X-Ray Production and Quality

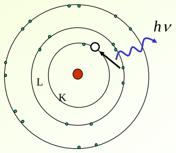
Chapter 9

F.A. Attix, Introduction to Radiological
Physics and Radiation Dosimetry

Introduction

- Physics of x-ray generation
  - Fluorescence x-rays
  - Bremsstrahlung x-rays
- Beam quality description
  - Hardness or penetrating ability
  - Energy spectral distribution
  - Biological effectiveness
  - X-ray filtration

Fluorescence X-Rays

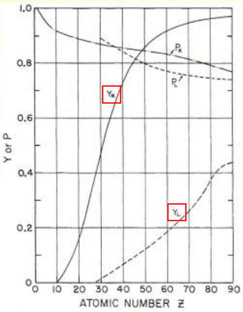


Video at https://www.youtube.com/watch?v=k8IoRx5G47A

- Produced when an inner shell electron in an atom is removed as a result of
  - Electron capture or internal conversion
  - Photoelectric effect
  - Hard collision with charged particle
- The minimum energy required to ionize the inner shell electron is E_b

Fluorescence Yield

- The probability that a fluorescence x-ray will escape the atom – fluorescence yield Y_K, Y_L
- $Y_K = 0$ for Z<10 (Ne)
- $Y_L = 0$ for Z<29 (Cu)
- $Y_M \sim 0$ for all Z



Initiating event

- The minimum energy supplied has to be > E_b
- When electrons are used to produce fluorescence, they appear against a very strong background of continuous bremsstrahlung spectrum
- To obtain a relatively pure fluorescence x-ray source have to use either heavy charged particles or x-rays (photoelectric effect)
- Both methods are used for trace-element fluorescence analysis

K-Fluorescence Photon Energy

- Secondary event: an electron from higher level fills out the K-shell vacancy (not all transitions are allowed by quantum mech. selection rules)
- A fluorescence photon with the energy equal to the difference between the two energy levels may be emitted
- X-ray fluorescence has a very narrow energy line; often used for calibration of spectrometers
- No angular correlation to the direction of the incident particle

K-Fluorescence Photon Energy

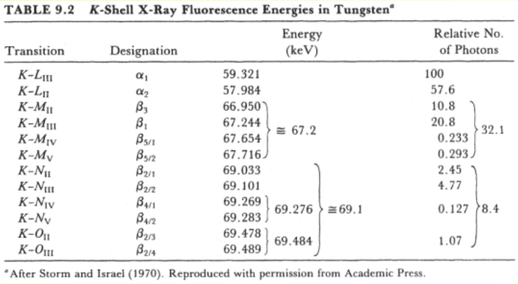


TABLE 9.2 *K*-Shell X-Ray Fluorescence Energies in Tungsten[a]

Transition	Designation	Energy (keV)			Relative No. of Photons	
$K\text{-}L_{III}$	α_1	59.321			100	
$K\text{-}L_{II}$	α_2	57.984			57.6	
$K\text{-}M_{II}$	β_3	66.950	≅ 67.2		10.8	32.1
$K\text{-}M_{III}$	β_1	67.244			20.8	
$K\text{-}M_{IV}$	$\beta_{5/1}$	67.654			0.233	
$K\text{-}M_{V}$	$\beta_{5/2}$	67.716			0.293	
$K\text{-}N_{II}$	$\beta_{2/1}$	69.033		≅69.1	2.45	8.4
$K\text{-}N_{III}$	$\beta_{2/2}$	69.101			4.77	
$K\text{-}N_{IV}$	$\beta_{4/1}$	69.269	69.276		0.127	
$K\text{-}N_{V}$	$\beta_{4/2}$	69.283				
$K\text{-}O_{II}$	$\beta_{2/3}$	69.478	69.484		1.07	
$K\text{-}O_{III}$	$\beta_{2/4}$	69.489				

[a] After Storm and Israel (1970). Reproduced with permission from Academic Press.

With typical spectroscopic resolution can observe two doublets separated by 10 keV

Angular distribution

- Fluorescence: emitted isotropically with respect to both energy and intensity
- Bremsstrahlung: emitted anisotropically, more forwardly directed, especially with increasing energy
- Generally true irrespective of Z,T, or target thickness

Angular distribution

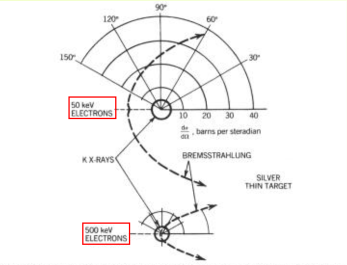
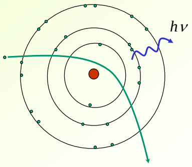


FIGURE 9.2. Comparison of the directional distributions of *K* x-rays (solid curves) and bremsstrahlung (dashed curves) for 50- and 500-keV electrons incident on a thin silver target. The relative magnitudes are shown in terms of differential cross sections for *K*-shell ionization and bremsstrahlung production, per unit solid angle in which the photons are emitted. (From Dick

Bremsstrahlung production



- In ~2-3 % of the cases in which the electron passes near the nucleus, an inelastic radiative interaction occurs in which an x-ray is emitted
- The electron is not only deflected, but gives a significant fraction (up to 100%) of its KE to the photon
- Such x-rays are referred to as bremsstrahlung, the German word for “braking radiation”

Video
https://www.youtube.com/watch?v=GBj1ln6x3LY

Bremsstrahlung production efficiency

- Thin target: the beam energy and stopping power is not changed; little energy is deposited
- Thick target: electron spends most of its energy or is completely stopped
- Bremsstrahlung production yield for thin target:

$$\frac{(dT/\rho dx)_r}{(dT/\rho dx)} = \frac{(dT/\rho dx)_r}{(dT/\rho dx)_c + (dT/\rho dx)_r} \cong \frac{TZ}{n+TZ}$$

- n depends on energy and material

Bremsstrahlung production efficiency

- High-Z targets convert a larger fraction of electron’s energy into bremsstrahlung
- Most of the energy is spent in collision interactions, resulting in target heating
- Tungsten is a common choice for target due to high Z and high melting point
- Production efficiency depends on energy:
  - At 100 keV 1% goes into bremsstrahlung
  - At 1 GeV 99% goes into bremsstrahlung

Bremsstrahlung production efficiency

Overall x-ray production efficiency is small for thin targets

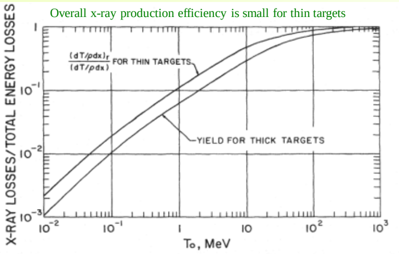
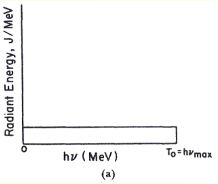


FIGURE 9.4. Fraction of electron energy losses that are spent in bremsstrahlung x-ray production in thin (upper curve) or thick (lower curve) tungsten targets (data after Berger and Seltzer, 1983). Upper curve: Eq. (9.2); lower curve: radiation yield (fraction of the incident electron kinetic energy T_0 that goes into x-ray production as the particle slows to a stop in a thick target).

Bremsstrahlung energy spectrum

Thin target, $T_0 << m_0c^2$



Maximum photon energy $h\nu_{max} = T_0$
Duane and Hunt's law

- For thin target the radiant-energy spectrum is constant over the whole energy range
- T_0 is the kinetic energy of the incident electron

Bremsstrahlung energy spectrum for thin target

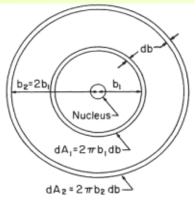


FIGURE 9.6. Classical explanation of the thin-target x-ray spectrum generated by nonrelativistic electrons. Consider a beam of electrons of kinetic energy T_0 entering the page perpendicularly, and each passing the nucleus at some distance (impact parameter) b. The differential interaction cross section when $b = b_1$ is proportional to the area $dA_1 = 2\pi b_1$ db. For $b = b_2 = 2b_1$, $dA_2 = 2\,dA_1$. Thus twice as many photons (N_2) come from interactions in dA_2 as the N_1 from dA_1. If the magnitude of the interaction (i.e., the x-ray quantum energy $h\nu$ produced) is assumed to be proportional to $1/b$, then $h\nu_1 = 2\,h\nu_2$. Therefore $N_1 h\nu_1 = N_2 h\nu_2$, and the x-ray radiant-energy spectrum should be flat, as it is observed to be.

If assume:

- the probability of interaction ~ b (through dA)
- the intensity of interaction (energy loss) $hv \propto 1/b$

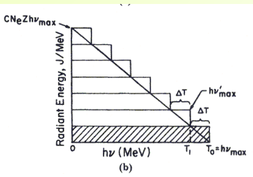
$\Rightarrow Nhv = \text{const}$

Bremsstrahlung energy spectrum

- Thick target can be considered as a stack of thin targets, adequate in aggregate depth to stop the electron beam; the electrons lose their kinetic energy gradually by many small collision interactions
- Radiative losses are negligible as a mechanism for reducing the beam energy for $T_0 \ll m_0c^2$

Bremsstrahlung energy spectrum

Thick target, $T_0 << m_0c^2$



Bremsstrahlung radiant-energy spectrum from electrons of incident energy $T_0 \ll m_0c^2$.

- The collision stopping power increases as 1/T for decreasing energy
- The foils in the stack are assumed to become progressively thinner with increasing depth, so that each one reduces the beam energy by the same amount

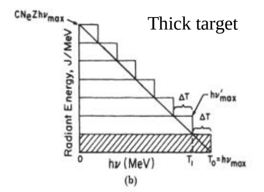
Bremsstrahlung energy spectrum

- For N_e incident electrons the radiant energy generated in a thick target can be crudely estimated by:

$$R \cong 1 \times 10^{-3} N_e Z T_0^2 \ \text{MeV}$$

- Constant has units of MV^{-1}
- Comparison with radiation yield in Appendix E, where $Y(T_0) = R/T_0N_e$ shows agreement within 30% with tabulated values

Bremsstrahlung energy spectrum



The array of rectangular areas (representing the x-ray outputs of individual foils comprising the thick target) can be fitted by a triangular envelope called the *Kramers spectrum*, described by formula

$$R'(h\nu) = CN_eZ(h\nu_{\max} - h\nu)$$

- Here *R*’ is the differential radiant-energy spectral distribution of bremsstrahlung generated in the thick target

Bremsstrahlung energy spectrum

- The area under triangle represents the total radiant energy of the unfiltered bremsstrahlung

$$R = \frac{C}{2}N_eZ(h\nu)^2_{\max} = \frac{C}{2}N_eZT_0^2$$

- Comparing with analytical expression for *R*, find that C/2=1x10^{-3}. In Joules:

$$R \cong 1.6\times10^{-16}N_eZT_0^2 \quad \mathrm{J}$$

Bremsstrahlung energy spectrum

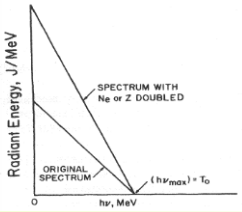


FIGURE 9.7a. Effect of doubling N_e or Z on the unfiltered bremsstrahlung x-ray spectrum.

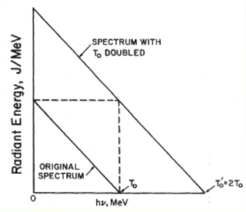


FIGURE 9.7b. Effect of doubling $T_0 = h\nu_{\max}$ on the unfiltered bremsstrahlung x-ray spectrum.

- Graphic effect of changing the parameters
- Never observed experimentally due to self-absorption within the target

Bremsstrahlung energy spectrum

- For $T_0 \gg m_0c^2$ (relativistic electrons) have to use more general Bethe-Heitler formula for radiative stopping power
- Resulting photon output spectrum can be expressed through differential cross-section

$$\frac{d\sigma_r}{d(h\nu)} \propto \frac{B_r}{h\nu}$$

- Here B_r is gradually decreasing dimensionless function (tabulated)

Bremsstrahlung energy spectrum

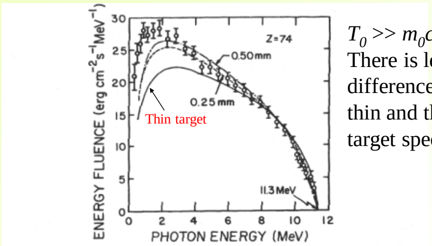


FIGURE 9.8. Bremsstrahlung intensity (energy-flux density) spectrum in the 0° direction for 11.3-MeV electrons on a 1.5-mm tungsten target, as measured with a Compton spectrometer (points). The Bethe–Heitler thin-target spectrum, modified by the photon absorption in window materials, is shown by the solid curve (lower). The dashed curves show corresponding theoretical *spectra for 10-mil ($\frac{1}{4}$-mm) and* 20-mil ($\frac{1}{2}$-mm) tungsten targets corrected for attenuation in the target material as well. (After Motz et al., 1953. Reproduced with permission from J. W. Motz

$T_0 \gg m_0c^2$

There is less difference between thin and thick target spectra

Bremsstrahlung directional dependence

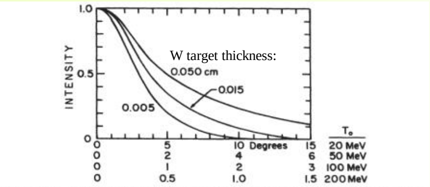


FIGURE 9.9. Ratio of bremsstrahlung intensity at angle θ to that at θ = 0 for three thicknesses of tungsten target, as calculated by Schiff (1946). Note that the curves for different energies T_0 differ only by a scale factor that is inversely proportional to T_0. (Reproduced by permission of The American Physical Society.)

- The x rays tend to be emitted with an appreciable sideways component for low-energy electron beams
- More forward peaked as T_0 is increased

Bremsstrahlung directional dependence

- The forward peaking at high energies necessitates the use of a conical beam-flattening filter in linac x-ray beams for radiotherapy applications, even though the phenomenon becomes less pronounced for thicker targets
- Such a filter attenuates the beam less strongly as a function of distance away from its central axis, thus producing a beam of more uniform intensity over a useful area
- Exact alignment of such a filter is critical

X-ray filtration

- Unfiltered x-ray beam contains both fluorescence (characteristic of the target) and bremsstrahlung x-rays
- The principle result of adding filter is the removal photons at energies where attenuation coefficient is largest (since the photoelectric interaction coefficient varies approximately as $1/(h\nu)^3$ in this energy range)
- Can narrow the spectral distribution progressively; beam is "hardened" by filtration

X-ray filtration

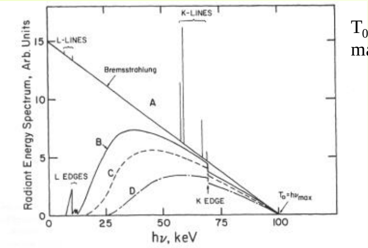


FIGURE 9.10. X-ray spectrum from 100-keV electrons on a thick tungsten target. Upper curve A: Unfiltered. B: Filtered through 0.01 mm W in escaping the target. C: Additionally filtered through 2 mm Al. D: Filtered through 0.15 mm Cu and 3.9 mm Al in addition to inherent target filtration. To avoid confusion, the K-fluorescence lines are not shown in curves B, C, and D, but are attenuated from their heights in curve A in the same proportion as the bremsstrahlung is attenuated at the same energies.

$T_0 < 300$ keV, mainly PE effect

X-ray filtration

- The most common x-ray filtering media are lead, tin, copper, and aluminum, which may be used alone or in combination
- The advantage of combination filters is that they are generally capable of narrowing the spectrum to any desired degree while preserving more of the x-ray output than can be achieved with a single filtering material
- The higher-Z filters provide strong filtering action but cause discontinuities at the shell binding energies. The lower-Z filters tend to smooth the resulting spectrum

X-ray filtration

- It is important in using combination filters that they be positioned in the beam in *descending order* of Z, going in the direction of the rays
- This allows each filter to remove the fluorescence x rays that originate in the higher-Z filter upstream from it
  - Aluminum ($h\nu_K$ = 1.5 keV) is best for the final filter
  - Copper ($h\nu_K$ = 9 keV) fluorescence also is low enough in energy not to be detectable in most cases.
  - Neither tin ($h\nu_K$ = 29 keV) nor lead ($h\nu_K$ = 85 KeV) should be used without sufficient following filtration to remove the fluorescence, unless those photons are desired as part of the output

X-ray filtration

- At higher energies ($T_0 > 300$ keV) the photoelectric effect becomes less important than the Compton effect, and the total absorption is less energy-dependent
- The filtering of an x-ray spectrum generated by MV electrons mainly removes the photons below a few hundred keV without greatly modifying the spectral shape at higher energies
- The use of a thick high-Z filter such as lead on MV x-ray beam tends to filter out the highest-energy photons (>4 MeV) through pair production, as well as the lowest through the photoelectric effect

X-ray beam quality

- The quality of the beam can be specified either in terms of its spectrum or its attenuation characteristics in a medium
- It is possible (with some limitations) to derive an x-ray spectrum from the shape of an attenuation curve, which is a "signature" for the related spectrum in a given medium
- Thus attenuation data can be used to characterize x-ray beams

Attenuation curves and HVLs

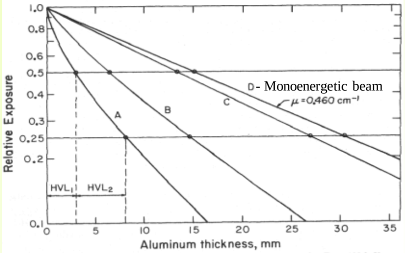


FIGURE 9.12. Approximate exposure-attenuation curves in aluminum for T_0 = 100-keV x-rays from a thick tungsten target, filtered by (A) 2 mm Al; (B) 0.15 mm Cu + 3.9 mm Al; and (C) 2 mm Sn + 0.5 mm Cu + 4 mm Al. (From data of Seelentag et al., 1979.) Also shown for comparison is the attenuation of 100-keV photons in aluminum (curve D). The first and second half-value layers are shown for curve A.

Half-value layers

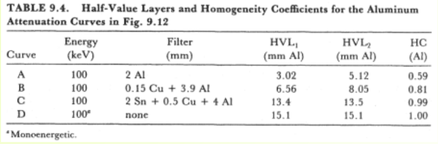


TABLE 9.4. Half-Value Layers and Homogeneity Coefficients for the Aluminum Attenuation Curves in Fig. 9.12

Curve	Energy (keV)	Filter (mm)	HVL_1 (mm Al)	HVL_2 (mm Al)	HC (Al)
A	100	2 Al	3.02	5.12	0.59
B	100	0.15 Cu + 3.9 Al	6.56	8.05	0.81
C	100	2 Sn + 0.5 Cu + 4 Al	13.4	13.5	0.99
D	100[a]	none	15.1	15.1	1.00

[a] Monoenergetic.

- The attenuation curves, can be reasonably well specified for radiological applications in terms of their *first and second half-value layers*, HVL_1, and HVL_2
- HVL_1, is defined as the thickness required to reduce the exposure by half, in narrow-beam geometry; HVL_2 is the thickness necessary to reduce it by half again under the same conditions
- *Homogeneity coefficient* (HC), defined as the ratio HVL_1/HVL_2. This approaches unity as the spectrum is narrowed by filtration

Half-value layers

- Another quantity that is sometimes used in beam-quality specification is the *equivalent photon energy*, $h\nu_{eq}$, defined as the quantum energy of a monoenergetic beam having the same HVL_1, as the beam being specified
- The equivalent photon energy gives no more information than HVL_1, but gives it in a form that is especially useful in describing heavily filtered beams approaching monochromaticity

Half-value layers

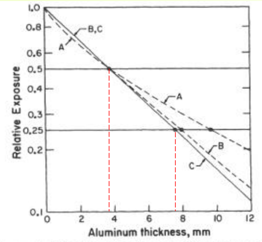


FIGURE 9.13. An example of the attenuation of three dissimilar x-ray beams having practically the same value of HVL_1 (3.8 mm) in aluminum. Curve A: 100 kV, 3 mm Al filter, HVL_2 = 5.82 mm. Curve B: 50 kV, 0.1 mm Pb + 4 mm Al filter, HVL_2 = 4.13 mm. Curve C: monoenergetic 37-keV beam, HVL_2 = 3.8 mm. Note that 37 keV is the equivalent photon energy for both beams A and B. (HVL data after Seelentag et al., 1979.)

Attenuation curves

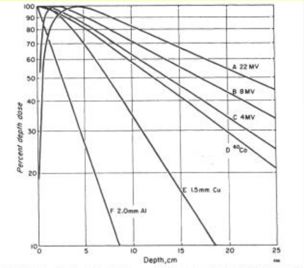
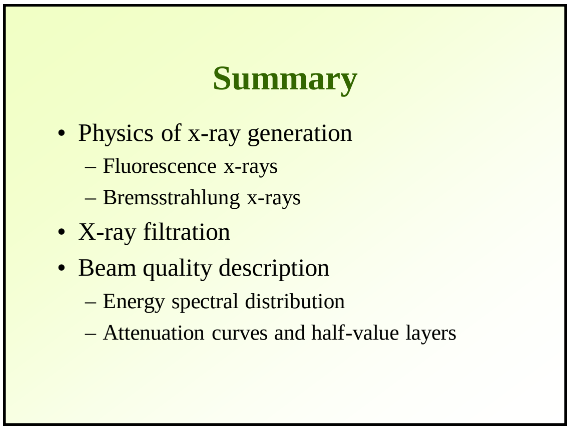


FIGURE 9.15. Variation of absorbed dose vs. depth in a water phantom for the following 10 × 10-cm² photon beams: A, 22 MV, 70-cm SSD; B, 8 MV, 100-cm SSD; C, 4 MV, 100-cm SSD; D, ^{60}Co γ-rays, 100-cm SSD; E, 200 keV, HVL_1 = 1.5 mm Cu, 50-cm SSD; F, 120 kV, HVL_1 = 2.0 mm Al, 15-cm SSD. (From Johns and Cunningham, 1983. Reproduced with permission from

- For high-energy beams (T_0>300keV) HVLs, etc. are irrelevant
- For MV beams photon spectra can be contaminated with secondary electrons
- Depth of d_{max} increases with energy as the range of secondary electrons increases



Summary

- Physics of x-ray generation
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  - Bremsstrahlung x-rays
- X-ray filtration
- Beam quality description
  - Energy spectral distribution
  - Attenuation curves and half-value layers