Overview: As this course is currently taught it might as well be named “Numerical Methods in Linear Algebra”. The focus of the course is on matrix factorizations and the algorithms used to find them. The stability and efficiency of the various algorithms should be compared throughout the course. MATLAB or any of its clones (octave, etc.) can be useful in this context. How much computer activity you require of the students is left to your discretion. The text has a number of exercises designed to be done using a computer.

The two problems driving the course are linear systems and eigenvalue problems. The four fundamental subspaces of a matrix help to organize the material – what solutions to a problem can be expected and what information can be read from the various factorizations – and should be introduced as early as possible. These subspaces should be trotted back throughout the semester as refinements are made possible by the introduction of orthogonal subspaces, projectors, etc.

The text has minimal discussion of iterative methods – primarily for eigenvalue problems. A serious treatment will require some outside material. This again is left to your discretion.

A final note: it’s recommended that abstract vector spaces be downplayed (or eliminated) in favor of real (and complex when needed) column spaces.

Linear Systems: The factorizations and algorithms that should be stressed are:

nonsingular: LU factorization using Gaussian elimination, and PLU factorization using Gaussian elimination with partial pivoting.

overdetermined: QR factorization using modified Gram-Schmidt.

underdetermined: Singular Value Decomposition. There isn’t enough time in the semester to derive a numerically sound algorithm for the SVD, so a reasonable approach is to use the eigenvalues of $A^TA$.

Eigenvalue Problems: This topic must be covered before the SVD can be handled, and, as in the case of the SVD, there isn’t really time to derive any numerically sound algorithms.

On the other hand the use of determinants should be downplayed – or even eliminated entirely. Eigenvalues (and eigenvectors) for special matrices – $2 \times 2$ matrices, triangular matrices, even block triangular matrices if you’re ambitious – can be calculated easily without determinants, and should be
enough to communicate the general ideas. The bound on the number of eigenvalues can be done via the linear independence of eigenvectors corresponding to distinct eigenvalues.

The factorizations that should be emphasized here are (for general matrices) the Schur decomposition $A = UTU^*$, and (for nondefective matrices) the eigenvalue/eigenvector revealing decomposition $A = XDX^{-1}$, and (for symmetric matrices) the orthogonal diagonalization $A = QDQ^T$.

Exams: It’s suggested that there be 2 or 3 midterms plus a final exam.

Schedule: The following syllabus and schedule is meant only as a suggestion. It leaves enough time to accommodate additions. You may, of course, freely modify or ignore it as you choose.

Chapter 1: (9 hours)

§1.1: (Linear Systems)
§1.2: (Row Reduction)
§1.3: (Vector Equations)
§1.4: (Matrix eqn $Ax = b$)
§1.5: (Solution Sets)
§1.7: (Linear Independence)
§1.8: (Linear Transformations)

Chapter 2: (9 hours)

§2.1: (Matrix Operations)
§2.2: (Matrix Inverse)
§2.3: (Invertible Matrices)
§2.4: (Partitioned Matrices)
§2.5: (Matrix Factorizations)
§2.8: (Subspaces of $\mathbb{R}^n$)
§2.9: (Dimension and Rank)

Chapter 5: (8 hours)

§5.1: (Eigenvectors/values)
§5.2: (Characteristic Equation)
§5.3: (Diagonalization)
§5.8: (Iterative methods)

Chapter 6: (8 hours)

§6.1: (Inner Product)
§6.2: (Orthogonal Sets)
§6.3: (Orthogonal Projections)
§6.4: (Gram–Schmidt)
§6.5: (Least–Squares)
Chapter 7:  (6 hours)
§7.1:  (Diagonalization of Symmetric Matrices)
§7.2:  (Quadratic Forms)
§7.4:  (Singular Value Decomposition)