Chapter 1
Basic Concepts

Radiation Dosimetry I


Units

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Unit</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>electron</td>
<td>energy absorbed from ionizing radiation per unit mass of matter</td>
</tr>
<tr>
<td>dE/dx</td>
<td>energy loss</td>
<td>energy absorbed from ionizing radiation per unit path length</td>
</tr>
<tr>
<td>I</td>
<td>current</td>
<td>number per second</td>
</tr>
<tr>
<td>D</td>
<td>absorbed dose</td>
<td>energy absorbed from ionizing radiation per unit mass of matter</td>
</tr>
</tbody>
</table>

Example 1

For how many seconds a current of 2 mA must flow into 100 nF capacitor to produce a 50 V potential difference across the capacitor?

A. 1
B. 2
C. 2.5
D. 5
E. 25

\[ I = \frac{V}{C} \]

\[ V = 50 \text{ V} \]

\[ C = 100 \text{ nF} \]

\[ I = \frac{50}{100 \times 10^{-9}} = 5 \times 10^{7} \text{ A} \]

Example 2

2-cc-volume (cm³) ionization chamber is placed in a radiation field of 100 R/s. What is the current generated in amperes (ρ_air = 0.0013 g/cm³)?

A. 5.1 x 10⁻¹¹
B. 6.3 x 10⁻¹⁰
C. 5.1 x 10⁻⁹
D. 6.7 x 10⁻⁸
E. 5.1 x 10⁻⁷

\[ I_R = \frac{2.58 \times 10^{-4}}{22.4} \text{ C/kg} \]

\[ 100 \text{ R/s} = 100 \times 2.58 \times 10^{-4} \text{ C/kg} \times 200 \times 10^{-4} \text{ C/kg} \]

\[ m_{cap} = \frac{V}{I} \times \rho_{air} = 2 \text{ cm}^3 \times 0.0013 \text{ g/cm}^3 = 0.00026 \text{ g} = 2.6 \times 10^{-6} \text{ kg} \]

\[ I = \frac{2.58 \times 10^{-4} \text{ C/kg} \times 2.6 \times 10^{-6} \text{ kg}}{6.71 \times 10^{-6} \text{ A}} \]
Atoms

- **Isotopes**: have the same number of protons, but different number of neutrons
  - Same atomic number $Z$, and number of electrons
  - Same chemical properties
  - Different mass number $A$
- **Isotones**: have the same number of neutrons ($A-Z$)
- **Isobars**: have the same atomic mass $A$ (total number of protons + neutrons)
- There is redundancy in full notation $A_Z^X$
  - Atomic number $Z$ determines the element $X$
  - *Isomers*: the same $A$ and $Z$, different nuclear energy state (stable vs. metastable, or excited); notation: $A_Z^X$

Atomic energy levels

- Electron orbits have defined energies (levels)
- The innermost is K (up to two electrons with opposite spins), next is L (up to eight electrons), etc.
- Filled outer shell – chemically inert atom

Atomic energy levels

- System of nuclear energy levels
- Nuclear transitions produce photons (γ-rays) and particles in MeV energy range

Nucleus and its energy levels

- X-rays arise from transitions to K, L, M levels (eV to keV energy range)

Mass and energy

- Photon energy: $E = h\nu = hc/\lambda$.
- Mass-to-energy conversion: $E = mc^2$
- Relativistic mass: $m = \sqrt{1 - v^2/c^2}$
- Rest mass $m_0$
- Kinetic energy: $K.E. = mc^2 - m_0c^2$

Mass-to-energy conversion factors:
- 1 electron mass = 0.511 MeV
- 1 amu = 931.5 MeV
Example 3
From the following table of particle rest masses, calculate the gamma energy emitted when a proton captures a neutron to create a deuteron. 1 amu corresponds to the rest mass energy of 931.5 MeV.

<table>
<thead>
<tr>
<th>particle rest mass</th>
<th>amu</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proton</td>
<td>1.00727</td>
</tr>
<tr>
<td>Neutron</td>
<td>1.00866</td>
</tr>
<tr>
<td>Deuteron</td>
<td>2.01355</td>
</tr>
</tbody>
</table>

A. 1.875 MeV  
B. 2.02 MeV  
C. 2.22 MeV  
D. 2.38 MeV  
E. 4.03 MeV

Example 4

• Find the velocity of an electron accelerated through the potential difference of 5 MeV.

KE=5MeV

\[ E = m_e c^2 \left( \frac{1}{\sqrt{1-v^2/c^2}} - 1 \right) \]

v/c = 0.995

v ~ 0.995 \times 3 \times 10^8 \text{ m/s}

Exponential behavior

\[ \frac{dN}{dt} = -\lambda N \quad \Rightarrow \quad N = N_0 e^{-\lambda t} = N_0 2^{-\lambda t_{\lambda}} \]

The sign determines the process: decay or growth \( \lambda \).

\( \lambda \) - transformation constant

\( t_{\lambda} \) - average life; \( t_{\lambda/h} \) - half-life;

\( \lambda = 1/t_{\lambda} = \ln 2/t_{\lambda} = 0.693/t_{\lambda} \)

Exponential behavior

<table>
<thead>
<tr>
<th>Process</th>
<th>Variable</th>
<th>Constant of Proportionality</th>
<th>Useful Relations</th>
<th>Useful Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radioactive decay of atoms, ( N )</td>
<td>time, ( t )</td>
<td>transformation constant, ( \lambda )</td>
<td>mean life, ( T_{\lambda} )</td>
<td>N = N_0 e^{-\lambda t}</td>
</tr>
<tr>
<td>Growth of investment, ( N )</td>
<td>time, ( t )</td>
<td>interest rate, ( r )</td>
<td>doubling time, ( T_{\lambda} )</td>
<td>N = N_0 (1 + r)^t</td>
</tr>
<tr>
<td>Growth of pop. of cells, ( N )</td>
<td>time, ( t )</td>
<td>growth constant, ( k )</td>
<td>doubling time, ( T_{\lambda} )</td>
<td>N = N_0 e^{kt}</td>
</tr>
<tr>
<td>Killing of cells, ( N ), by radiation</td>
<td>dose, ( D )</td>
<td>killing constant, ( \alpha )</td>
<td>dose to kill ( 50% ), ( D_{50} )</td>
<td>N = N_0 e^{-\alpha D}</td>
</tr>
<tr>
<td>Accumulation of a trace of photon, ( N )</td>
<td>time, ( t )</td>
<td>absorption coefficient, ( \mu )</td>
<td>mean free path, ( \lambda_0 )</td>
<td>N = N_0 e^{-\mu t}</td>
</tr>
</tbody>
</table>

• If more than one process takes place:

\[ \frac{dN}{dt} = (\dot{\lambda}_1 + \dot{\lambda}_2 + \ldots + \dot{\lambda}_n)N \]

Example 5

In the case of simultaneous physical decay and biological clearance, when \( T_p \) is the physical half-life and \( T_b \) is the biological half-life, the effective half-life \( T_{\text{eff}} \) is generally equal to:

A. \( T_p + T_b \)  
B. \( 1/T_p + 1/T_b \)  
C. \( T_p \times T_b \)  
D. \( 1/(1/T_p + 1/T_b) \)  
E. \( \sqrt{T_p \times T_b} \)

\[ \frac{dN}{dt} = -\lambda_{\text{eff}} N = -\frac{1}{T_{\text{eff}}} N = -\frac{1}{T_p} N - \frac{1}{T_b} N \]

\[ J_{\text{coup}} = \sqrt{\left( \frac{1}{T_{\text{eff}}} \right)^2 - \left( \frac{1}{T_p} \right)^2} \]

Since \( T_{\text{eff}} = \frac{t_{\lambda/b}}{\ln 2} \)

\[ T_{\text{eff}} = \sqrt{\left( \frac{1}{T_p} \right)^2 + \left( \frac{1}{T_b} \right)^2} \]
Chapter 2
The Production and Properties of X Rays
Radiation Dosimetry I

http://www.utoledo.edu/med/depts/radther

X-ray tube design

- Filament is heated, releasing electrons via thermionic emission ($V_f \sim 10V$, $I_f \sim 4A$, resulting in $T>2000^\circ C$)
- X-rays are produced by high-speed electrons bombarding the target
- Typically < 1% of energy is converted to x-rays; the rest is heat

Figure 2-1 (a). Schematic diagram of x-ray tube and circuit

X-ray tube current

- Electron cloud near the filament creates space charge region, opposing the release of additional electrons
- Increase in tube voltage increases tube current; limited by filament emission
- High filament currents and tube voltage of 40 to 140kV must be used

Figure 2-1 (b). Tube current as function of tube voltage. Curve 3: tube operating at a lower current

X-ray tube: power source

- The source of electrical power is usually ac (easier to transmit through power lines)
- X-ray tubes are designed to operate at a single polarity: positive anode, negative cathode
- Need to manipulate available power source (suppress or rectify wrong polarity)
- The highest x-ray production efficiency can be achieved at a constant potential

Alternating currents and voltages

- Phase changes from 0 to 360° during the 1 cycle time of 1/60 s
- Negative wave is suppressed or rectified
- Averaging: $V_{av} \approx \frac{2}{\pi} V_0$ or $V_{av} = \frac{1}{\sqrt{2}} V_0$

Alternating currents and voltages

Current flows in one direction: series of pulses

Figure 2-3 (b) High voltage from secondary of transformer with peak value of 120 kV. The inverse part of the cycle is ABC. (c) Tube current for the circuit of Figure 2-2 when the x-ray tube has the characteristics of curve 2 of Figure 2-3 and the tube voltage is given by Figure 2-3M. The intensity of the resulting x-ray pulse calculated assuming x-ray production is proportional to $V^2$ is also given. Curve 6 is the current pulse using the saturation curve 3 of Figure 2-3b.
Rectification

Three phase units

- Need to increase pulse repetition rate to deliver high x-ray flux in a short period of time
- Three phase units: voltage between any pair of 3 wires

Tube potential is almost constant, with a “ripple”

Example 1

- Which type of x-ray generator produces the highest effective tube voltage, assuming the peak voltage is applied across the tube?

A. One-phase
B. Three-phase
C. Constant potential
D. The effective voltage is the same for all types above

In C - effective voltage = peak voltage

Example 2

Ratio of the turns in a transformer is N. Given an input RMS (primary) voltage, what is the peak output (secondary) voltage?

Faraday’s law:

\[ V_c = N \frac{d\Phi}{dt}; \quad V_p = N_x \frac{d\Phi}{dt} \]

\[ V_c = N_x \Rightarrow N \]

\[ V_p = N \sqrt{2} V_{rms} \]

\[ V_{rms} = \frac{V_c}{\sqrt{2}} \]

Example 3

- What energy (kJ) is imparted to a rotating anode (0.25 kg) during a 2 s exposure that produced a temperature of 2500°C. Specific heat of tungsten is 0.035 kcal/kg°C, and 1 cal = 4.186 J

A. 17.9
B. 45.7
C. 87.5
D. 91.5
E. 182.9

Duration of exposure is irrelevant

\[ E = cm\Delta T \]

\[ 0.035 \times 4.186 \times 10^3 \times 0.25 \times 2500 = 91.5 \times 10^3 = 91.5 \text{ kJ} \]
Diagnostic x ray tubes

- X-rays that are emitted from the target travel through different thickness of cathode material
- **Heel effect**: radiation intensity toward the cathode side of the x-ray tube is higher than on the anode side
- Cathode is typically mounted over the thicker part of the patient to balance the amount of transmitted photons on the imager

Rating of diagnostic tubes

- Focal spot loading determines the maximum permissible exposure: there is a maximum power that can be tolerated before target starts melting ($T_{\text{melting}} = 3400^\circ\text{C}$ for tungsten)
- Anode cooling and housing cooling rates determine the number of exposures that may be given in a sequence

Rating of diagnostic tubes

- The combination of current and voltage must lie to the left of the appropriate curve
- The maximum duration of a single exposure depends on spot size, anode rotation speed, current, voltage, power supply type

X ray tubes for radiotherapy

- Mostly for superficial treatments
- No need for a small spot source
- The instantaneous energy input is small (about 1/10) but the average energy input is ~ 10 times greater compared with a diagnostic tube
- Due to much higher energy (>200keV) of electrons bombarding the target, there is a problem of secondary electrons emerging from the target
  - Solution: the target is placed in a “hood” - hollow tube with copper shielding intercepting the secondary electrons

X-ray spectra

- Characteristic radiation from the tungsten target
- Continuous white radiation (Bremsstrahlung)
- End point energy (105keV)
Interactions of electrons with the target to give x rays

Most probable: no x-rays produced

Breaking radiation

Figure 6.14. Typical electron interactions with a target. (a) Electron suffers no interaction, giving rise to delta rays and eventually heat. (b) The electron ejects a K electron, giving rise to characteristic radiation. (c) Collision between an electron and a nucleus, leading to bremsstrahlung of energy h. The electron emerges from the "collision" with energy $E - h$. (d) Rare collision when the electron is completely stopped in one collision, giving rise to a photon of energy $E = h$.

Characteristic radiation

<table>
<thead>
<tr>
<th>Transition</th>
<th>Symbol</th>
<th>Energy (keV)</th>
<th>Relative Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>K/L/M/H/...</td>
<td>K/L/M/H/...</td>
<td>69.891/67.244/66.000/</td>
<td>7/21/11/100</td>
</tr>
<tr>
<td>K/L/M/H/...</td>
<td>K/L/M/H/...</td>
<td>11.674/11.283/9.982/9.817</td>
<td>10/24/18/17</td>
</tr>
<tr>
<td>K/L/M/H/...</td>
<td>K/L/M/H/...</td>
<td>57.984/19.092/17.470/17.975</td>
<td>58/24/100/52</td>
</tr>
</tbody>
</table>

Table 6.3

From Sturm and Israel (SI)

- Different transitions have different probabilities, according to quantum mechanics selection rules (some transitions are forbidden).

Bremsstrahlung interaction

• Thin target approximation: one collision per electron
• Thick target approximations: $I(E) = C Z (E_{max} - E)$

Figure 6.15. Relative energy of (a) I, in each photon energy interval produced when a stream of monoenergetic electrons of energy $E$ is incident on a thin target. The electron energy drops to that of the primary electrons produced by the collisions. The bremsstrahlung spectrum is similar to that for electron energies of $E$, $E_0$, and $E_{max}$. The target material is not important in this approximation. Figure 6.16. Relative energy of I (b) and $I_E$ (c) for a thin target, compared to that of a thick target.

Example 4

- The energy levels of K, L, and M shells in tungsten are -69.5, -11.0, and -2.5 keV. What photon energies will be present in its characteristic X-ray spectrum?

A. 67.2, 58.5, 8.5 keV
B. 72.0, 60.5, 13.5 keV
C. 69.5, 11.0, 2.5 keV
D. Continuous spectrum from 2.5 to 69.5 keV
E. Continuous spectrum below 2.5 keV

Answers:

A. Characteristic only
B. Bremsstrahlung only
C. Both A and B
D. Neither A nor B

Example 5

- A target material has the following binding energies: K:30 keV, L:4 keV, M:0.7 keV. If 40.0 keV electrons are fired at the target, what kind of x-rays can have the following energies?

- 6-1: 34 keV
- 6-2: 26 keV
- 6-3: 40.7 keV

A. Characteristic only
B. Bremsstrahlung only
C. Both A and B
D. Neither A nor B

Answers:

- 6-1: B
- 6-2: C
- 6-3: D

Example 6

- X-rays are produced via bremsstrahlung interactions of high-energy electrons within a target
- Efficiency is low, most energy goes into target heating
- Continues spectrum includes characteristic x-ray due to target material
- Required high voltage (~50-100 kV) to accelerate electrons
- Power source: ac to dc conversion

Summary