Chapter 1
Basic Concepts

Radiation Dosimetry I

Text: H.E Johns and J.R. Cunningham,
The physics of radiology, 4th ed.

Units

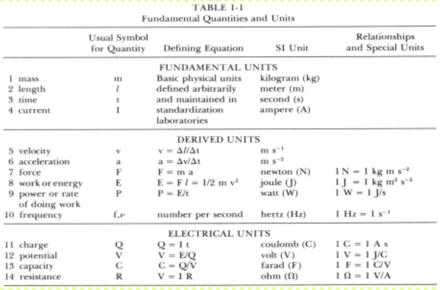
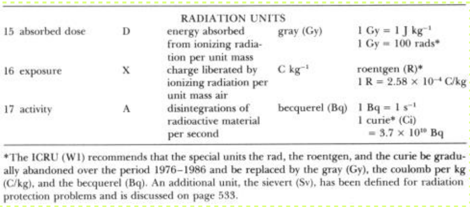


TABLE 1-1
Fundamental Quantities and Units

	Usual Symbol for Quantity	Defining Equation	SI Unit	Relationships and Special Units
FUNDAMENTAL UNITS				
1 mass	m	Basic physical units defined arbitrarily and maintained in standardization laboratories	kilogram (kg)	
2 length	l		meter (m)	
3 time	t		second (s)	
4 current	I		ampere (A)	
DERIVED UNITS				
5 velocity	v	$v = \Delta l/\Delta t$	m s^{-1}	
6 acceleration	a	$a = \Delta v/\Delta t$	m s^{-2}	
7 force	F	$F = m\ a$	newton (N)	1 N = 1 kg m s^{-2}
8 work or energy	E	$E = F\ l = 1/2\ m\ v^2$	joule (J)	1 J = 1 kg m^2 s^{-2}
9 power or rate of doing work	P	$P = E/t$	watt (W)	1 W = 1 J/s
10 frequency	f,ν	number per second	hertz (Hz)	1 Hz = 1 s^{-1}
ELECTRICAL UNITS				
11 charge	Q	$Q = I\ t$	coulomb (C)	1 C = 1 A s
12 potential	V	$V = E/Q$	volt (V)	1 V = 1 J/C
13 capacity	C	$C = Q/V$	farad (F)	1 F = 1 C/V
14 resistance	R	$V = I\ R$	ohm (Ω)	1 Ω = 1 V/A

- Special unit of energy: electron volt eV
- 1 eV=1.602x10^{-19} C x 1 volt= 1.602x10^{-19} J

Units



RADIATION UNITS				
15 absorbed dose	D	energy absorbed from ionizing radiation per unit mass	gray (Gy)	1 Gy = 1 J kg^{-1} 1 Gy = 100 rads*
16 exposure	X	charge liberated by ionizing radiation per unit mass air	C kg^{-1}	roentgen (R)* 1 R = 2.58 × 10^{-4} C/kg
17 activity	A	disintegrations of radioactive material per second	becquerel (Bq)	1 Bq = 1 s^{-1} 1 curie* (Ci) = 3.7 × 10^{10} Bq

*The ICRU (W1) recommends that the special units the rad, the roentgen, and the curie be gradually abandoned over the period 1976–1986 and be replaced by the gray (Gy), the coulomb per kg (C/kg), and the becquerel (Bq). An additional unit, the sievert (Sv), has been defined for radiation protection problems and is discussed on page 533.

- ***Absorbed dose***: describes energy deposition in water phantom, detector, patient, etc.
- ***Exposure***: describes the ability of radiation to ionize air, used for energies < 3 MeV (below typical radiotherapy range)
- ***Activity***: describes radioactive isotopes; 1 Ci is close to the activity of 1 g of radium (^{226}Ra)

Example 1

- For how many seconds a current of 2 μA must flow into 100 nf capacitor to produce a 50 V potential difference across the capacitor?

A. 1
B. 2
[✓ C. 2.5]
D. 5
E. 25

$$C = \frac{Q}{V} = \frac{I \cdot t}{V}$$

$$t = \frac{CV}{I} = \frac{100\times10^{-9}\,\mathrm{f}\cdot 50\mathrm{V}}{2\times10^{-6}\,\mathrm{A}} = 2.5\,\mathrm{s}$$

Example 2

- 2-cc-volume (cm^3) ionization chamber is placed in a radiation field of 100 R/s. What is the current generated in amperes (ρ_{air} = 0.0013 g/cm^3)?

A. 5.1 x 10 $^{-11}$
B. 6.3 x 10 $^{-10}$
C. 5.1 x 10 $^{-9}$
[✓ D. 6.7 x 10 $^{-8}$]
E. 5.1 x 10 $^{-7}$

$$1\mathrm{R} = 2.58\times10^{-4}\ \mathrm{C/kg}$$

$$100\ \mathrm{R/s} = 100\cdot 2.58\times10^{-4}\ \mathrm{C/(kg\cdot s)} = 2.58\times10^{-2}\mathrm{C/(kg\cdot s)}$$

$$m_{air} = V_{air}\,\rho_{air} = 2\ \mathrm{cm}^3\cdot 0.0013\,\mathrm{g/cm}^3 = 0.0026\,\mathrm{g} = 2.6\times10^{-6}\mathrm{kg}$$

$$I = 2.58\times10^{-2}\mathrm{C/(kg\cdot s)}\cdot 2.6\times10^{-6}\mathrm{kg} = 6.71\times10^{-8}\ \mathrm{A}$$

Atoms

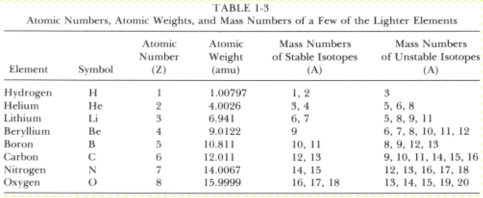


TABLE 1-3
Atomic Numbers, Atomic Weights, and Mass Numbers of a Few of the Lighter Elements

Element	Symbol	Atomic Number (Z)	Atomic Weight (amu)	Mass Numbers of Stable Isotopes (A)	Mass Numbers of Unstable Isotopes (A)
Hydrogen	H	1	1.00797	1, 2	3
Helium	He	2	4.0026	3, 4	5, 6, 8
Lithium	Li	3	6.941	6, 7	5, 8, 9, 11
Beryllium	Be	4	9.0122	9	6, 7, 8, 10, 11, 12
Boron	B	5	10.811	10, 11	8, 9, 12, 13
Carbon	C	6	12.011	12, 13	9, 10, 11, 14, 15, 16
Nitrogen	N	7	14.0067	14, 15	12, 13, 16, 17, 18
Oxygen	O	8	15.9999	16, 17, 18	13, 14, 15, 19, 20

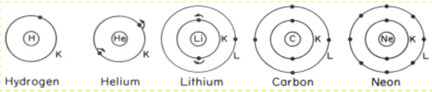
- Atomic number Z: number of electrons (protons)
- Mass number A: total number of protons + neutrons
- Atomic weight (mass): Carbon 12 has atomic mass 12.0000 amu (6 protons + 6 neutrons)
- Typical notation:

$^{A}_{Z}$Element

Atoms

- *Isotopes*: have the same number of protons, but different number of neutrons
  - Same atomic number Z, and number of electrons
  - Same chemical properties
  - Different mass number A
- *Isotones*: have the same number of neutrons (A-Z)
- *Isobars*: have the same atomic mass A (total number of protons + neutrons)
- There is redundancy in full notation $^{A}_{Z}X$
  - Atomic number Z determines the element X
- *Isomers*: the same A and Z, different nuclear energy state (stable vs. metastable, or excited); notation: $^{A}_{Z}{}^{m}X$

Atomic energy levels



- Electron orbits have defined energies (levels)
- The innermost is K (up to two electrons with opposite spins), next is L (up to eight electrons), etc.
- Filled outer shell – chemically inert atom

Atomic energy levels

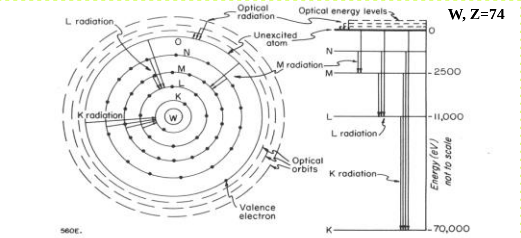


Figure 1-2. Schematic diagram of the tungsten atom showing the shells on the left and an energy level diagram on the right. The energy scale in eV is not drawn to scale. X-radiation arises through transitions of electrons to the K, L, and M shells. Optical radiation arises by transitions of the valence electron from optical orbits to the O shell.

X-rays arise from transitions to K, L, M levels (eV to keV energy range)

Nucleus and its energy levels

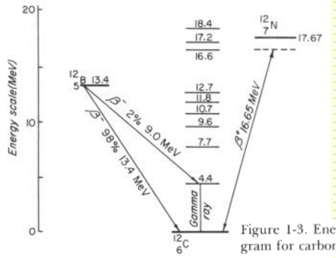


Figure 1-3. Energy level diagram for carbon 12.

- System of nuclear energy levels
- Nuclear transitions produce photons (γ-rays) and particles in MeV energy range

Mass and energy

- Photon energy: E = $h\nu = hc/\lambda$
- Mass-to-energy conversion: E = mc^2
- Relativistic mass: $$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$
- Rest mass m_0
- Kinetic energy: K.E. = $mc^2 - m_0c^2$

Mass and energy

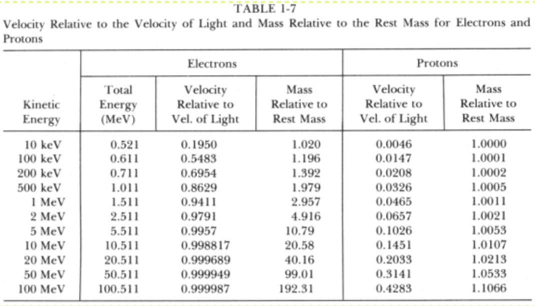


TABLE 1-7

Velocity Relative to the Velocity of Light and Mass Relative to the Rest Mass for Electrons and Protons

	Electrons			Protons	
Kinetic Energy	Total Energy (MeV)	Velocity Relative to Vel. of Light	Mass Relative to Rest Mass	Velocity Relative to Vel. of Light	Mass Relative to Rest Mass
10 keV	0.521	0.1950	1.020	0.0046	1.0000
100 keV	0.611	0.5483	1.196	0.0147	1.0001
200 keV	0.711	0.6954	1.392	0.0208	1.0002
500 keV	1.011	0.8629	1.979	0.0326	1.0005
1 MeV	1.511	0.9411	2.957	0.0465	1.0011
2 MeV	2.511	0.9791	4.916	0.0657	1.0021
5 MeV	5.511	0.9957	10.79	0.1026	1.0053
10 MeV	10.511	0.998817	20.58	0.1451	1.0107
20 MeV	20.511	0.999689	40.16	0.2033	1.0213
50 MeV	50.511	0.999949	99.01	0.3141	1.0533
100 MeV	100.511	0.999987	192.31	0.4283	1.1066

Mass-to-energy conversion factors:
1 electron mass = 0.511 MeV
1 amu = 931.5 MeV

Example 3

From the following table of particle rest masses, calculate the gamma energy emitted when a proton captures a neutron to create a deuteron. 1 amu corresponds to the rest mass energy of 931.5 MeV

particle rest mass.	**amu**
Proton	1.00727
Neutron	1.00866
Deuteron	2.01355

A. 1.875 MeV
B. 2.02 MeV
[✓ C.] 2.22 MeV
D. 2.38 MeV
E. 4.03 MeV

$$E_\gamma = E_{rmP} + E_{rmN} - E_{rmD} =$$
$$(1.00727 + 1.00866 - 2.01355) \times 931.5\,\text{MeV} =$$
$$0.00238 \times 931.5\,\text{MeV} = 2.21697\,\text{MeV} \approx 2.22\,\text{MeV}$$

Example 4

- Find the velocity of an electron accelerated through the potential difference of 5 MeV.

$$KE = 5\text{MeV}$$
$$E = m_0 c^2 \cdot \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right)$$
$$5\,\text{MeV} = 0.511\,\text{MeV} \cdot \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right)$$
$$v^2/c^2 \sim 1 - 0.01 = 0.99$$
$$v/c \sim 0.995$$
$$v \sim 0.995 \cdot 3 \times 10^8 \;\; \text{m/c}$$

Exponential behavior

$$\frac{dN}{dt} = -\lambda N = -\frac{1}{t_a} N$$
$$N = N_0 e^{-\lambda t} = N_0 e^{-t/t_a} = N_0 2^{-t/t_h}$$

The sign determines the process: decay or growth λ

λ - transformation constant

t_a – average life; t_h – half-life;

$\lambda = 1/t_a = \ln 2/t_h = 0.693/t_h$

Exponential behavior

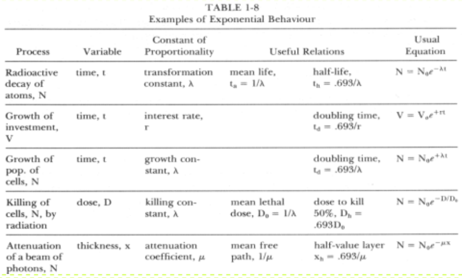


TABLE 1-8
Examples of Exponential Behaviour

Process	Variable	Constant of Proportionality	Useful Relations		Usual Equation
Radioactive decay of atoms, N	time, t	transformation constant, λ	mean life, $t_a = 1/\lambda$	half-life, $t_h = .693/\lambda$	$N = N_0 e^{-\lambda t}$
Growth of investment, V	time, t	interest rate, r		doubling time, $t_d = .693/r$	$V = V_0 e^{+rt}$
Growth of pop. of cells, N	time, t	growth constant, λ		doubling time, $t_d = .693/\lambda$	$N = N_0 e^{+\lambda t}$
Killing of cells, N, by radiation	dose, D	killing constant, λ	mean lethal dose, $D_0 = 1/\lambda$	dose to kill 50%, $D_h = .693D_0$	$N = N_0 e^{-D/D_0}$
Attenuation of a beam of photons, N	thickness, x	attenuation coefficient, μ	mean free path, $1/\mu$	half-value layer $x_h = .693/\mu$	$N = N_0 e^{-\mu x}$

- If more than one process takes place: $$\frac{dN}{dt} = -(\lambda_1 + \lambda_2 + \cdots + \lambda_n) N$$

Exponential behavior

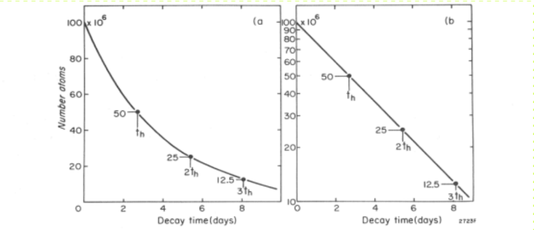


Figure 1-4. Graphs showing the exponential decay of a source of 10^8 atoms of ^{198}Au with half-life of 2.69 days. The vertical scale on the left is linear while the one on the right is logarithmic.

$$N = N_0 2^{-t/t_h}$$

Example 5

In the case of simultaneous physical decay and biological clearance, when T_p, is the physical half-life and T_b is the biological half-life, the effective half-life T_{eff} is generally equal to:

A. $T_p + T_b$
B. $1/T_p + 1/T_b$
C. $T_p \times T_b$
[✓ D.] $1/(1/T_p + 1/T_b)$
E. $\sqrt{T_p \times T_b}$

$$\frac{dN}{dt} = -\frac{1}{t_{a.eff}} N = -\frac{1}{t_{a.p}} N - \frac{1}{t_{a.b}} N$$
$$t_{a.eff} = 1 \Big/ \left(\frac{1}{t_{a.p}} + \frac{1}{t_{a.b}} \right)$$

Since $T_{eff} = t_{a.eff} / \ln 2$

$$T_{eff} = 1 \Big/ \left(\frac{1}{T_p} + \frac{1}{T_b} \right)$$

Chapter 2
The Production and Properties of X Rays

Radiation Dosimetry I

Text: H.E Johns and J.R. Cunningham, The physics of radiology, 4th ed.

http://www.utoledo.edu/med/depts/radther

X-ray tube design

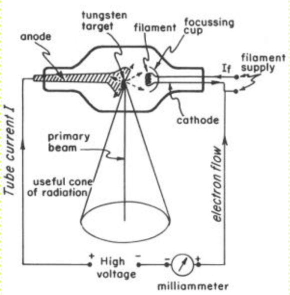


Figure 2-1 (a). Schematic diagram of x-ray tube and circuit

- Filament is heated, releasing electrons via thermionic emission (V_f ~ 10V, I_f ~ 4A, resulting in T>2000°C)
- X-rays are produced by high-speed electrons bombarding the target
- Typically < 1% of energy is converted to x-rays; the rest is heat

X-ray tube current

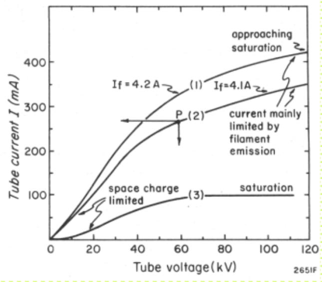


Figure 2-1 (b). Tube current as function of tube voltage. Curve 3: tube operating at a lower current

- Electron cloud near the filament creates space charge region, opposing the release of additional electrons
- Increase in tube voltage increases tube current; limited by filament emission
- High filament currents and tube voltage of 40 to 140kV must be used

X-ray tube: power source

- The source of electrical power is usually ac (easier to transmit through power lines)
- X-ray tubes are designed to operate at a single polarity: positive anode, negative cathode
- Need to manipulate available power source (suppress or rectify wrong polarity)
- The highest x-ray production efficiency can be achieved at a constant potential

Alternating currents and voltages

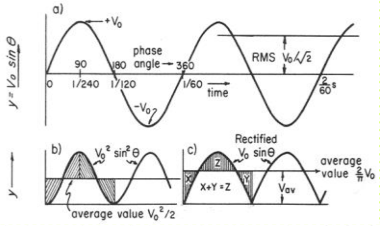


Figure 2-3. (a) Diagram to illustrate the variations with time (or phase angle) of an alternating voltage at 60 Hz. (b) Diagram to show that the RMS value is $V_0/\sqrt{2}$. (c) Diagram to illustrate that the average value of a sinusoidal rectified voltage (or current) is $2/\pi$ times the peak value.

- Phase changes from 0 to 360° during the 1 cycle time of 1/60 s
- Negative wave is suppressed or rectified
- Averaging: $V_{av} = \frac{2}{\pi} V_0$ or $V_{rms} = \frac{1}{\sqrt{2}} V_0$

Alternating currents and voltages

Current flows in one direction: series of pulses

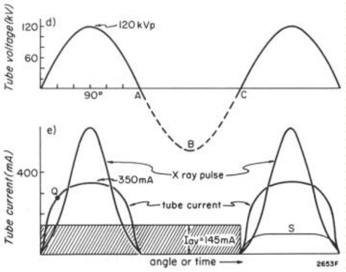


Figure 2.3 (d) High voltage from secondary of transformer with peak value of 120 kV. The inverse part of the cycle is ABC. (e) Tube current for the circuit of Figure 2-2 when the x ray tube has the characteristics of curve 2 of Figure 2-1b and the tube voltage is given by Figure 2-3d. The intensity of the resulting x ray pulse calculated assuming x ray production is proportional to IV^2 is also given. Curve S is the current pulse using the saturation curve 3 of Figure 2-1b.

Rectification

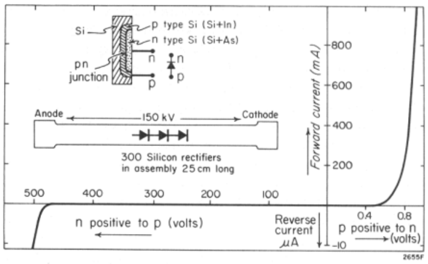
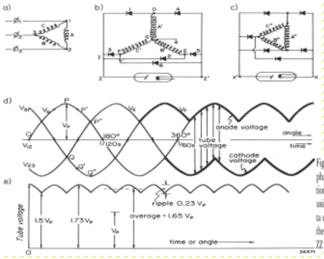


Figure 2-4. Schematic diagram of a p,n silicon rectifier. The operating characteristics are for the MR2272, a typical silicon rectifier. Note that the voltage and current scales are different for the forward and reverse directions. In the insert is shown a Machlett rectifier, which consists of some 300 rectifiers in series in a tube 25 cm long and capable of withstanding an inverse voltage of 150 kV.

Three phase units

- Need to increase pulse repetition rate to deliver high x ray flux in a short period of time
- Three phase units: voltage between any pair of 3 wires



Tube potential is almost constant, with a “ripple”

Figure 2-6. Diagram to illustrate 3 phase power and its use in x ray generators. (a) Three phase power line 1,2,3 connected to three primary windings A, B, C in a delta configuration. (b) Connections of the secondaries A', B', C' in Y configuration to an x ray tube using 6 rectifiers giving a “6 pulse” system. (c) Secondaries A", B", C", connected in a delta configuration to also give a 6 pulse system. (d) Wave forms for either V_{12}, V_{23}, V_{31}, or the voltages on D, E, and F, relative to ground. (e) Tube voltage, that is the voltage across ZZ', as a function of time.

Example 1

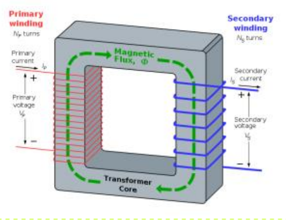
- Which type of x-ray generator produces the highest effective tube voltage, assuming the peak voltage is applied across the tube?

A. One-phase
B. Three-phase
[✓ C.] Constant potential
D. The effective voltage is the same for all types above

In C - effective voltage = peak voltage

Example 2

Ratio of the turns in a transformer is N. Given an input RMS (primary) voltage, what is the peak output (secondary) voltage?



Faraday’s law:

$$V_S = N_S \frac{d\Phi}{dt}; \quad V_P = N_P \frac{d\Phi}{dt}$$

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} = N$$

$$V_S = N\sqrt{2} \cdot V_{P\,RMS}$$

$$V_{RMS} = V_0 / \sqrt{2}$$

Diagnostic x ray tubes

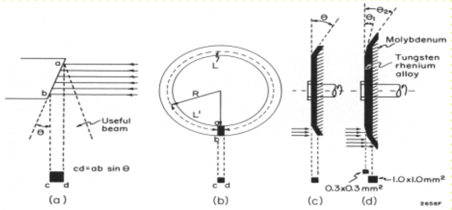


Figure 2-7. Diagrams illustrating anode construction and focal spots. (a) Line focus fixed anode. (b) Line focus rotating anode, viewed from the end. (c) Line focus rotating anode viewed from the side. (d) Alternative arrangement of rotating anode using two separate “tracks” at different angles as in the Siemens “Biangulix” series.

- The objective is to deliver x ray beam from a point source in a very short time
- To overcome heating need to spread electrons over some area that appears as a point

Example 3

- What energy (kJ) is imparted to a rotating anode (0.25 kg) during a 2 s exposure that produced a temperature of 2500ºC. Specific heat of tungsten is 0.035 kcal/kgºC, and 1 cal = 4.186 J

A. 17.9
B. 45.7
C. 87.5
[✓ D.] 91.5
E. 182.9

Duration of exposure is irrelevant

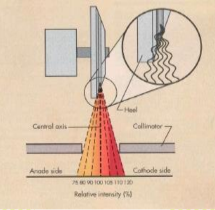
$$E = cm\Delta T =$$

$$0.035 \cdot 4.186 \times 10^3 \cdot 0.25 \cdot 2500 =$$

$$91.5 \times 10^3 = 91.5\,\text{kJ}$$

Diagnostic x ray tubes

Image from
http://www.slideshare.net/jdtomines/x-ray-tube



Intensity profile

- X-rays that are emitted from the target travel through different thickness of cathode material
- *Heel effect*: radiation intensity toward the cathode side of the x-ray tube is higher than on the anode side
- Cathode is typically mounted over the thicker part of the patient to balance the amount of transmitted photons on the imager

Rating of diagnostic tubes

- Focal spot loading determines the maximum permissible exposure: there is a maximum power that can be tolerated before target starts melting ($T_{melting}$=3400^{o}C for tungsten)
- Anode cooling and housing cooling rates determine the number of exposures that may be given in a sequence

Rating of diagnostic tubes

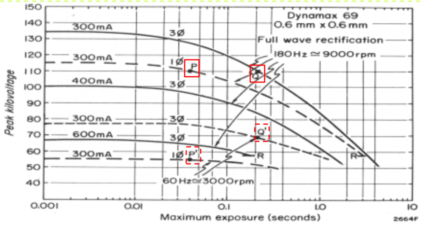
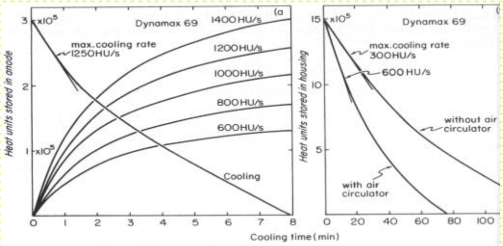


Figure 2-11. Curves giving the maximum kV$_p$ for a given exposure for a Machlett Dynamax 69 under a number of different excitation conditions. Focal spot 0.6 mm × 0.6 mm, full wave rectification. Data for single exposures.

- The combination of current and voltage must lie to the left of the appropriate curve
- The maximum duration of a single exposure depends on spot size, anode rotation speed, current, voltage, power supply type

Rating of diagnostic tubes



- Heat stored in the anode and its cooling rate limits the number of exposures given in a sequence
- Heat unit HU characterizes the energy deposited within the anode in a single exposure

X ray tubes for radiotherapy

- Mostly for superficial treatments
- No need for a small spot source
- The instantaneous energy input is small (about 1/10) but the average energy input is ~ 10 times greater compared with a diagnostic tube
- Due to much higher energy (>200keV) of electrons bombarding the target, there is a problem of secondary electrons emerging from the target
  - Solution: the target is placed in a "hood" - hollow tube with copper shielding intercepting the secondary electrons

X-ray spectra

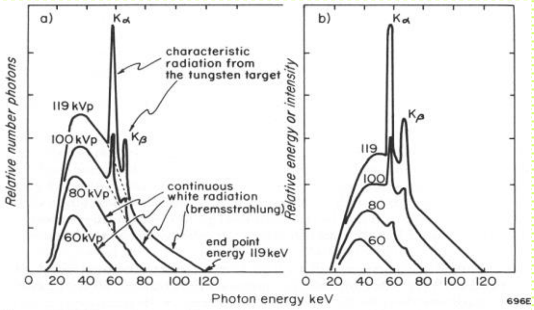
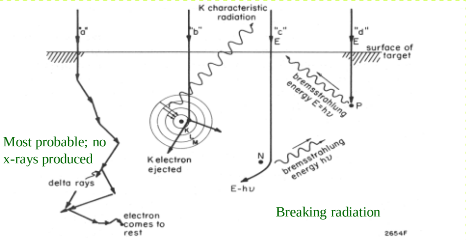


Figure 2-13. Observed spectra from a diagnostic x ray tube excited at 60, 80, 100, and 119 kV$_p$, due to Yaffe (Y1), added filter 2 mm A1.

Interactions of electrons with the target to give x rays



Most probable; no x-rays produced

Breaking radiation

Figure 2-14. Typical electron interactions with a target. (a) Electron suffers ionizational losses, giving rise to delta rays and eventually heat. (b) The electron ejects a K electron, giving rise to characteristic radiation. (c) Collision between an electron of energy E and a nucleus, leading to bremsstrahlung of energy hν. The electron recedes from the "collision" with energy E − hν. (d) Rare collision when the electron is completely stopped in one collision, giving rise to a photon of energy E = hν.

Characteristic radiation

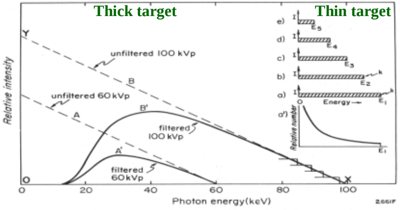
TABLE 2-3
Principal Emission Lines in keV for Tungsten and Molybdenum

K Lines Tungsten				L Lines Tungsten			
Transition	Symbol	Energy (keV)	Relative Number	Transition	Symbol	Energy (keV)	Relative Number
K-N$_{II}$N$_{III}$	Kβ_2	69.081	7	L$_I$-N$_{III}$	Lγ_5	11.674	10
K-M$_{III}$	Kβ_1	67.244	21	L$_{II}$-N$_{IV}$	Lγ_1	11.285	24
K-M$_{II}$	Kβ_3	66.950	11	L$_{III}$-N$_V$	Lβ_2	9.962	18
K-L$_{III}$	Kα_1	59.321	100	L$_I$-M$_{III}$	Lβ_3	9.817	37
K-L$_{II}$	Kα_2	57.984	58	L$_{II}$-M$_{IV}$	Lβ_1	9.670	127
K lines Molybdenum				L$_I$-M$_{II}$	Lβ_4	9.523	29
K-M$_{II}$M$_{III}$	Kβ_{31}	19.602	24	L$_{III}$-M$_V$	Lα_1	8.395	100
K-L$_{III}$	Kα_1	17.479	100	L$_{III}$-M$_{IV}$	Lα_2	8.333	11
K-L$_{II}$	Kα_2	17.375	52				

From Storm and Israel (S1)

- Different transitions have different probabilities, according to quantum mechanics selection rules (some transitions are forbidden)

Bremsstrahlung interaction



Thick target

Thin target

Figure 2-15. Relative energy or intensity I, in each photon energy interval produced when a beam of monoenergetic electrons of energy E_1 bombard a thin target. The distribution a' is the data of a converted to a number distribution. Curves b, c, d, and e are thin target intensity spectra similar to a but for electron energies of E_2, E_3, E_4, and E_5. The main diagram shows *thick target* spectra (dotted lines A and B) produced by the superposition of many thin target spectra when the target is bombarded with 60 and 100 keV electrons. The solid curves A' and B' were obtained from A and B by taking into account the attenuation of 2 mm A1.

- Thin target approximation: one collision per electron
- Thick target approximations: $I(E) = C Z (E_{max} - E)$

Example 4

- The energy levels of K, L, and M shells in tungsten are -69.5, -11.0, and -2.5 keV. What photon energies will be present in its characteristic X-ray spectrum?

[✓ A.] 67.0, 58.5, 8.5 keV

Photon energies are equal to the differences between corresponding energy levels

B. 80.5, 72.0, 13.5 keV
C. 69.5, 1 1 .O, 2.5 keV
D. Continuous spectrum from 2.5 to 69.5 keV
E. Continuous spectrum below 2.5 keV

Example 5

- A target material has the following binding energies: K=30 keV, L=4 keV, M=0.7 keV. If 40.0 keV electrons are fired at the target, what kind of x-rays can have the following energies?
- 6-1: 34 keV
- 6-2: 26 keV
- 6-3: 40.7 keV

A. Characteristic only
B. Bremsstrahlung only
C. Both A and B
D. Neither A nor B

Answers:
- 6-1: B
- 6-2: C
- 6-3: D

Summary

- X-rays are produced via bremsstrahlung interactions of high-energy electrons within a target
  - Efficiency is low, most energy goes into target heating
  - Continues spectrum includes characteristic x-ray due to target material
- Required high voltage (~50-100 kV) to accelerate electrons
  - Power source: ac to dc conversion