – reduces to Thomson's expression
- Needed at high photon energy

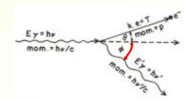
Compton interaction: Cross sections
Interaction cross section



Thomson's

$\frac{d_e\sigma_{K-N}}{d\Omega_\varphi}$ shows probability for a photon to be scattered at an angle φ

Photons with high energies tend to scatter in the forward direction ($\varphi \sim 0$)



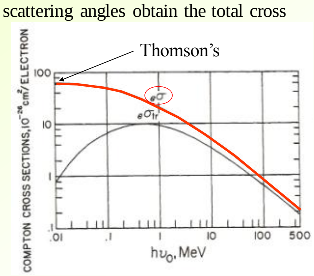
Compton interaction: Cross sections
Interaction cross section

Klein-Nishina: Compton scattering on free electron (includes Dirac's quantum relativistic theory)

Integrating over all photon scattering angles obtain the total cross section

$${}_e\sigma_{K-N} = 2\pi r_0^2\{...\}$$

{...} - depends on incident photon energy: higher energy => lower interaction probability



Compton interaction: Cross sections
Energy-transfer cross section

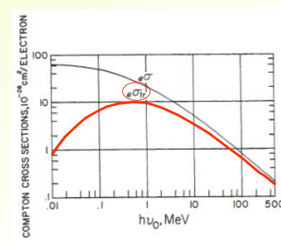
Total cross section –> fraction of energy diverted into Compton interactions –> fraction of energy transferred to electrons –> **dose**

Energy transfer cross section

$${}_e\sigma_{tr} = {}_e\sigma \cdot \frac{T}{h\nu}$$

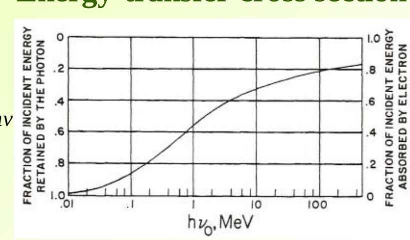
Average kinetic energy of recoiling electrons:

$$\overline{T} = h\nu \cdot \frac{{}_e\sigma_{tr}}{{}_e\sigma}$$



Compton interaction: Cross sections
Energy-transfer cross section

$\overline{h\nu'}/h\nu$



$\overline{T}/h\nu$

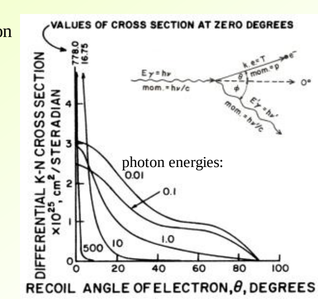
For $h\nu$ = 1.6 MeV – half of the photon energy is transferred to the electron ($\overline{T}$ = 0.8 MeV)

Compton interaction: Cross sections
Other cross sections

Differential K-N cross section for electron scattering at angle θ, per unit solid angle, per electron

$$\frac{d_e\sigma_{K-N}}{d\Omega_\theta}$$

For high photon energies electrons are preferentially forward scattered ($\theta = 0$)



Compton interaction: Cross sections
Other cross sections

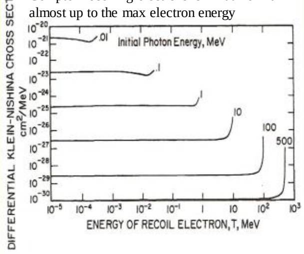
Probability that a single photon will have Compton interaction and transfers energy between T and $T+dT$

$$\frac{d_e\sigma_{K-N}}{dT} \text{ in } cm^2 MeV^{-1} e^{-1}$$

Energy distribution of electrons, averaged over all θ

T_{max} -> $h\nu$-0.2555 MeV for high $h\nu$

The distribution of kinetic energies given to the Compton recoiling electrons is ~ flat from 0 almost up to the max electron energy



Compton interaction: Mass attenuation coefficient

Cross section per electron (no Z dependence due to free electron assumption) $\quad {}_e\sigma \propto Z^0$

Cross section per atom $\quad {}_a\sigma \propto Z \cdot {}_e\sigma$

Cross section per unit mass (mass attenuation coefficient) $\quad \frac{\sigma}{\rho} = \frac{N_A Z}{A} \cdot {}_e\sigma$

N_A – Avogadro's constant; Z – number of electrons per atom; A – number of grams per mole of material; ρ – density in g/cm3

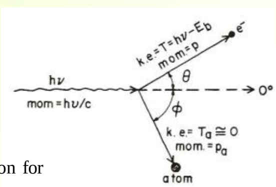
Photoelectric effect: Kinematics

Most important at low photon energies

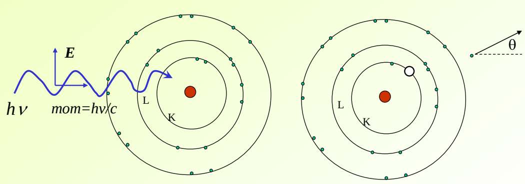
- Interaction with atomic-shell electrons tightly bound with potential energy $E_b < h\nu$
- Photon is completely absorbed
- Kinetic energy to electron:

$$T = h\nu - E_b$$

independent of scattering angle
- Atom acquires some momentum
- No universal analytical expression for cross sections



Photoelectric effect basics

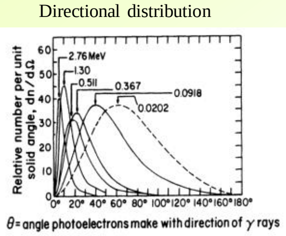


- Photon transfers its momentum $h\nu/c$ plus some transversal momentum due to the perpendicular electric field in the electromagnetic wave
- Final state = free electron + hole in the atomic shell

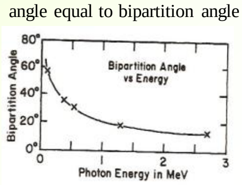
Photoelectric effect: Directional distribution

For higher photon energies electrons tend to scatter in forward direction (θ =0 is forbidden since it is perpendicular to the vector E)

Directional distribution



Half of all electrons is ejected within a forward cone of half angle equal to bipartition angle



Photoelectric effect: Cross sections
Interaction cross section

Total interaction cross section per atom, in cm^2/atom

$${}_a\tau \cong k \frac{Z^n}{(h\nu)^m}$$

$k = Const$

m, n – energy dependent

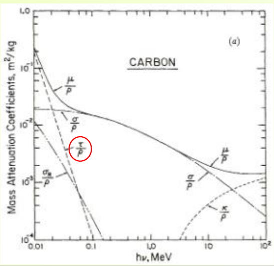
$m \cong 3,\ n \cong 4$ at $h\nu = 0.1$ MeV $\qquad \tau \cong \frac{Z^4}{(h\nu)^3}$

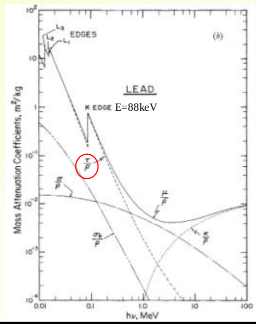
Mass attenuation coefficient

$$\frac{\tau}{\rho} \cong \left(\frac{Z}{h\nu}\right)^3$$

Photoelectric effect: Mass attenuation coefficient

$$\frac{\tau}{\rho} \cong \left(\frac{Z}{h\nu}\right)^3$$





Photoelectric effect: Cross sections
Energy-transfer cross section

Fraction of energy transferred to all electrons

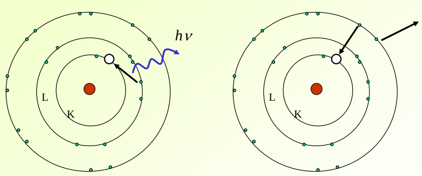
$$\frac{T}{h\nu} = \frac{h\nu - E_b}{h\nu}$$

Vacancy created by a photon in the inner shell has to be filled through Auger process, additionally contributing to kerma.

Final result:

$$\frac{\tau_{tr}}{\rho} = \frac{\tau}{\rho}\left[\frac{h\nu - P_K Y_K \cdot h\bar{\nu}_K - (1-P_K)P_L Y_L \cdot h\bar{\nu}_L}{h\nu}\right]$$

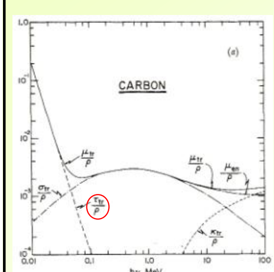
Photoelectric effect: Atom relaxation

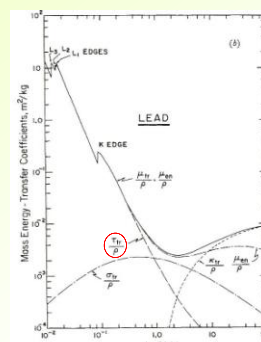


Excited atom relaxes its energy by
- fluorescence (emission of photons) or
- Auger process (emission of electron)

when the higher energy shell electrons move downward

Mass energy-transfer coefficients

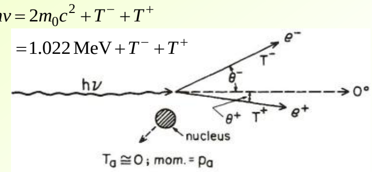




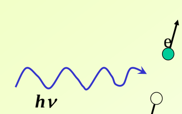
Pair production

- Photon is absorbed giving rise to electron and positron
- Occurs predominantly in Coulomb force field
  - usually near atomic nucleus
  - sometimes in a field of atomic electron
- Minimum photon energy required $2m_0c^2 = 1.022\,\text{MeV}$

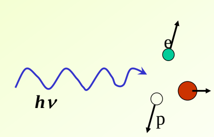
$$h\nu = 2m_0c^2 + T^- + T^+$$
$$= 1.022\,\text{MeV} + T^- + T^+$$



Pair production: Third body is needed



- There exists a reference frame where the total momentum of electron and positron is zero
- But photon momentum is always hν/c
- Third body needed for momentum conservation: electron or nucleus



Pair production in Nuclear Coulomb Force Field

Total cross section per atom

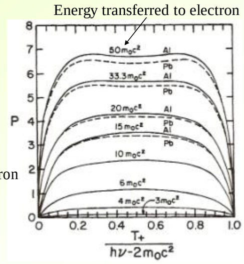
$$_a\kappa = \sigma_0 Z^2 \overline{P}$$

P – function of $h\nu$ and Z

$$\sigma_0 = \frac{r_0^2}{137} = \frac{1}{137}\left(\frac{e^2}{m_0c^2}\right)^2 = 5.80\text{x}10^{-28}\ \text{cm}^2/\text{electron}$$

Nuclear attraction and repulsion tend to give the positron slightly more energy than the electron, the difference being less than $0.0075Z$ MeV

Energy transferred to electron



Fraction of energy transferred to positron

Pair production in Electron Coulomb Force Field

Triplet production – higher threshold $4m_0c^2 = 2.044\,\text{MeV}$ required for conservation of momentum

Ratio of cross section for all electrons of the atom to nuclear cross section of the same atom is small:

$$\frac{\kappa(electron)}{\kappa(nucleus)} \cong \frac{1}{CZ}$$

C – parameter depending on energy, close to 1

For Pb the ratio is ~1%

Pair production: Cross sections

Total cross section for pair production per unit mass:

$$\left(\frac{\kappa}{\rho}\right)_{pair} = \left(\frac{\kappa}{\rho}\right)_{nuclear} + \left(\frac{\kappa}{\rho}\right)_{electron}$$

Pair production energy transfer coefficient:

$$\frac{\kappa_{tr}}{\rho} = \frac{\kappa}{\rho}\left(\frac{h\nu - 2m_0c^2}{h\nu}\right)$$

Rayleigh (coherent) scattering

- Photon is scattered by combined action of whole atom
- Photons do not lose energy, redirected through only a small angle
- No charged particles receive energy, no excitation produced
  => No contribution to kerma or dose

Atomic cross section: $\dfrac{\sigma_R}{\rho} \propto \dfrac{Z}{(h\nu)^2}$

Typical ratios of Rayleigh to total attenuation coefficient σ_R/μ

Element	$h\nu$ = 0.01 MeV	0.1 MeV	1.0 MeV
C	0.07	0.02	0
Cu	0.006	0.08	0.007
Pb	0.03	0.03	0.03

Photonuclear Interactions

- Photon with energy exceeding few MeV excites nucleus, which emits proton or neutron
- Contributes to kerma and dose
- Relative amount less that 5% of pair production
- Usually not included in dosimetry consideration
- Important for shielding design (neutrons)

Total coefficients for attenuation, energy transfer and absorption

Total mass attenuation coefficient for photon interactions - add probabilities for photoelectric effect, Compton effect, pair production and Rayleigh scattering

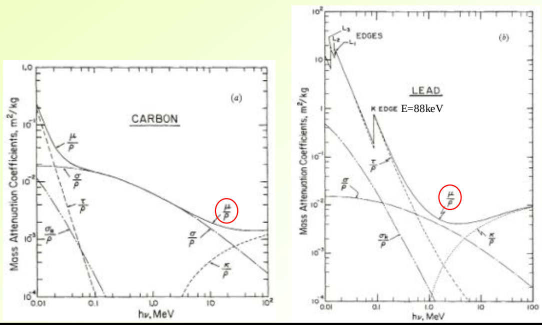
$$\frac{\mu}{\rho} = \frac{\tau}{\rho} + \frac{\sigma}{\rho} + \frac{\kappa}{\rho} + \frac{\sigma_R}{\rho}$$

Total mass energy-transfer coefficient:

$$\frac{\mu_{tr}}{\rho} = \frac{\tau_{tr}}{\rho} + \frac{\sigma_{tr}}{\rho} + \frac{\kappa_{tr}}{\rho}$$

$$= \frac{\tau}{\rho}\left[\frac{h\nu - p_K Y_k h\bar{\nu}_K}{h\nu}\right] + \frac{\sigma}{\rho}\left[\frac{\overline{T}}{h\nu}\right] + \frac{\kappa}{\rho}\left[\frac{h\nu - 2m_0c^2}{h\nu}\right]$$

Mass attenuation coefficients



Mass energy-absorption coefficient

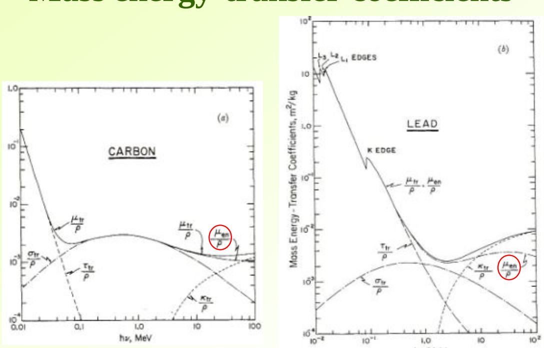
$$\frac{\mu_{en}}{\rho} = \frac{\mu_{tr}}{\rho}(1-g)$$

g - average fraction of secondary electron energy lost in radiative interactions

For low Z and high $h\nu$, g --> 0

Appendix D

Mass energy-transfer coefficients



Summary

- Compton effect
- Photoelectric effect
- Pair production

$\Big\}$ the most important

- Rayleigh (coherent) scattering – no energy transferred to the medium
- Photonuclear interactions – relevant at high energies