Engineering Structures 198 (2019) 109474

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Specialized strut-and-tie method for rapid strength prediction of bridge pier caps

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ARTICLE INFO

Keywords:
Bent cap
Deep beam
Load rating
Nonlinear analysis
Pier cap
Rehabilitation
Sectional method
Shear capacity
Strengthening
Strut and tie

ABSTRACT

Deep pier caps possess additional shear strength due to the formation of the strut action which cannot be captured by the traditional sectional method. The strut-and-tie method (STM) is a suitable method for capturing the deep beam action. The application of the STM, however, has many challenges including creating and optimizing a valid truss model, performing an indeterminate truss analysis, calculation of nodal and elemental stress limits, etc. The objective of this study is to explore innovative strategies to reduce the complexity of the STM by developing a strut-and-tie methodology to rapidly and accurately predict the shear capacities of deep pier caps. A graphical solution algorithm and associated computer code is developed to generate and analyze efficient strut-and-tie models while intuitively educating engineers in the correct use of the methodology. The accuracy of the methodology is assessed by modeling eight existing bridge pier caps with a general-purpose strut-and-tie method. In addition, nonlinear finite element analyses of the same pier caps are performed for an in-depth investigation and comparisons of the governing behaviors, strengths, and modes of failure with those obtained from the proposed methodology. Although not valid for deep beams, the sectional method calculations are performed to demonstrate the consequences of using it. The relationship between the shear span-to-depth ratio and the shear strength predictions from all three methods are compared for twenty-one regions. The proposed methodology has general applicability for modeling deep pier caps and is shown to provide similar modeling time and effort to the sectional method with accuracies comparable to those obtained from the nonlinear finite element analysis.

1. Introduction

There are millions of bridges worldwide, each with multiple pier caps. 'Pier caps' or 'bent caps' transfer the load from the girders to the columns. Most pier caps are deep beams, which possess additional shear strength due to the formation of the strut action. Bridge design codes, such as AASHTO LRFD Bridge Design Specifications [1], only started to include the deep beam methods in 1994. Consequently, most in-service bridges were not originally designed considering the deep beam effects and thus possess a hidden reserve shear strength. Modifications to the bridges, such as bridge deck expansions, addition of lanes, and an increase in design truck load, have taken place over the years, frequently causing the bridges to exceed their original design loads. Thus, there is a need for appropriate analysis methodologies that can accurately predict the true load capacities of these structures while considering the deep beam action.

Pier caps are special structures due to their geometry and position of the girder loads, which results in relatively small shear span-to-depth ratios and thus making them a deep beam. The Hooke-Euler-Bernoulli theory of bending [2–4], employed as a part of the sectional method, cannot accurately capture the behavior of the deep beams; hence, it is not an appropriate analysis method for deep beams [e.g.,5]. Strut-and-tie method (STM) and nonlinear finite element analysis (NLFEA) method are two suitable options for the analysis of deep beams [1]. STM is a conceptually simple methodology; however, its application presents many challenges such as the development of a valid truss model, performing of iterative solutions to optimize the model, and checking nodal stresses, all of which requires an advance knowledge and a labor-intensive geometric solution process.

Over the past few decades, various researchers and design codes have proposed equations and geometrical rules for creating effective STM [1,6–21]. However, these equations and rules are developed for general structure modeling and require expert-level STM knowledge and an iterative solution for application to specific types of structures. In addition, very few computer programs are available to apply these equations and rules for structure modeling [22–24]. These factors

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https://doi.org/10.1016/j.engstruct.2019.109474
Received 9 March 2019; Received in revised form 12 July 2019; Accepted 1 August 2019

create major challenges and limit the prevalent usage of the STM in practice.

The main objective of this study is to explore innovative strategies to reduce the complexities in creating and analyzing strut-and-tie models for only bridge pier caps. A mathematical algorithm and associated computer code are developed for this purpose. The proposed methodology seeks to provide a graphical representation to generate valid and efficient STM models while intuitively educating practicing engineers in the correct use of the method. The proposed methodology is verified using eight existing pier caps located in Ohio, USA. In addition, NLFEA is undertaken to investigate the flow of stresses and obtain an in-depth understanding on the nonlinear load and deformation responses of pier caps. A secondary objective is to assess the consequences of using the sectional method for analyzing deep beams. The shear strength predictions obtained from the sectional method, proposed methodology and the NLFEA are compared for this purpose.

2. Methodology

2.1. Creation of a valid truss model and optimization for efficiency

The STM model geometry determination is the most critical and challenging step in obtaining accurate analysis results. A well-defined methodology is developed in this study for the creation and optimization of an efficient STM model for bridge pier caps. The proposed methodology is based on the principle that "the loads applied on the structure is transferred to the supports using the shortest paths." According to this, if a load is applied on a cantilever span, it is directly transferred to the nearest support (i.e., bridge pier or column). If a load is applied directly above a support, it is directly transferred to the support below it; similarly, if a load is applied at any point in the clear span of the pier cap, it is transferred to adjacent supports in certain proportions (see Fig. 1). The top and bottom horizontal members of the model are located at the centroid of the top and bottom steel reinforcements. The top nodes are located under every load while the bottom nodes are located at the reaction points. At each column, the supports are divided to create a uniform stress distribution throughout the column cross-section as illustrated in Fig. 1. The reactions are located at the center of each column proportionate to the loads as computed by Eq. (1). The nodes are connected with force paths following the above principle to create the STM model (i.e., truss model) of the pier cap.

$$\frac{R_1}{W_1} = \frac{R_2}{W_2} = \frac{R_3}{W_3} \tag{1}$$

where R_1, R_2, and R_3 are the reactions at each support proportioning, and w_1, w_2 and w_3 are the corresponding proportioned support width.

The STM model is created by a set of compressive forces (i.e., struts) and tension forces (i.e., ties) as shown in Fig. 2(a). An idealized

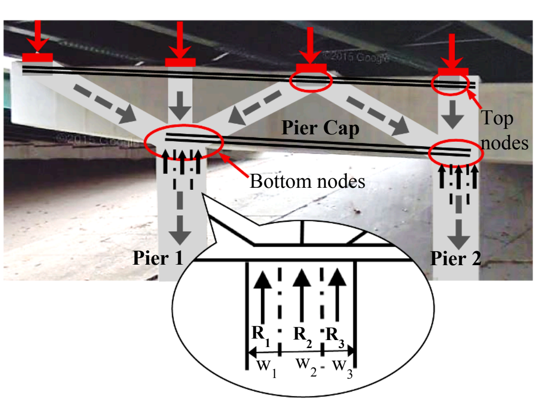


Fig. 1. STM model theory for the proposed methodology (Pier Cap A).

prismatic-shaped strut is considered for the proposed methodology. This strut incorporates a uniform width, and hence a constant capacity along its length as shown with a single line in Fig. 2(b).

A truss model developed for a given multi-column pier cap is typically statically indeterminate. The proposed methodology employs the matrix stiffness method to calculate the truss member forces using a constant stiffness for each element. The axial forces in each element are determined considering the global system and the capacity is determined considering the individual element as per AASHTO LRFD [1]. The factored axial force in each element (i.e., strut or tie) is checked with factored capacity as per Eq. (2). More details on the proposed methodology can be found elsewhere [25,26].

$$\sum_{i=1}^{N} \psi_i F_i \leqslant \phi R_n \tag{2}$$

where ψ_i is the corresponding load factor, F_i is the service load for each member (kips or kN), ϕ is the strength reduction factor, and R_n is the nominal strength of each member (kips or kN).

The methodology developed for determining the capacities of elements for Pier Cap B in Fig. 3 is presented in Fig. 4. The nodal capacity assessments, which are typically one of the most complex aspects of STM, are integrated into the proposed methodology as shown in the flowchart. AASHTO LRFD [1] provisions are employed when determining the strut, tie and nodal capacities. A utilization ratio (abbreviated as UR, which is the ratio of the force or demand to the capacity or strength) is calculated for each STM element. URs reflect the condition (i.e., overloaded or reserve capacity) of the pier cap under the application of given girder loads. If the UR is less than 1.0, it indicates a reserve capacity; otherwise, it indicates an overload. For example, an UR of 0.69 indicates that the pier cap has 69% of its capacity in use and has approximately 31% reserve capacity remaining.

In Fig. 4, α_s is the smaller angle between the compressive strut and adjoining tension ties (strut angle), f_{cu} is the limiting compressive (effective) stress (ksi or MPa), f_c' is the uniaxial compressive strength of concrete (ksi or MPa), f_y is the yield strength of reinforcing bars (ksi or MPa), A_{st} is the total area of longitudinal rebar in the tie (in.^{2} or mm^2), A_{cs1} is the area of prismatic strut at Node 1 (in.^{2} or mm^2), A_{cs2} is the area of prismatic strut at Node 2 (in.^{2} or mm^2), A_v is the area of shear reinforcement within a distance s (in.^{2} or mm^2), and s is the spacing of transverse reinforcement (i.e., stirrups) (in. or mm).

The load capacities and URs are strictly dependent on the STM model created. The vertical members (i.e., the vertical ties representing the shear stirrups) can be provided for each inclined member to modify the STM model and create an alternative, and potentially more efficient, model if the available stirrup quantity is sufficient. The proposed methodology permits performing iterations to assess the URs for all possible cases. The process of obtaining the maximum capacities or the minimum URs is commonly referred to as the optimization of the STM. One example is shown on Fig. 5, which is the output obtained from the proposed methodology. Model 4 is the most efficient model with the minimum governing UR (also see Fig. 6). If there are not enough stirrups provided, vertical ties will show an UR larger than 1.0 (e.g., overloaded). In such cases, the models with no vertical ties will provide more optimized models (which is not the case in Fig. 5; see [25,26] for such cases). Thus, it is important to perform the optimization process at every vertical tie location.

2.2. Creation of a mathematical algorithm and associated computer code (STM-CAP)

In order to allow for an efficient use of the proposed methodology, an automated solution algorithm is developed. The algorithm uses the Visual Basic programming language and provides graphical solutions to help the analyst better understand the system and identify any potential input errors. The code is embedded in Microsoft Excel to develop a

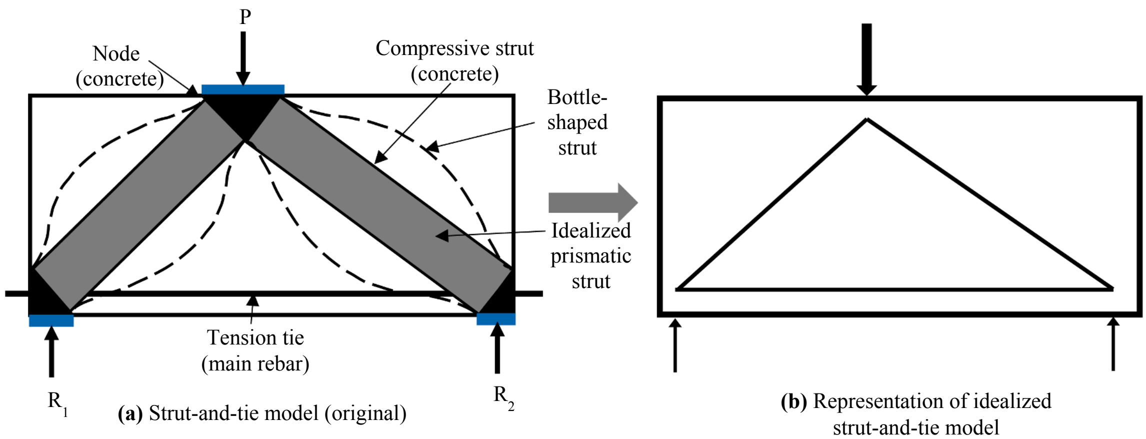


Fig. 2. Strut-and-tie model for a pier cap span.

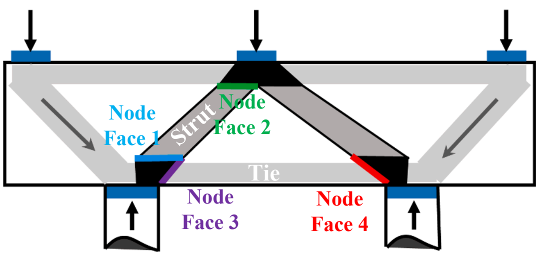


Fig. 3. Sample STM model of the proposed methodology (Pier Cap B).

program named STM-CAP (Strut-and-Tie Method for pier CAPs) [25,26]. The program is divided into several sections covering various aspects of the input parameters, calculation details, and analysis result output. The flowchart for input, analysis process, and the output is presented in Fig. 7.

In Fig. 7, f_c' is the uniaxial compressive strength of concrete (ksi or MPa), f_y is the yield strength of reinforcing bars (ksi or MPa), a/d is the shear span-to-depth ratio, ϕ_c is the resistance factor for concrete, and ϕ_s is the resistance factor for reinforcing steel.

The proposed methodology is developed for the analysis of deep pier caps subjected to static girder loads for both symmetrical (up to eight columns) and asymmetrical caps (up to four columns). Most common multi-column bridge pier caps fall within these limits. A graphical to-scale sketch of the beam is dynamically generated, and the required input fields are created. The solution algorithm first determines if a pier cap is deep or not. Based on the load application points and the geometry, the shear span-to-depth ratios for every region are calculated. If the ratios are less than 2.0, the cap beam is considered deep. The subsequent input includes the material properties and resistance factors. The bearing or base plate dimensions are required to calculate the widths of the struts and perform nodal bearing checks. The reinforcement anchorage and development length checks are conducted to ensure that the longitudinal bars are adequately developed; otherwise, strength of the ties are reduced proportional to the lack of the required development length (which may also be manually adjusted). The proposed STM model with the utilization ratios is then generated along with the analysis results e.g., Fig. 5(a). The model is shown color-coded where red represents the ties, blue represents the struts, and their intersections represent the nodes. The largest UR is named as the governing UR. The model can be optimized by activating (i.e., input of one) or deactivating (i.e., input of zero) vertical ties for each span. After each modification, a re-analysis is performed and corresponding URs are displayed. The analyst can then compare the URs among these models and pick the model with the smallest governing UR (i.e., the most optimized model). If a span contains amounts of stirrups sufficient to pick up the compressive force coming from a strut, the activation of ties will be beneficial (see Fig. 5 as an example); otherwise, an overload at the activated tie will appear, indicating that no tie option can be more optimized – the analyst should compare the largest URs from all models to decide. It should be noted that all models created will be valid but with different effectiveness. Consequently, the developed solution algorithm and associated STM-CAP program enables engineers to develop efficient models by creating and assessing several valid models, while educating them on the correct use of the strut-and-tie method. The entire modeling and analysis process can be completed in a short period of time similar to that of the traditional sectional analysis method.

3. Verification of the methodology

Eight existing pier caps, which are representative of the caps located in Ohio, USA, are modeled using the proposed methodology, a general-purpose computer-aided strut-and-tie method, a NLFEA method, and the sectional method. The Ohio Department of Transportation selected these structures and provided the structural drawings for use in this study. These pier caps are supported by three to eight columns and subjected to the loads coming from four to ten girders. The caps have shear span-to-depth ratios (a/d) ranging from 0.45 to 3.0 and include symmetric and almost-symmetric configurations (modelled as symmetric in this study) with cantilever and non-cantilever ends. The results obtained from each method are compared to quantify the major influences and assess the accuracy of the proposed method through the use of the STM-CAP program.

3.1. Proposed methodology

All eight pier caps are modeled using STM-CAP program following the principles discussed above. Modeling details are provided elsewhere [25,26].

3.2. General-purposed computer-aided strut-and-tie method

The results obtained from the proposed methodology are verified by the computer-aided strut-and-tie program CAST, a general-purpose strut-and-tie modeling method used for the analysis and design of disturbed regions or any configuration of deep beams [21,24,27]. Identical models are created and the utilization ratios, member forces, and reactions from each model are compared. A sample comparison is shown in Fig. 9 for the sample pier cap shown in Fig. 8.

Similarly, the URs from all eight bridges are compared and, in most cases, the results are found to be identical. In some rare cases, the difference in the utilization ratios between the two methods is up to 5%. One of the reasons for the discrepancies is the geometrical

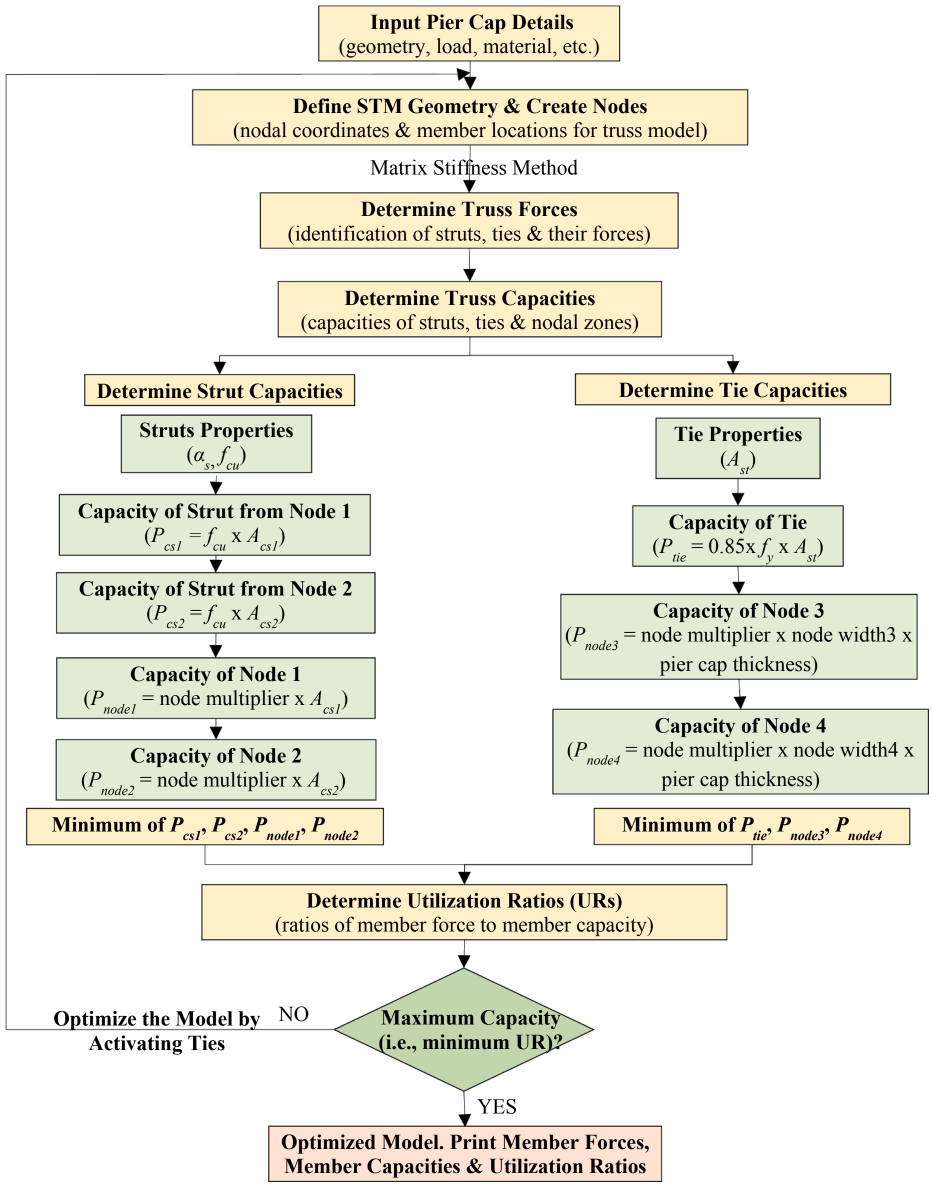


Fig. 4. Flowchart of the methodology.

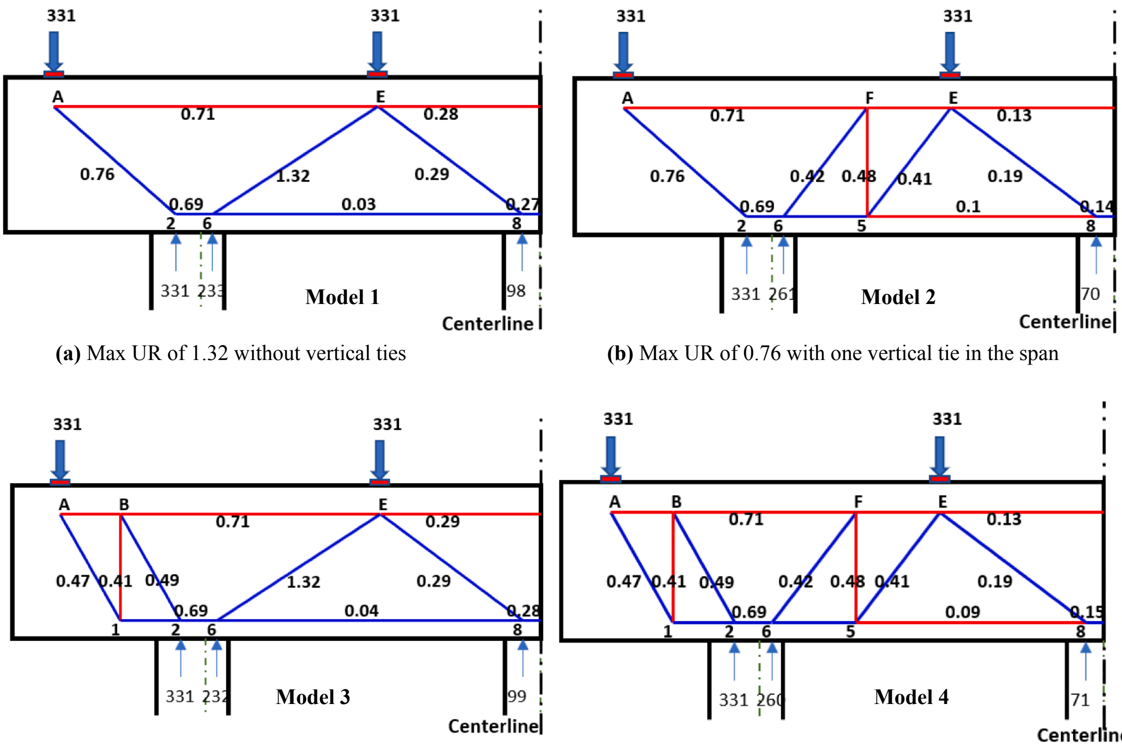
simplifications made in CAST, which used a grid with a constant spacing. STM-CAP permitted more accurate input of the bridge geometry (e.g., a girder spacing of 13 ft and 11.5 in). Another reason may involve round-off errors. Verification with hand calculations indicate that STM-CAP is more accurate in cases of such discrepancies.

3.3. Nonlinear finite element analysis

Nonlinear finite element analyses (NLFEA) are performed for an in-depth investigation and obtain the complete response simulations of the pier caps in terms of the load capacity, deformation response, cracking behavior, and failure modes. Senturk [28,29] verified the program VecTor2 [30,31] through large-scale experimental studies and concluded that VecTor2 provides accurate shear capacity estimates for deep cap beams in a comparison with other analysis methods. VecTor2 is a non-linear finite element analysis program for two-dimensional structures based on the Disturbed Stress Field Model [32], which is the extension of the Modified Compression Field Theory [33]. The DSFM is based on a smeared, rotating crack approach and models concrete as an orthotropic material while accounting for shear slip deformations across cracks. The modified Park-Kent model [34] was adopted for the post-peak concrete response, whereas the Popovics model [35] was used for the pre-peak compression response. VecTor2 also accounts for the important second-order material behaviors such as compression softening [36], tension stiffening [37], and dowel action [38], which are found to be important when modeling pier caps (see Fig. 10).

Five of the pier caps are modeled to determine their in-depth response, using VecTor2 (see Fig. 11 for one example). Three regions (shown with pink[1], blue and orange colors) are created to represent different shear reinforcement ratios, which are considered smeared (ρ_y) inside concrete. The fourth region (shown with yellow) represents the clear cover while the fifth region (shown with gray) represents the columns. Discrete truss bars are used to model the horizontal main reinforcement. Only one-half of the beams are modelled benefiting from the symmetry by defining vertical roller restraints along the right-side edge nodes. The strength reduction factors as per AASHTO LRFD [1] are used for the concrete and reinforcing steel material properties. Factored loads are applied, which are increased proportionally and

[1] For interpretation of color in Figs. 11 and 12, the reader is referred to the web version of this article.



(a) Max UR of 1.32 without vertical ties

(b) Max UR of 0.76 with one vertical tie in the span

(c) Max UR of 1.32 with one vertical tie in the cantilever

(d) Max UR of 0.71 with two vertical ties

Fig. 5. Optimization of the STM model for Pier Cap 1 (STM-CAP output is shown).

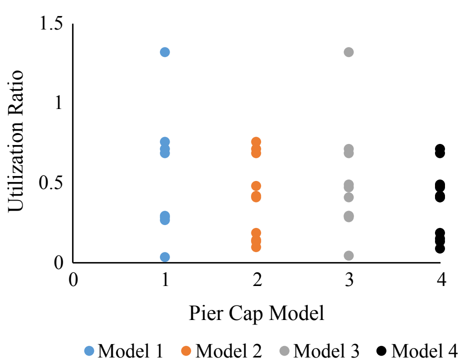


Fig. 6. UR comparison for STM optimization for Pier Cap 1.

monotonically from zero.

The crack patterns and stress distributions of the concrete and reinforcement at factored load conditions are obtained. To compare the results with the STM-CAP, equivalent utilization ratios were calculated. The utilization ratios for struts correspond to the concrete elements and is calculated as the average stress over a region divided by the compressive strength of the concrete as shown with circles in Fig. 12(a) for one of the pier caps. Unlike concrete, where failure occurs over a certain region, failure in rebar is considered to occur due to the rupture within a single finite element. Thus, utilization ratios of ties are calculated as the average stress in the most stressed rebar element divided by the yield strength of rebar as shown with ellipses in Fig. 12(b). The visualization of struts and ties formed is clearly indicated by highly stressed and cracked regions from NLFEA in Fig. 12(c), where the cracks are shown with red lines. The utilization ratios obtained from the proposed STM-CAP and the NLFEA are compared as shown in Fig. 12(d). As expected, the NLFEA provided smaller URs (i.e., higher capacities) due to the consideration of many advanced behaviors discussed above and load re-distribution as certain parts of the beam yields or cracks. The governing behavior and the mode of failure agreed well in both methods; thus, verifying the load path formed in the strut and tie analysis method proposed.

3.4. Sectional method

The sectional analysis is not a valid method for deep beams, and thus should not be used. It is employed in this study, however, to demonstrate the consequences of using it. The shear utilization ratios are obtained for twenty-one sections with different a/d ratios of the previously discussed pier caps. The factored shear capacities are calculated using the sectional provisions contained in [1], which are fundamentally based on the Modified Compression Field Theory [33]. A sample sectional shear capacity calculation is presented in Table 1 for the pier cap shown in Fig. 13.

3.5. Comparison of results

The utilization ratios predicted by the proposed STM method, NLFEA, and the sectional method for corresponding a/d ratios are presented in Fig. 14.

The plot includes twenty-one regions with the shear span-to-depth ratios (a/d) ranging from 0.45 to 3.0. Most regions are deep with a/d ratios less than 2.0. The comparison of blue and orange curves demonstrates that the proposed STM methodology predicts lower utilization ratios (i.e., higher shear capacities) than the sectional method. For lower a/d ratios (e.g., at around 0.50), the proposed methodology predicts three times higher shear capacities. With the increase in a/d ratio (i.e., as the beams get shallower), the discrepancy between the two methods diminish. The predictions of both methods converge at an a/d ratio of 2.8 to 3.0. These results show that the sectional method consistently underestimates the shear capacities or deep beams. The comparison of green and orange curves demonstrates that the NLFEA consistently predicts lower URs than the proposed methodology (40% on average). This can be attributed to the fact that the STM is based on the lower-bound theorem; thus, the proposed methodology terminates

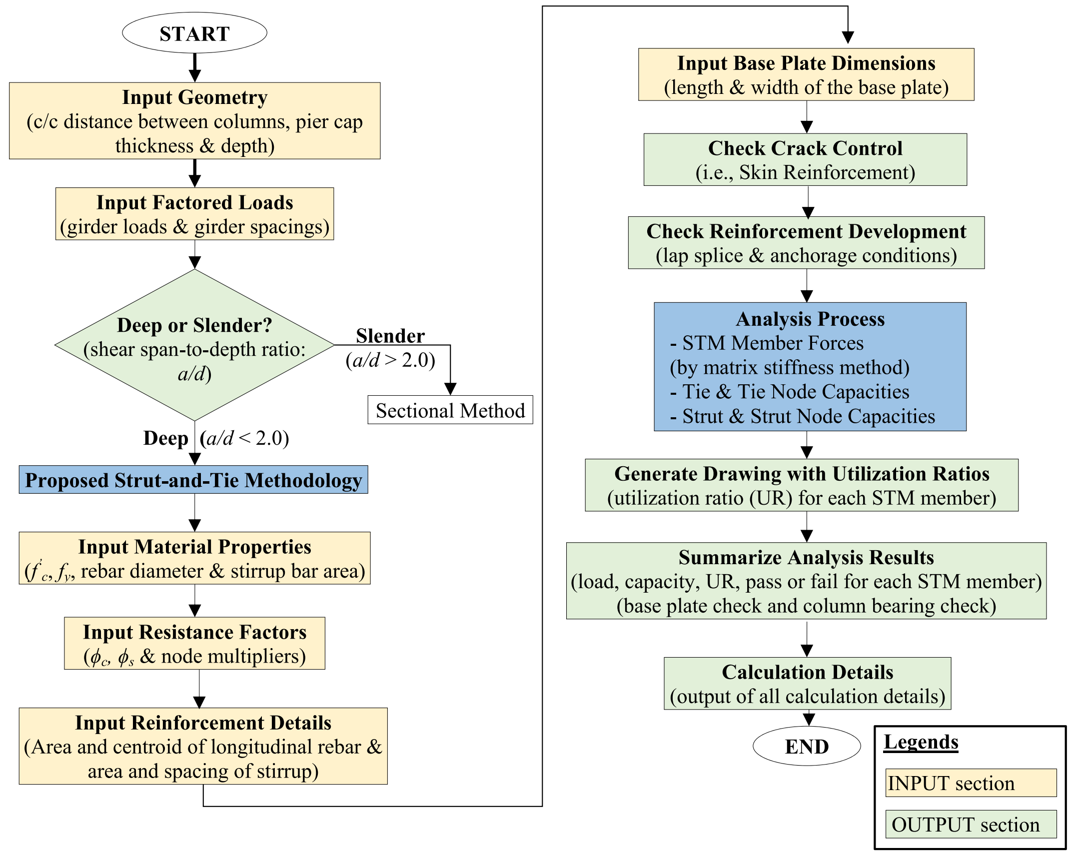
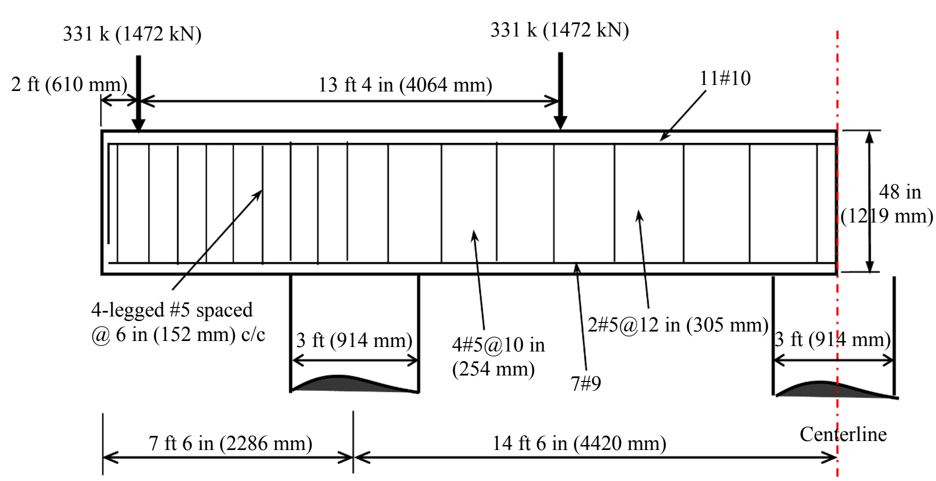


Fig. 7. Flowchart of the solution algorithm.

the analysis at the first yielding or crushing whereas the NLFEA continues the analysis, considering concrete cracking, reinforcement yielding, nonlinear deformations, and redistribution of stresses. The predictions of both methods converge at an a/d ratio of 0.5 (i.e., very deep beam region) and 3.0 (i.e., slender beam region).

4. Summary, conclusions and recommendations

This study proposes a strut-and-tie methodology and the associated computational algorithm to create and analyze specialized strut-and-tie models for bridge pier caps. The methodology determines a valid geometry for the strut-and-tie model, conducts the structural analysis, calculates a utilization ratio for each member, and identifies overloaded members if present. The computational algorithm is coded to create a computer tool called STM-CAP (Strut-and-Tie Method for pier CAPs) to overcome the difficulties encountered in the practical application of the STM. In addition to providing a capability to rapidly apply the developed methodology to pier caps with up to eight columns, STM-CAP graphically presents the generated strut-and-tie model and allows the analyst to optimize the model through the use of vertical ties. This approach not only provides an efficient model but also intuitively educates the practicing engineers about the correct use of the strut-and-



Thickness of cap: 36 in (914 mm) Material: f'_c = 4 ksi (27.5 MPa) and f_y = 60 ksi (413 MPa)

Fig. 8. Sample pier cap details (Pier Cap 1).

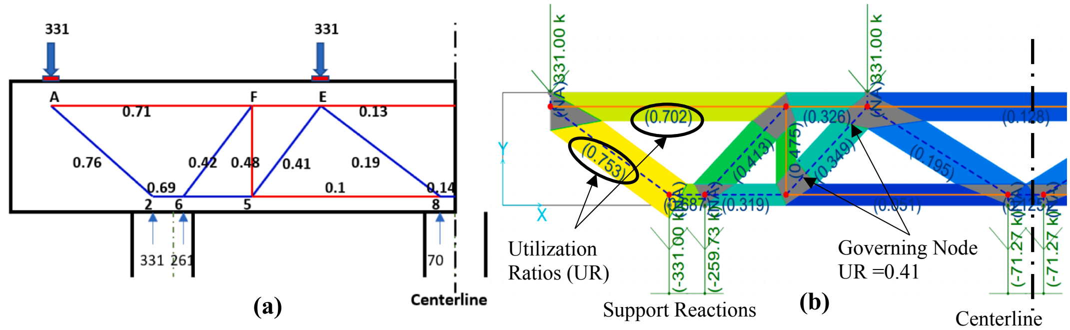
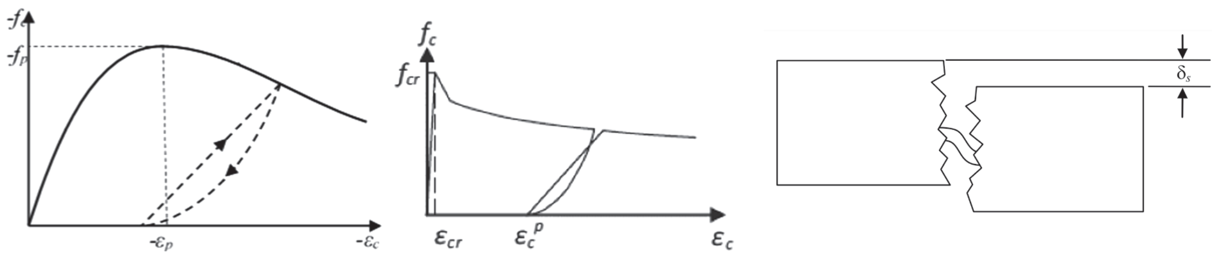


Fig. 9. Utilization ratios for the sample pier cap obtained from (a) proposed methodology (b) computer-aided strut-and-tie.



(a) Concrete pre-peak and post-peak response [31, 33, 36]

(b) Tension stiffening (Modified Bentz 2003) [31, 37]

(c) Rebar dowel action (Tassios-Crack Slip) [31, 38]

Fig. 10. Some of the material behavior models employed in VecTor2.

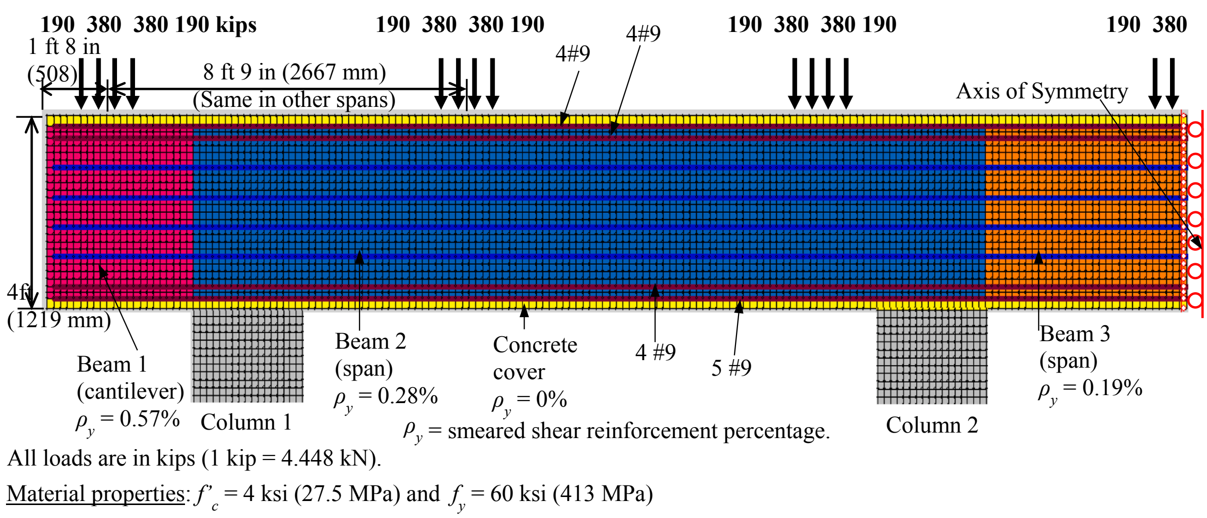


Fig. 11. A sample pier cap model (Pier Cap 4) using VecTor2.

tie methodology. The proposed methodology is verified by modeling eight bridge pier caps with a computer-aided strut-and-tie method. Nonlinear finite element analyses (NLFEA) are performed for an in-depth investigation and to obtain the complete response simulations of the pier caps in terms of the load capacity, deformation response, cracking behavior, and failure modes. Sectional analyses are also performed for comparison purposes. The results of the studies conducted support the following conclusions:

1. Most bridge cap beams are deep beams and exhibit nonlinear strain distributions. Analysis methods capable of representing the deep beam action are required to obtain accurate strength predictions;
2. The strut-and-tie method (STM) provides a viable analysis method for the strength prediction of deep pier caps;
3. The proposed STM methodology and associated computer code (STM-CAP) is shown to produce valid and efficient models while intuitively educating the analyst and overcoming the challenges encountered in the practical applications of the STM;
4. The sectional method systematically underestimates the shear capacities of deep pier caps. The deeper the pier cap, the higher the discrepancy between the calculated shear capacities. For shear span-to-depth (a/d) ratios of 0.50, the proposed STM predicts up to three times higher shear load capacities than the sectional method. As a/d ratio increases, the predictions from both models converges until an a/d ratio of 3.0;
5. Nonlinear finite element analysis (NLFEA) is useful in determining the complete load and deformation response of pier caps, including the cracking behavior and sequence of nonlinear phenomena;
6. NLFEA predicts higher shear capacities for deep regions than the STM. For a/d ratios between 1.5 and 2.0, NLFEA predicted up to two times larger shear load capacities. As the a/d ratio decreased (very deep members) or increased (slender members), the results from the

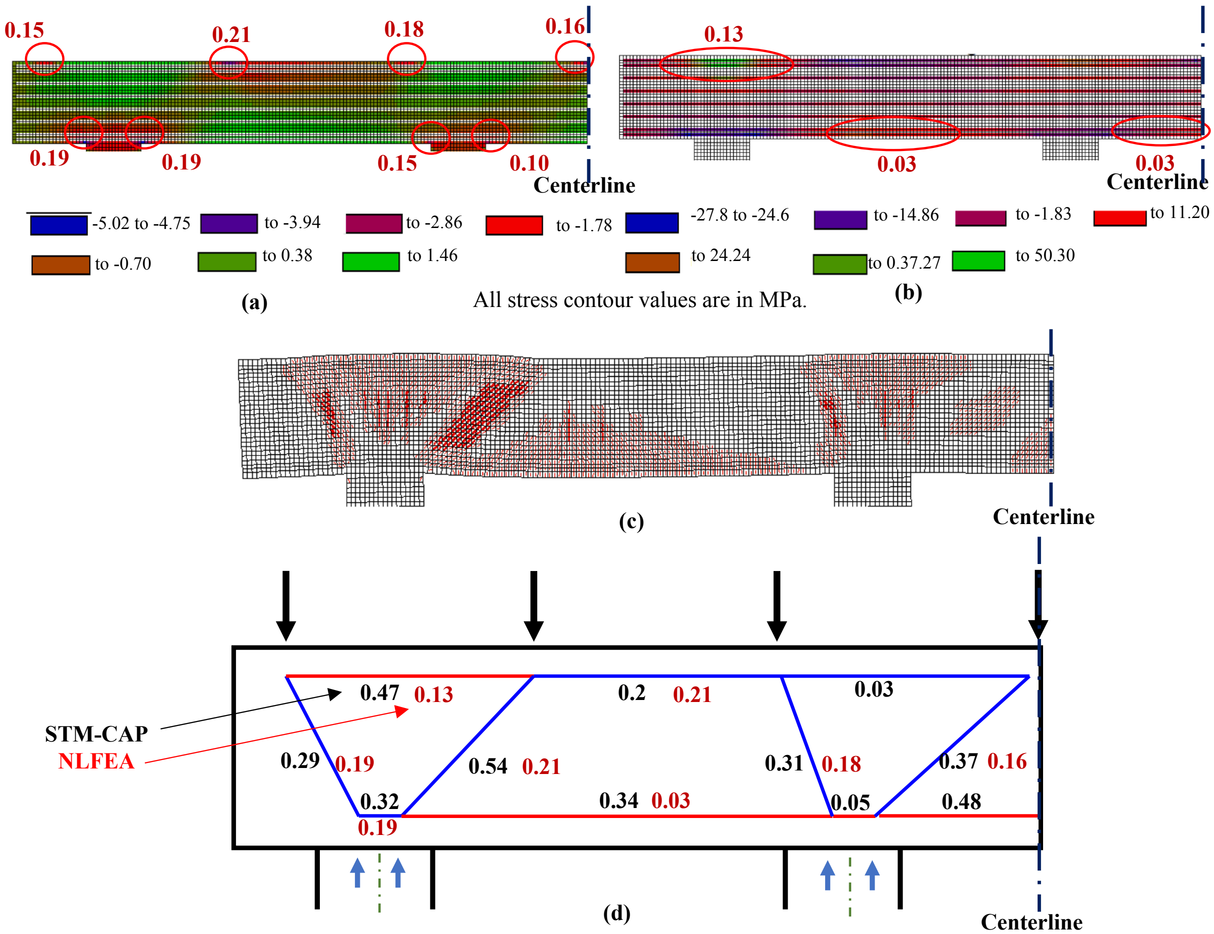


Fig. 12. Pier Cap 4 (a) utilization ratios from concrete stresses (b) utilization ratios from rebar stresses (c) crack pattern (d) combined utilization ratio comparisons for the proposed STM method and NLFEA.

nonlinear FEM and STM converged. The utilization ratios from the NLFEA is determined to be 40% on average of those from the STM;

7. The proposed STM methodology provides a good compromise between accuracy and complexity as compared to the sectional method and the NLFEA. While it is as simple and fast as the sectional method, it provides an accuracy more consistent and closer to the NLFEA.
8. If a pier cap is found overloaded by the proposed STM methodology, it is recommended to perform a NLFEA in an attempt to obtain higher and more accurate shear capacities. The NLFEA should be conducted by an experienced analyst using a computer software capable of simulating all expected material behaviors.
9. The proposed methodology can be expanded, in future studies, to include hammerhead (i.e., tee type) pier caps or pier caps with more than 8 columns.

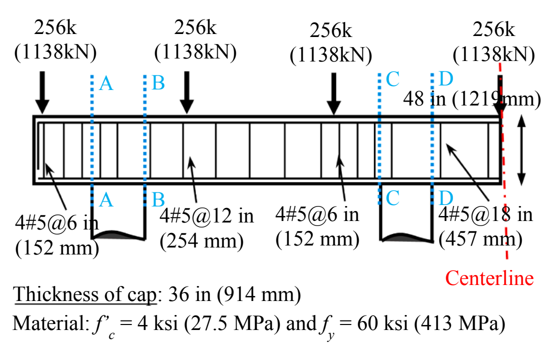


Fig. 13. A sample pier cap (Pier Cap 1) and associated transverse reinforcement details.

Table 1
Shear URs for various sections of Pier Cap 1.

Sectional method								STM-CAP
Section	Stirrup	V_c(kN)	V_s(kN)	V_n(kN)	V_u(kN)	Shear Force (kN)	Utilization ratio	Utilization ratio
A-A	4#5@ 6″	787	2153	2490	2647	1139	0.43	0.29
B-B	4#5@ 12″	787	1076	1864	1677	1139	0.68	0.54
C-C	4#5@ 6″	787	2153	2490	2647	1139	0.43	0.31
D-D	4#5@ 18″	787	716	1504	1352	569	0.42	0.37

V_c and V_s = shear strength due to concrete and stirrups respectively.
V_n and V_u = nominal and ultimate shear strength due to concrete respectively.

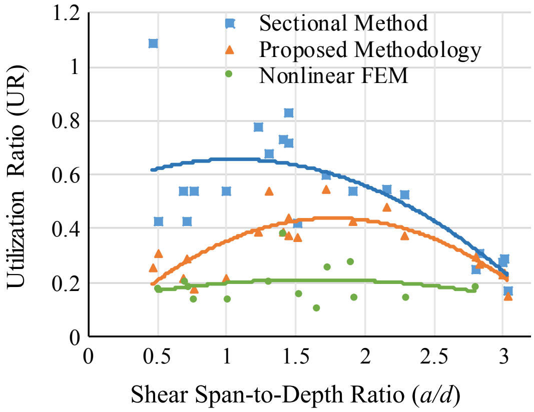


Fig. 14. Utilization ratio vs. a/d ratios obtained using different analysis technique.

Acknowledgements

The authors would like to thank the Ohio Department of Transportation, United States, (ODOT) for funding and supporting this research. The authors are grateful to the technical liaisons of ODOT: Mr. Matthew Blythe, P.E. and Ms. Andrea Parks, P.E. for their feedback and support, and to Ms. Michelle Lucas for managing the project. In addition, the authors acknowledge the efforts of MS student Anish Sharma for conducting the nonlinear finite element analyses contained in this study. The authors are also grateful to the Department of Civil and Environmental Engineering at the University of Toledo, Ohio, United States for providing the facilities required to conduct this research.

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