USER BULLETIN 3: DETERMINATION OF UNSUPPORTED LENGTH RATIO L/D$_b$

The unsupported length ratio (L/Db) is calculated by the automatic Sectional modeler spreadsheet using Dhakal and Maekawa [1]. This method consists in the calculation of an equivalent stiffness ratio (k_{eq}) that is used as input in a table that associates this ratio with the number of spaces between ties over the buckling length (n). This number is multiplied by the distance between ties and divided by the diameter of the bar to result in the desired unsupported ratio.

Equivalent stiffness ratio (k_{eq})

The equivalent stiffness ratio is the division of the normalized stiffness of the rebar (k) by the tie stiffness (k_t).

$$k_{eq} = \frac{k}{k_t}$$

The normalized stiffness of the rebar is

$$k = \frac{\pi^4 E_r I}{s^3}$$

Where $E_r I$ is the reduced flexural rigidity of the rebar and is $E_r I = 0.5 E_s I \sqrt{(f_y/400)}$; I is the second moment of inertia of the rebar and is $I = \pi D^4/64$; s is the tie spacing.

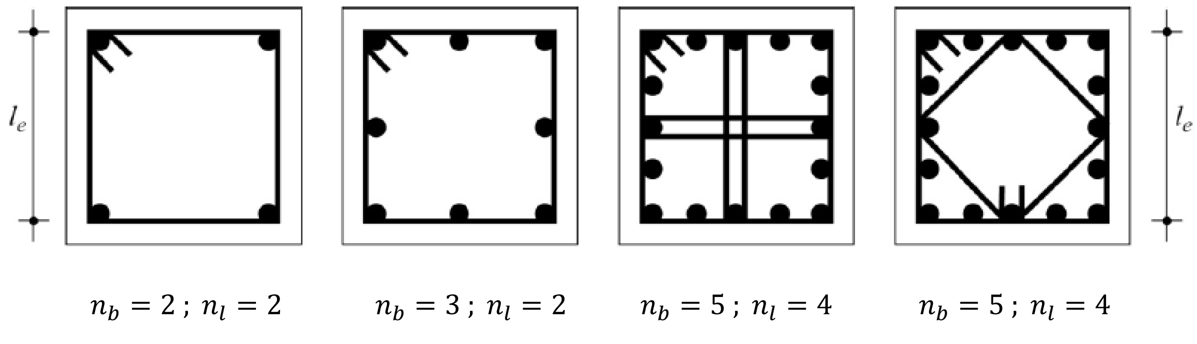
The tie stiffness is:

Rectangular sections $k_t = \frac{E_t A_t}{l_e} \frac{n_l}{n_b}$

Circular Sections $k_t = \frac{2 E_t A_t}{D_{core}}$

where E_t is the modulus of elasticity of the tie; A_t is the area of the tie; l_e is the length of the tie leg; D_{core} is the diameter of the cross section's core; n_b is the number of longitudinal bars supported by these tie legs, and n_l is the number of tie legs parallel to the lateral load.

The length of the tie leg (l_e) is exemplified as follows:



$n_b = 2$; $n_l = 2$ $n_b = 3$; $n_l = 2$ $n_b = 5$; $n_l = 4$ $n_b = 5$; $n_l = 4$

Figure 1 - Number of bars and ties exemplified.

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Number of spaces between ties over the buckling length (n)

The equivalent stiffness ratio is used as the input for table 1 that associates this ratio to a number of spaces between ties.

Stable buckling mode, n	k_{eq}
1	> 0.75
2	0.7500 - 0.1649
3	0.1649 - 0.0976
4	0.0976 - 0.0448
5	0.0448 - 0.0084
6	0.0084 - 0.0063
7	0.0063 - 0.0037
8	0.0037 - 0.0031
9	0.0031 - 0.0013
10	0.0013 - 0.0009

Table 1 - Determination of n

Unsupported Length Ratio

The buckling length is determined by $L = n.s$ and the unsupported length ratio is calculated as follows:

$$Unsupported\ Length\ Ratio = \frac{L}{D_b} = \frac{n.s}{D_b}$$

where D_b is the diameter of the rebar.

Different methods must be considered based on the purpose of which the spreadsheet is being used for. Therefore, three main cases were considered in this spreadsheet, namely, circular, beam/column and slab.

Circular

With hoop reinforcement

For the circular section with hoop reinforcement, the equivalent stiffness ratio is defined for the entire diameter of the cross section as being $k_{eq} = k_t/k$, as shown previously, and the unsupported length ratio can be calculated following the steps shown previously.

No hoop reinforcement

In this case, the unsupported length ratio is considered to be D/D_b, where D is the diameter of the cross section.

Beam/Column

The beam/column presents two different types of unsupported length ratio. The first one is representative of the buckling contribution on the edge's bars due to the bending effect. The second one considers the buckling contribution on the core's bars due to pure compression (Figure 1).

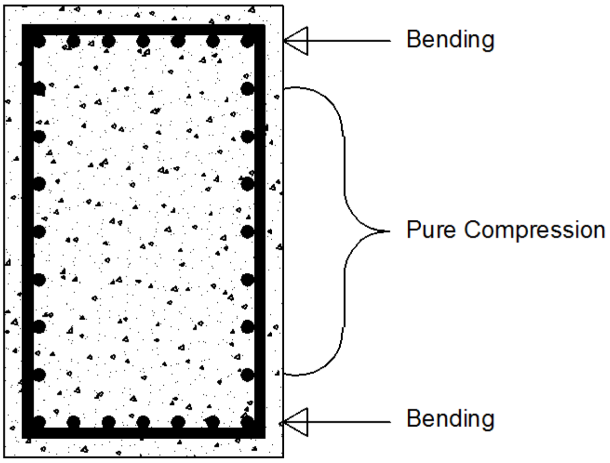


Figure 2 - Bending and pure compression bars

With transverse reinforcement

For the bending unsupported length ratio, the equivalent stiffness ratio is calculated considering the length of the tie leg (l_e) of the tie that is restraining the edge's bars in the tie stiffness (Figure 2(a), next page).

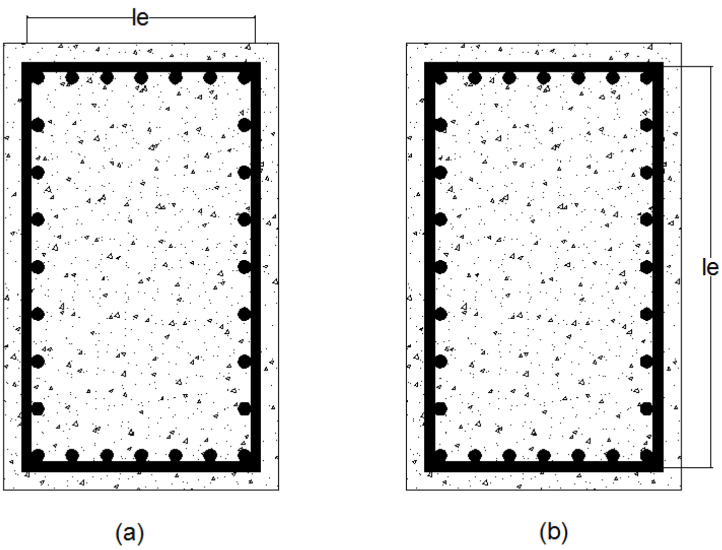


Figure 3 - Tie leg for bending and pure compression cases

For the pure compression unsupported length ratio, the equivalent stiffness ratio is calculated considering the length of the tie leg (l_e) of the tie that is restraining the core's bars in the tie stiffness

(Figure 2(b)). Since pure compression acts on all core's bars, the number of longitudinal bars supported by the ties (exemplified in Figure 1) are multiplied by 2.

For both cases, the unsupported length ratio calculation follows its normal procedure.

No transverse reinforcement

In this case, the unsupported length ratio is section height divided by bar diameter.

Slab

With shear reinforcement

The unsupported length ratio is due to bending of the section.

No shear reinforcement

The unsupported length ratio is three times section height divided by bar diameter.

Slenderness ratio

The slenderness ratio (r_b) indicates approximately the reduction of normalized stress (f_i/f_y) on the modified compressive stress-strain curve due to buckling [2]. Its calculation follows:

$$r_b = \frac{L}{D}\sqrt{\frac{f_y}{100}}$$

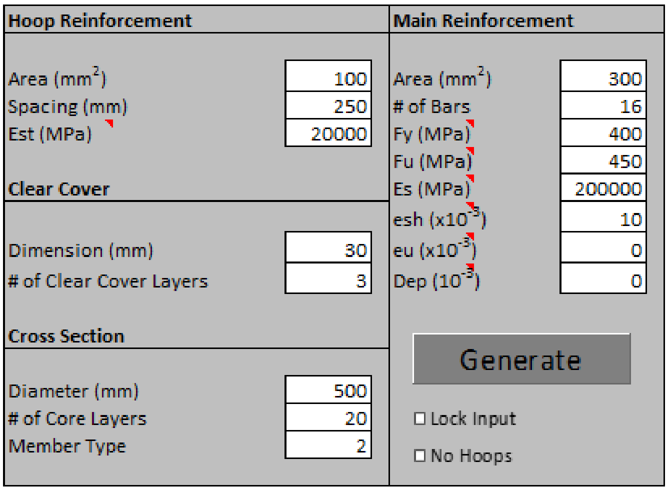
Table 2 associates the value of the slenderness ratio with the buckling effect level.

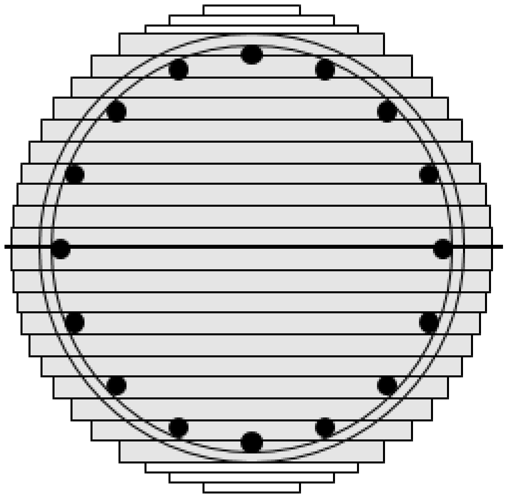
r_b	f_i/f_y	Buckling effect level
< 8	-	No effect
8 - 16	1.0 - 0.8	Small
16 - 34	0.8 - 0.4	High
34 - 50	0.4 - 0.2	Very High

Table 2 - Determination of r_b and its relation with normalized intermediate stress ratio

EXAMPLES

1. Circular





$$I = \frac{\pi D^4}{64} = \frac{\pi x 19.54^4}{64} = 7155.96\ mm^4$$

$$E_r I = 0.5 E_s I \sqrt{(f_y/400)} = 0.5x200000x7155.96x\sqrt{(400/400)} = 715\ 596\ 382.2\ Nmm^2$$

$$k = \frac{\pi^4 E_r I}{s^3} = \frac{\pi^4 x 715\ 596\ 382.2}{250^3} = 4461.16\ N/mm$$

$$k_t = \frac{2E_t A_t}{D_{core}} = \frac{2E_t A_t}{D - 2xClearCover - TieDiameter} = \frac{2x200000x100}{500 - 2x30 - 11.28} = 93300.99\ N/mm$$

$$k_{eq} = \frac{k_t}{k} = \frac{93300.99}{4461.16} = 20.91$$

By entering with k_{eq} in table 1, we get the n = 1. Therefore, the unsupported length ratio is:

$$\frac{L}{D_b} = \frac{n.s}{D_b} = \frac{1x250}{19.54} = 12.79$$

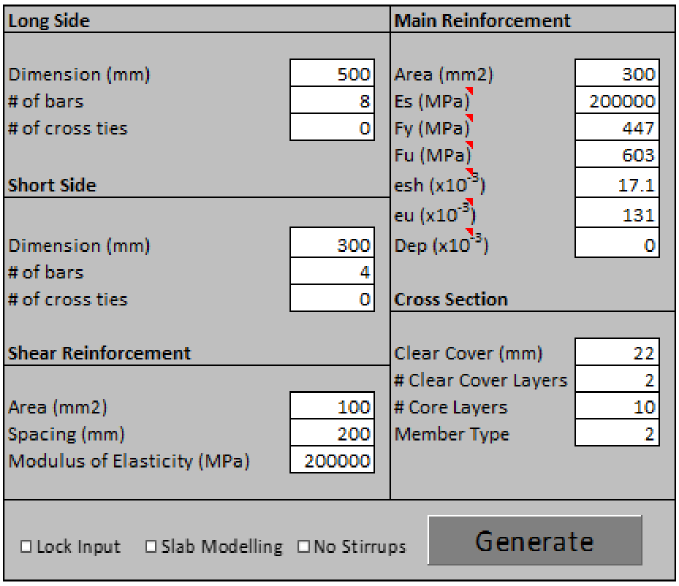
The slenderness ratio is:

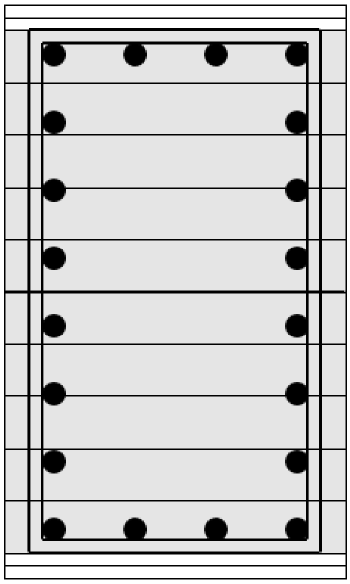
$$r_b = \frac{L}{D}\sqrt{\frac{f_y}{100}} = 12.79\sqrt{\frac{400}{100}} = 25.59$$

Based on Table 2, the table below gives high buckling effect for all reinforcement layers.

N	L/Db	n	r_b	Buckling Effects
1	12.79	1	25.6	High
2	12.79	1	25.6	High
3	12.79	1	25.6	High
4	12.79	1	25.6	High
5	12.79	1	25.6	High
6	12.79	1	25.6	High
7	12.79	1	25.6	High
8	12.79	1	25.6	High
9	12.79	1	25.6	High

2. Beam/Column





$$I = \frac{\pi D^4}{64} = \frac{\pi x 19.54^4}{64} = 7155.96\ mm^4$$

$$E_r I = 0.5 E_s I \sqrt{(f_y/400)} = 0.5x200000x7155.96x\sqrt{(447/400)} = 756\,469\,993.6\ Nmm^2$$

$$k = \frac{\pi^4 E_r I}{s^3} = \frac{\pi^4 x 756\,469\,993.6}{200^3} = 9210.88\ N/mm$$

For bending unsupported length:

$$k_t = \frac{E_t A_t}{l_e} \frac{n_l}{n_b} = \frac{200000x100}{300 - 2x22 - 11.28} \frac{2}{4} = 40863.03\ N/mm$$

$$k_{eq} = \frac{k_t}{k} = \frac{40863.03}{9210.88} = 4.44$$

For pure compression unsupported length:

$$k_t = \frac{E_t A_t}{l_e} \frac{n_l}{n_b} = \frac{200000x100}{500 - 2x22 - 11.28} \frac{2}{2x8} = 5621.52\ N/mm$$

$$k_{eq} = \frac{k_t}{k} = \frac{5621.52}{9210.88} = 0.61$$

By entering with k_{eq} in table 1, we get n = 1 for the bending ratio and n=2 for the pure compression ratio. Therefore, the unsupported length ratio is:

For bending unsupported length

$$\frac{L}{D_b} = \frac{n.s}{D_b} = \frac{1x200}{19.54} = 10.23$$

For pure compression unsupported length

$$\frac{L}{D_b} = \frac{n.s}{D_b} = \frac{2x200}{19.54} = 20.47$$

The slenderness ratio is:

For bending ratio

$$r_b = \frac{L}{D}\sqrt{\frac{f_y}{100}} = 10.23\sqrt{\frac{447}{100}} = 21.63$$

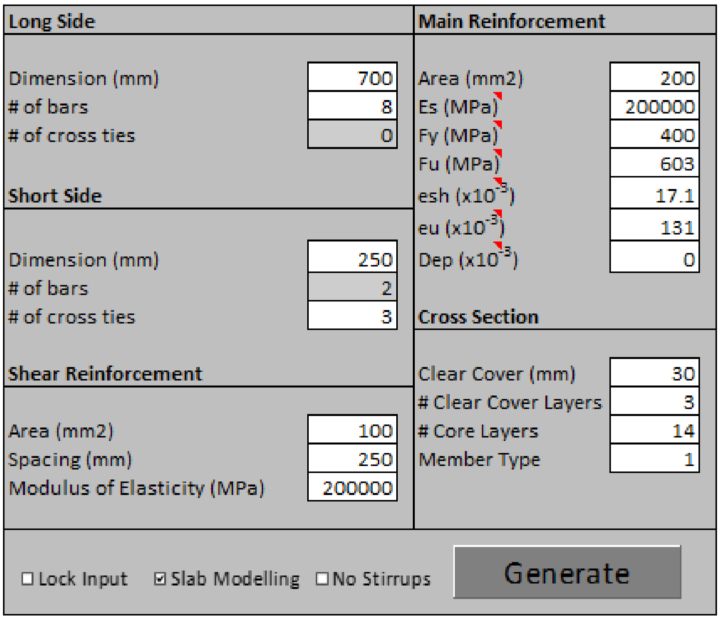
For pure compression ratio

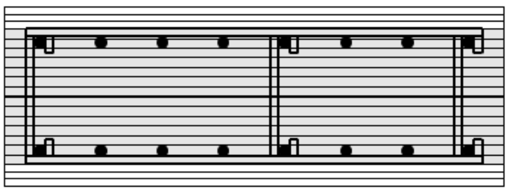
$$r_b = \frac{L}{D}\sqrt{\frac{f_y}{100}} = 20.47\sqrt{\frac{447}{100}} = 43.26$$

Based on Table 2, the table below gives high buckling effect for the reinforcement layers with bending unsupported length and very high buckling effect for the reinforcement layers with pure compression unsupported length.

N	L/Db	n	rb	Buckling Effects
1	10.23	1	21.6	High
2	20.47	2	43.3	Very High
3	20.47	2	43.3	Very High
4	20.47	2	43.3	Very High
5	20.47	2	43.3	Very High
6	20.47	2	43.3	Very High
7	20.47	2	43.3	Very High
8	10.23	1	21.6	High

3. Slab





$$I = \frac{\pi D^4}{64} = \frac{\pi x 15.96^4}{64} = 3184.94\ mm^4$$

$$E_r I = 0.5 E_s I \sqrt{(f_y/400)} = 0.5x200000x3184.94x\sqrt{(400/400)} = 318\ 494\ 140.5\ Nmm^2$$

$$k = \frac{\pi^4 E_r I}{s^3} = \frac{\pi^4 x 318\ 494\ 140.5}{250^3} = 1985.55\ N/mm$$

$$k_t = \frac{E_t A_t}{l_e}\frac{n_l}{n_b} = \frac{200000x100}{700 - 2x30}\frac{3}{8} = 11718.75\ N/mm$$

$$k_{eq} = \frac{k_t}{k} = \frac{11718.75}{1985.55} = 5.90$$

By entering with k_{eq} in table 1, we get the n = 1. Therefore, the unsupported length ratio is:

$$\frac{L}{D_b} = \frac{n.s}{D_b} = \frac{1x250}{15.96} = 15.66$$

The slenderness ratio is:

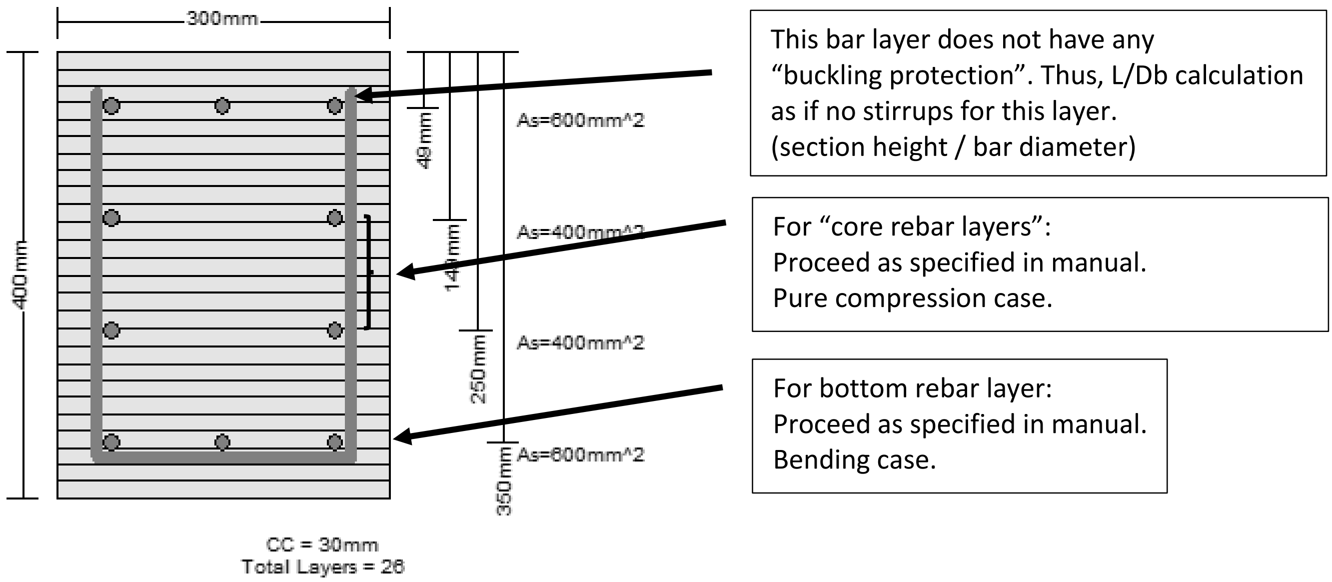
$$r_b = \frac{L}{D}\sqrt{\frac{f_y}{100}} = 15.66\sqrt{\frac{400}{100}} = 31.33$$

Based on Table 2, the table below gives high buckling effect for both reinforcement layers.

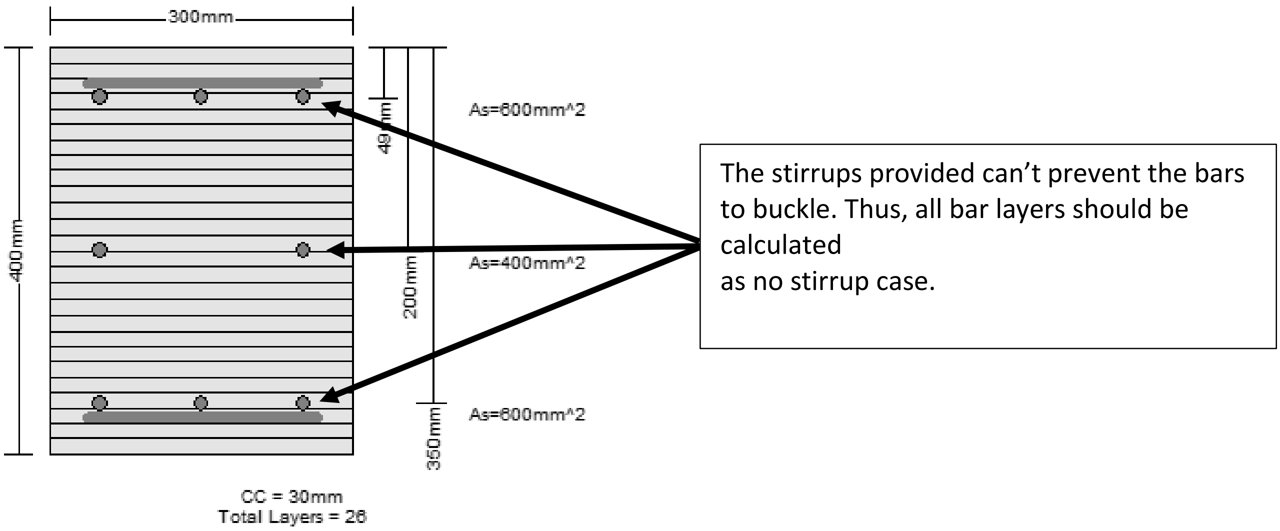
N	L/Db	n	rb	Buckling Effects
1	15.67	1	31.3	High
2	15.67	1	31.3	High

SPECIAL CASES

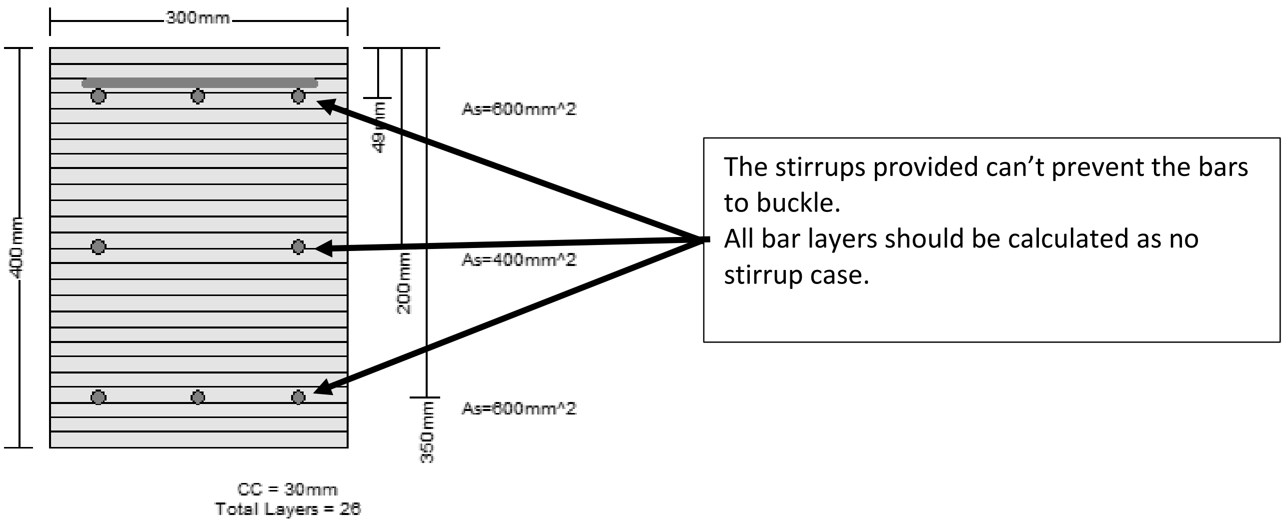
1. Open Stirrups Case



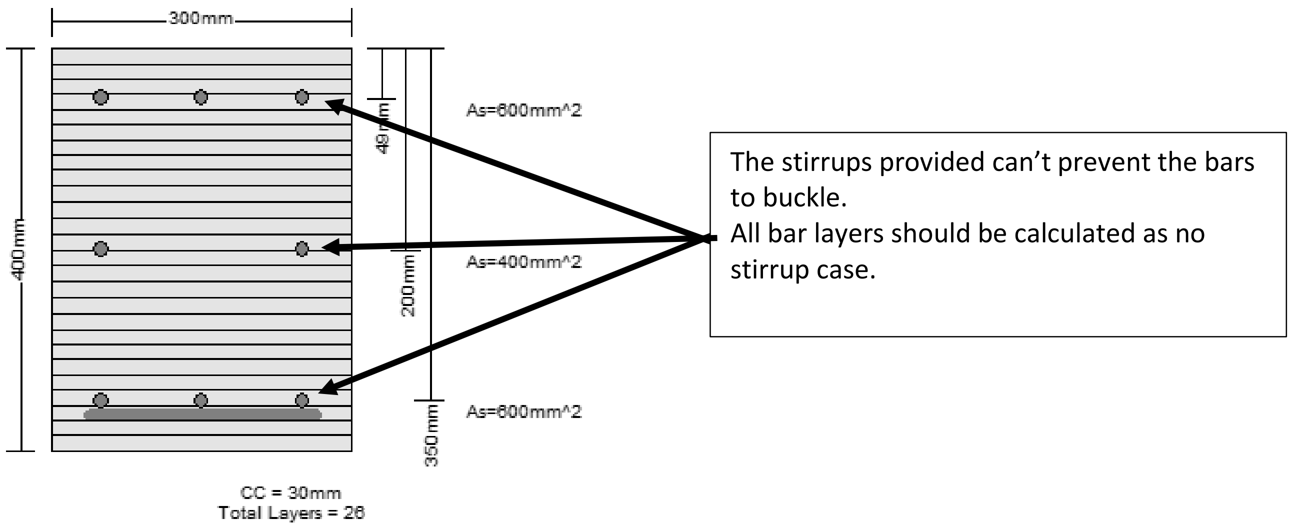
2. Top & Bottom Stirrups Case



3. Top Stirrups Case



4. Bottom Stirrups Case



REFERENCES

[1] Dhakal, R.P., and Maekawa, K., 2002c, “Path-dependent Cyclic Stress-Strain Relationship of Reinforcing bar including buckling,” Engineering Structures, Vol. 24, No. 11, pp. 1383-1396.

[2] Wong, P.S., Vecchio, F.J. and Trommels H., 2013, “VecTor2 & FormWorks User’s Manual”, Second Edition, pp. 155.