X-Ray Production and Quality

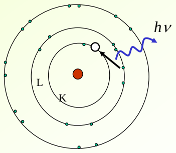
Chapter 9

F.A. Attix, Introduction to Radiological
Physics and Radiation Dosimetry

Outline

- Physics of x-ray generation
  - – Fluorescence x-rays
  - – Bremsstrahlung x-rays
- X-ray filtration
- Beam quality description
  - – Energy spectral distribution
  - – Attenuation curves and half-value layers

Fluorescence X-Rays



- Produced when an inner shell electron in an atom is removed as a result of
  - – Electron capture or internal conversion
  - – Photoelectric effect
  - – Hard collision with charged particle
- The minimum energy required to ionize the inner shell electron is E_b

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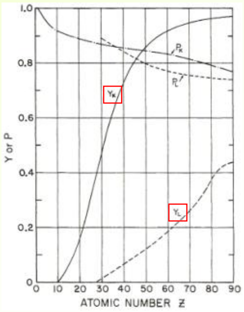
Fluorescence Yield

- The probability that a fluorescence x-ray will escape the atom – fluorescence yield Y_K, Y_L
- $Y_K = 0$ for Z<10 (Ne)
- $Y_L = 0$ for Z<29 (Cu)
- $Y_M \sim 0$ for all Z



Initiating event

- The minimum energy supplied has to be > E_b
- When electrons are used to produce fluorescence, they appear against a very strong background of continuous bremsstrahlung spectrum
- To obtain a relatively pure fluorescence x-ray source have to use either heavy charged particles or x-rays (photoelectric effect)
- Both methods are used for trace-element fluorescence analysis

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Initiating event

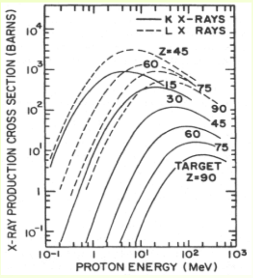


FIGURE 9.1. Atomic cross sections for fluorescence x-ray production by protons.

- When heavy charged particles are used the minimum energy required for K-shell ionization is:

$$T'_{\max} = \frac{4M_0 m_0 T}{(M_0 + m_0)^2} > (E_b)_K$$

- This estimate for a proton source results in energy ~460 x E_b
- Largely overestimates the required energy due to strong electron binding effectively increasing its mass

K-Fluorescence Photon Energy

- Secondary event: an electron from higher level fills out the K-shell vacancy (not all transitions are allowed by quantum mech. selection rules)
- A fluorescence photon with the energy equal to the difference between the two energy levels may be emitted
- X-ray fluorescence has a very narrow energy line; often used for calibration of spectrometers
- No angular correlation to the direction of the incident particle

K-Fluorescence Photon Energy

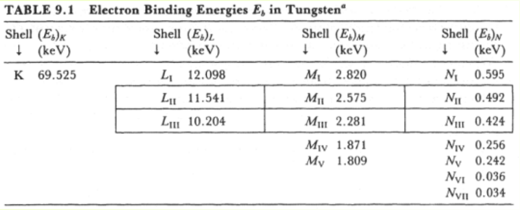


TABLE 9.1 Electron Binding Energies E_b in Tungsten[a]

Shell ↓	$(E_b)_K$ (keV)	Shell ↓	$(E_b)_L$ (keV)	Shell ↓	$(E_b)_M$ (keV)	Shell ↓	$(E_b)_N$ (keV)
K	69.525	L_I	12.098	M_I	2.820	N_I	0.595
		L_{II}	11.541	M_{II}	2.575	N_{II}	0.492
		L_{III}	10.204	M_{III}	2.281	N_{III}	0.424
				M_{IV}	1.871	N_{IV}	0.256
				M_V	1.809	N_V	0.242
						N_{VI}	0.036
						N_{VII}	0.034

Quantum mechanics selection rules (angular momentum conservation) allow transitions mainly from the levels shown in boxes

K-Fluorescence Photon Energy

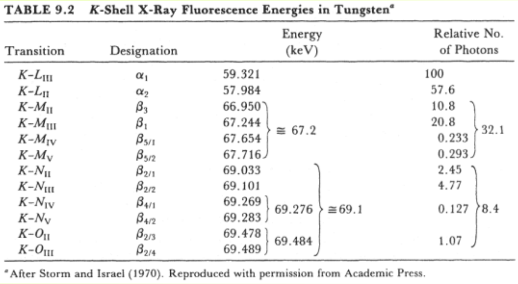


TABLE 9.2 *K*-Shell X-Ray Fluorescence Energies in Tungsten[a]

Transition	Designation	Energy (keV)			Relative No. of Photons	
$K\text{-}L_{III}$	α_1	59.321			100	
$K\text{-}L_{II}$	α_2	57.984			57.6	
$K\text{-}M_{II}$	β_3	66.950	$\cong$ 67.2		10.8	32.1
$K\text{-}M_{III}$	β_1	67.244			20.8	
$K\text{-}M_{IV}$	$\beta_{5/1}$	67.654			0.233	
$K\text{-}M_V$	$\beta_{5/2}$	67.716			0.293	
$K\text{-}N_{II}$	$\beta_{2/1}$	69.033		$\cong$ 69.1	2.45	8.4
$K\text{-}N_{III}$	$\beta_{2/2}$	69.101			4.77	
$K\text{-}N_{IV}$	$\beta_{4/1}$	69.269	69.276		0.127	
$K\text{-}N_V$	$\beta_{4/2}$	69.283				
$K\text{-}O_{II}$	$\beta_{2/3}$	69.478	69.484		1.07	
$K\text{-}O_{III}$	$\beta_{2/4}$	69.489				

[a] After Storm and Israel (1970). Reproduced with permission from Academic Press.

With typical spectroscopic resolution can observe two doublets separated by 10 keV

Angular distribution

- Fluorescence: emitted isotropically with respect to both energy and intensity
- Bremsstrahlung: emitted anisotropically, more forwardly directed, especially with increasing energy
- Generally true irrespective of Z,T, or target thickness

Angular distribution

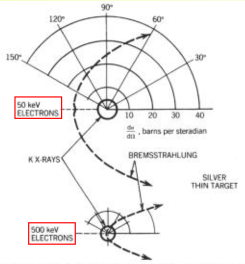


FIGURE 9.2. Comparison of the directional distributions of *K* x-rays (solid curves) and bremsstrahlung (dashed curves) for 50- and 500-keV electrons incident on a thin silver target. The relative magnitudes are shown in terms of differential cross sections for *K*-shell ionization and bremsstrahlung production, per unit solid angle in which the photons are emitted. (From Dick

Fluorescence yield vs. electron beam energy and Z

- No fluorescence production if $T<(E_b)_K$
- For $T>(E_b)_K$ all lines are generated with fixed relative strengths
- The efficiency of fluorescence production reaches maximum then decreases with T
- Depending on Z of the material can get more or less pure fluorescence beam with bremsstrahlung photons intermixing

Fluorescence yield vs. electron beam energy and Z

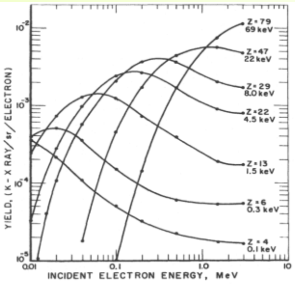


FIGURE 9.3. Dependence of *K* x-ray yield from thick targets of *Z* = 4 to 79 on incident electron energy. The approximate mean energy of the *K* fluorescence is also given for each curve. (From Sparrow

Fluorescence yield vs. electron beam energy

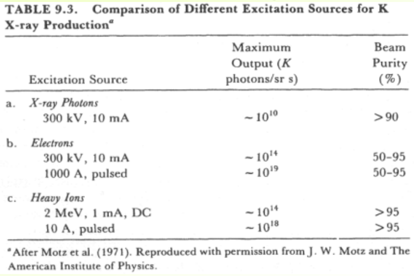


TABLE 9.3. Comparison of Different Excitation Sources for K X-ray Production[a]

Excitation Source	Maximum Output (*K* photons/sr s)	Beam Purity (%)
a. *X-ray Photons*		
300 kV, 10 mA	~10^{10}	>90
b. *Electrons*		
300 kV, 10 mA	~10^{14}	50–95
1000 A, pulsed	~10^{19}	50–95
c. *Heavy Ions*		
2 MeV, 1 mA, DC	~10^{14}	>95
10 A, pulsed	~10^{18}	>95

[a] After Motz et al. (1971). Reproduced with permission from J. W. Motz and The American Institute of Physics.

Method c produces the purest fluorescence beam

Bremsstrahlung production efficiency

- Thin target: the beam energy and stopping power is not changed; little energy is deposited
- Thick target: electron spends most of its energy or is completely stopped
- Bremsstrahlung production yield for thin target:

$$\frac{(dT/\rho dx)_r}{(dT/\rho dx)} = \frac{(dT/\rho dx)_r}{(dT/\rho dx)_c + (dT/\rho dx)_r} \cong \frac{TZ}{n+TZ}$$

- n depends on energy and material

Bremsstrahlung production efficiency

- High-Z targets convert a larger fraction of electron’s energy into bremsstrahlung
- Most of the energy is spent in collision interactions, resulting in target heating
- Tungsten is a common choice for target due to high Z and high melting point
- Production efficiency depends on energy:
  - At 100 keV 1% goes into bremsstrahlung
  - At 1 GeV 99% goes into bremsstrahlung

Bremsstrahlung production efficiency

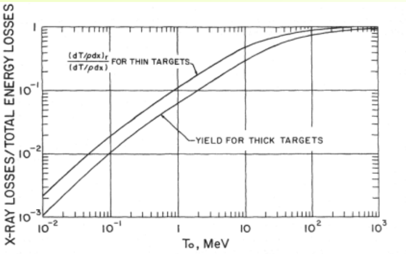


FIGURE 9.4. Fraction of electron energy losses that are spent in bremsstrahlung x-ray production in thin (upper curve) or thick (lower curve) tungsten targets (data after Berger and Seltzer, 1983). Upper curve: Eq. (9.2); lower curve: radiation yield (fraction of the incident electron kinetic energy T_0 that goes into x-ray production as the particle slows to a stop in a thick target).

Bremsstrahlung energy spectrum

$T_0 << m_0c^2$

Thin target

Maximum photon energy $h\nu_{max}=T_0$
Duane and Hunt’s law

Thick target

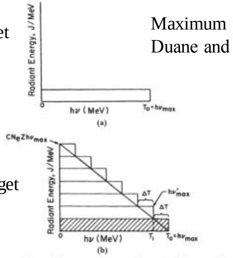


FIGURE 9.5. Bremsstrahlung radiant-energy spectrum from (*a*) a thin target, (*b*) a thick target irradiated by electrons of incident energy $T_0 \ll m_0c^2$.

Bremsstrahlung energy spectrum for thin target

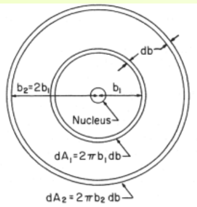


FIGURE 9.6. Classical explanation of the thin-target x-ray spectrum generated by nonrelativistic electrons. Consider a beam of electrons of kinetic energy T_0 entering the page perpendicularly, and each passing the nucleus at some distance (impact parameter) b. The differential interaction cross section when $b = b_1$ is proportional to the area $dA_1 = 2\pi b_1$ db. For $b = b_2 = 2b_1$, $dA_2 = 2\,dA_1$. Thus twice as many photons (N_2) come from interactions in dA_2 as the N_1 from dA_1. If the magnitude of the interaction (i.e., the x-ray quantum energy $h\nu$ produced) is assumed to be proportional to $1/b$, then $h\nu_1 = 2h\nu_2$. Therefore $N_1h\nu_1 = N_2h\nu_2$, and the x-ray radiant-energy spectrum should be flat, as it is observed to be.

If assume $hv \propto 1/b \Rightarrow Nhv = \text{const}$

Bremsstrahlung energy spectrum

- Thick target can be considered as a stack of thin targets, adequate in aggregate depth to stop the electron beam; the electrons lose their kinetic energy gradually by many small collision interactions
- The foils in the stack are assumed to become progressively thinner with increasing depth, so that each one reduces the beam energy by the same amount
- The collision stopping power increases as 1/T for decreasing energy
- Radiative losses are negligible as a mechanism for reducing the beam energy for $T_0 \ll m_0c^2$

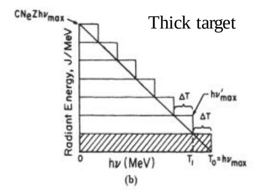
Bremsstrahlung energy spectrum

- Amount of radiant energy generated in a thick target can be crudely estimated by:

$$R \cong 1\times10^{-3} N_e Z T_0^2 \text{ MeV}$$

- Comparison with radiation yield in Appendix E, where $Y(T_0)=R/T_0N_e$ shows agreement within 30% with tabulated values

Bremsstrahlung energy spectrum



The array of rectangular areas (representing the x-ray outputs of all the imaginary individual foils comprising the thick target) can be fitted by a triangular envelope called the *Kramers spectrum,* having the formula

$$R'(hv) = CN_eZ(hv_{\max} - hv)$$

- Here *R'* is the Differential radiant-energy spectral distribution of bremsstrah-lung generated in the thick target of atomic number Z
- The area under triangle represents the total radiant energy of the unfiltered bremsstrahlung

Bremsstrahlung energy spectrum

- The area under triangle represents the total radiant energy of the unfiltered bremsstrahlung

$$R = \frac{C}{2} N_e Z (hv)^2_{\max} = \frac{C}{2} N_e Z T_0^2$$

- Comparing with analytical expression for R, find that C/2=1x10^{-3}. In Joules:

$$R \cong 1.6\times10^{-16} N_e Z T_0^2 \quad \text{J}$$

Bremsstrahlung energy spectrum

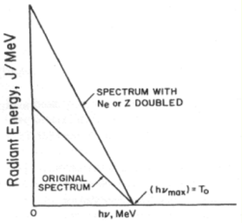


FIGURE 9.7*a*. Effect of doubling N_e or Z on the unfiltered bremsstrahlung x-ray spectrum.

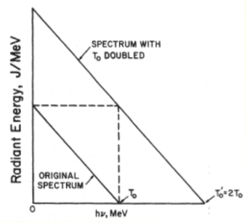


FIGURE 9.7*b*. Effect of doubling $T_0 = h\nu_{max}$ on the unfiltered bremsstrahlung x-ray spectrum.

Can observe graphic effect of changing the parameters

Bremsstrahlung energy spectrum

- For $T_0 >> m_0c^2$ (relativistic electrons) have to use more general Bethe-Heitler formula for radiative stopping power
- Resulting photon output spectrum can be expressed through differential cross-section

$$\frac{d\sigma_r}{d(h\nu)} \propto \frac{B_r}{h\nu}$$

- Here B_r is gradually decreasing dimensionless function (tabulated)

Bremsstrahlung energy spectrum

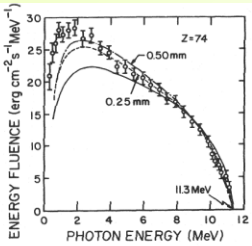


FIGURE 9.8. Bremsstrahlung intensity (energy-flux density) spectrum in the 0° direction for 11.3-MeV electrons on a 1.5-mm tungsten target, as measured with a Compton spectrometer (points). The Bethe–Heitler thin-target spectrum, modified by the photon absorption in window materials, is shown by the solid curve (lower). The dashed curves show corresponding theoretical *spectra for 10-mil ($\frac{1}{4}$-mm) and* 20-mil ($\frac{1}{2}$-mm) tungsten targets corrected for attenuation in the target material as well. (After Motz et al., 1953. Reproduced with permission from J. W. Motz

$T_0 >> m_0c^2$
There is less difference between thin and thick target spectra

Bremsstrahlung directional dependence

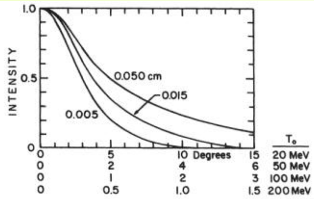


FIGURE 9.9. Ratio of bremsstrahlung intensity at angle θ to that at θ = 0 for three thicknesses of tungsten target, as calculated by Schiff (1946). Note that the curves for different energies T_0 differ only by a scale factor that is inversely proportional to T_0. (Reproduced by permission of The American Physical Society.)

The x rays tend to be emitted with an appreciable sideways component for low-energy electron beams, however, are more strongly forward as T_0 is increased

Bremsstrahlung directional dependence

- The forward peaking at high energies necessitates the use of a conical beam-flattening filter in linac x-ray beams for radiotherapy applications, even though the phenomenon becomes less pronounced for thicker targets
- Such a filter attenuates the beam less strongly as a function of distance away from its central axis, thus producing a beam of more uniform intensity over a useful area.
- Exact alignment of such a filter is critical

X-ray filtration

- Unfiltered x-ray beam contains both fluorescence (characteristic of the target) and bremsstrahlung x-rays
- The principle result of adding filter is the removal photons at energies where attenuation coefficient is largest (since the photoelectric interaction coefficient varies approximately as $1/(h\nu)^3$ in this energy range)
- Can narrow the spectral distribution progressively; beam is "hardened" by filtration

X-ray filtration

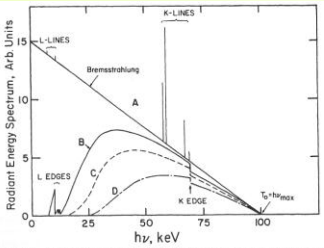


FIGURE 9.10. X-ray spectrum from 100-keV electrons on a thick tungsten target. Upper curve A: Unfiltered. B: Filtered through 0.01 mm W in escaping the target. C: Additionally filtered through 2 mm Al. D: Filtered through 0.15 mm Cu and 3.9 mm Al in addition to inherent target filtration. To avoid confusion, the K-fluorescence lines are not shown in curves B, C, and D, but are attenuated from their heights in curve A in the same proportion as the bremsstrahlung is attenuated at the same energies.

$T_0 < 300$ keV, mainly PE effect

X-ray filtration

- The most common x-ray filtering media are lead, tin, copper, and aluminum, which may be used singly or in combination.
- The advantage of combination filters is that they are generally capable of narrowing the spectrum to any desired degree while preserving more of the x-ray output than can be achieved with a single filtering material.
- The higher-Z filters provide strong filtering action but cause discontinuities at the shell binding energies. The lower-Z filters tend to smooth the resulting spectrum

X-ray filtration

- It is important in using combination filters that they be positioned in the beam in descending order of Z, going in the direction of the rays
- This allows each filter to remove the fluorescence x rays that originate in the higher-Z filter upstream from it
- Aluminum ($h\nu_K$ = 1.5 keV) is best for the final filter. Copper ($h\nu_K$ = 9 keV) fluorescence also is low enough in energy not to be detectable in most cases. However, neither tin ($h\nu_K$ = 29 keV) nor lead ($h\nu_K$ = 85 KeV) should be used without sufficient following filtration to remove the fluorescence, unless those photons are desired as part of the output

X-ray filtration

- At higher energies (T_0 > 300 keV) the photoelectric effect becomes less important than the Compton effect, and the total coefficient is less energy-dependent
- The filtering of an x-ray spectrum generated by MV electrons mainly removes the photons below a few hundred keV without greatly modifying the spectral shape at higher energies
- The use of a thick high-Z filter such as lead on MV x-ray beam tends to filter out the highest-energy photons (>4 MeV) through pair production, as well as the lowest through the photoelectric effect

X-ray beam quality

- The quality of the beam can be specified either in terms of its spectrum or its attenuation characteristics in a medium
- It is possible (with some limitations) to derive an x-ray spectrum from the shape of an attenuation curve, which is a "signature" for the related spectrum in a given medium
- Thus attenuation data can be used to characterize x-ray beams

Attenuation curves and HVLs

- To standardize such data the following conventions are generally followed:
  - Pure aluminum or copper is used as the attenuating medium, Al being preferred for T_0 < 120 keV and Cu for higher energies <0.5 MeV
  - Narrow-beam geometry is required (i.e., scattered rays from the attenuator must not reach the detector
  - The detector (e.g., ion chamber) must be air-equivalent, that is, must give a constant response per unit of exposure, independent of photon energy

Attenuation curves and HVLs

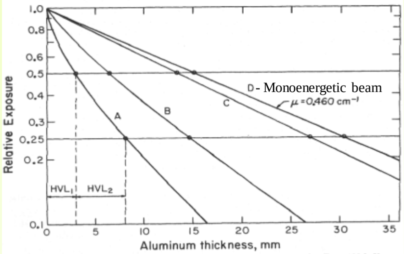


FIGURE 9.12. Approximate exposure-attenuation curves in aluminum for T_0 = 100-keV x-rays from a thick tungsten target, filtered by (A) 2 mm Al; (B) 0.15 mm Cu + 3.9 mm Al; and (C) 2 mm Sn + 0.5 mm Cu + 4 mm Al. (From data of Seelentag et al., 1979.) Also shown for comparison is the attenuation of 100-keV photons in aluminum (curve D). The first and second half-value layers are shown for curve A.

Half-value layers

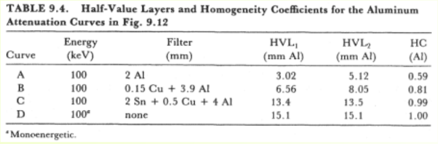


TABLE 9.4. Half-Value Layers and Homogeneity Coefficients for the Aluminum Attenuation Curves in Fig. 9.12

Curve	Energy (keV)	Filter (mm)	HVL_1 (mm Al)	HVL_2 (mm Al)	HC (Al)
A	100	2 Al	3.02	5.12	0.59
B	100	0.15 Cu + 3.9 Al	6.56	8.05	0.81
C	100	2 Sn + 0.5 Cu + 4 Al	13.4	13.5	0.99
D	100[a]	none	15.1	15.1	1.00

[a] Monoenergetic.

- The attenuation curves, can be reasonably well specified for radiological applications in terms of their *first and second half-value layers,* HVL_1, and HVL_2
- HVL_1, is defined as the thickness required to reduce the exposure by half, in narrow-beam geometry; HVL_2 is the thickness necessary to reduce it by half again under the same conditions
- *Homogeneity coefficient* (HC), defined as the ratio HVL_1/HVL_2. This approaches unity as the spectrum is narrowed by filtration to approach monochromaticity, in which case attenuation is exponential

Half-value layers

- Another quantity that is sometimes used in beam-quality specification is the *equivalent photon energy, hv_{eq}*, defined as the quantum energy of a monoenergetic beam having the same HVL_1, as the beam being specified
- The equivalent photon energy gives no more information than HVL_1, but gives it in a form that is especially useful in describing heavily filtered beams approaching monochromaticity

Half-value layers

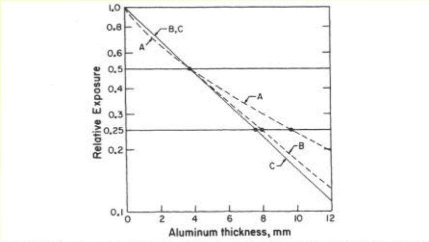


FIGURE 9.13. An example of the attenuation of three dissimilar x-ray beams having practically the same value of HVL_1 (3.8 mm) in aluminum. Curve A: 100 kV, 3 mm Al filter, HVL_2 = 5.82 mm. Curve B: 50 kV, 0.1 mm Pb + 4 mm Al filter, HVL_2 = 4.13 mm. Curve C: monoenergetic 37-keV beam, HVL_2 = 3.8 mm. Note that 37 keV is the equivalent photon energy for both beams A and B. (HVL data after Seelentag et al., 1979.)

Half-value layers

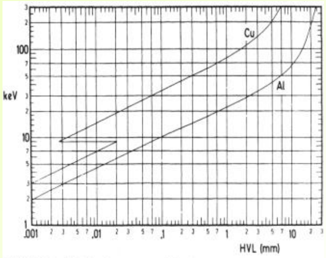


FIGURE 9.14. Equivalent photon energy vs. HVL_1 in copper or aluminum. (From Seelentag, et al., 1979. Reprinted with permission from the authors and the Gesellschaft für Strahlen- und Umweltforschung mbH, Munich.)

$$\frac{X}{X_0} = 0.5 = e^{-(\mu/\rho)_{eq} \cdot HVL_1 \cdot \rho}$$

$$\left(\frac{\mu}{\rho}\right)_{eq} = \frac{0.6931}{\rho \cdot HVL_1} \text{ cm}^2/\text{g}$$

Value of hv_{eq} corresponding to $(\mu/\rho)_{eq}$ can be obtained from Appendix D3

Attenuation curves

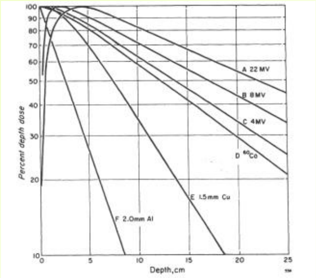


FIGURE 9.15. Variation of absorbed dose vs. depth in a water phantom for the following 10 × 10-cm^2 photon beams: A, 22 MV, 70-cm SSD; B, 8 MV, 100-cm SSD; C, 4 MV, 100-cm SSD; D, ^{60}Co γ-rays, 100-cm SSD; E, 200 keV, HVL_1 = 1.5 mm Cu, 50-cm SSD; F, 120 kV, HVL_1 = 2.0 mm Al, 15-cm SSD. (From Johns and Cunningham, 1983. Reproduced with permission from

Depth of Dmax increases with energy as the range of secondary electrons increases

Summary

- Physics of x-ray generation
  - Fluorescence x-rays
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  - Attenuation curves and half-value layers