Chapter 3
The Fundamentals of Nuclear Physics

Radiation Dosimetry I

Text: H.E Johns and J.R. Cunningham, The physics of radiology, 4th ed.

http://www.utoledo.edu/med/depts/radther

Outline

- Terms: activity, half life, average life
- Nuclear disintegration schemes
- Parent-daughter relationships
- Activation of isotopes

Natural radioactivity

- Particles inside a nucleus are in constant motion; can escape if acquire enough energy
- Most lighter atoms with Z<82 (lead) have at least one stable isotope
- All atoms with Z > 82 are radioactive and disintegrate until a stable isotope is formed
- Artificial radioactivity: nucleus can be made unstable upon bombardment with neutrons, high energy protons, etc.

Activity

- Activity – number of disintegrations per unit time; directly proportional to the number of atoms present

$$A = \frac{\Delta N}{\Delta t} = \lambda N = N_0 \lambda e^{-t/t_a} \longleftarrow$$

Average lifetime

t_a= 1.44 t_h

$$A = N_0 \lambda e^{-0.693t/t_h} = A_0 2^{-t/t_h} \longleftarrow$$

Half-life

- Units: Bq = 1/s, Ci=3.7x 10^{10} Bq

Activity

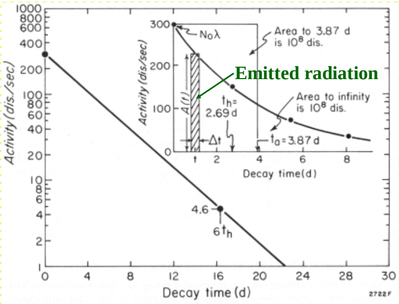


Figure 3-1. Decay of the activity of a ^{198}Au source containing 10^8 atoms.

Activity

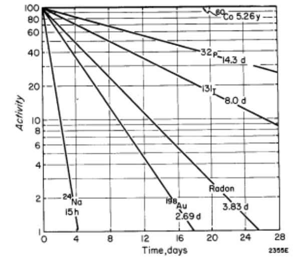


Figure 3-2. Decay of some commonly used isotopes plotted on double cycle semilog paper.

Example 1

- A prostate implant has a half-life of 17 days. What percent of the dose is delivered in the first day?

A. 0.5
B. 2
[✓ C. 4]
D. 15
E. 30

$$\frac{N}{N_0} = 2^{-t/t_h} \text{ or } \frac{N}{N_0} = e^{-\lambda t}$$

$$2^{-t/t_h} = 2^{-1/17} = 0.96$$

$$e^{-0.693t/t_h} = e^{-0.6931/17} = 0.96$$

$$\text{Delivered}: 4\%$$

Example 1A

- A prostate implant has a half-life of 17 days. If the initial dose rate is 10cGy/h, what is the total dose delivered?

$$D_{total} = \dot{D}_0 t_{avg}$$

A. 9
B. 29
[✓ C. 59]
D. 75
E. 300

$$D_{total} = \int_0^{\infty} \dot{D}_0 e^{-\frac{0.693t}{t_h}} dt = -\dot{D}_0 \frac{t_h}{0.693}\left[e^{-\frac{0.693t}{t_h}}\right]_0^{\infty} =$$

$$-\dot{D}_0 t_{avg}[0-1] = \dot{D}_0 t_{avg} =$$

$$10\times10^{-2}\frac{Gy}{h}\cdot\frac{17d}{0.693}\cdot 24\frac{h}{d} \approx 59Gy$$

Example 2

- Initial activity of an ^{123}I sample (t_h = 13 h) injected in the blood is 480 MBq. After 12 h measured activity is 20 MBq / *l*. Assuming the volume of the blood is 6 *l* find biological half-life, h.

A. 4
B. 6
C. 9
[✓ D. 11]
E. 13

$$A_0 = 480/6 = 80 \text{ MBq}/l$$

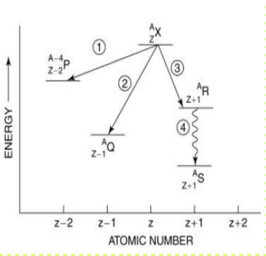
$$A_1 = A_0 2^{-t/t_{eff}} \ ; \quad A_1/A_0 = 1/4 = 2^{-2}$$

$$t_{eff} = 6 \text{ h}$$

$$1/t_{eff} = 1/t_I + 1/t_b$$

$$t_b = \frac{1}{1/t_{eff} - 1/t_I} = \frac{1}{1/6 - 1/13} \approx 11 \text{ h}$$

Decay schemes

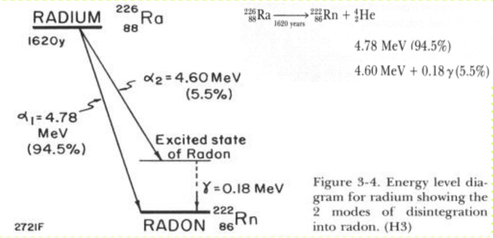


Hendee, Ritenour, Medical imaging physics, 4th edition, fig. 3-1, p.29

- A decay (disintegration) scheme depicts possible routes of radioactive decay for a nuclide:
  1) α decay
  2) β^+ (positron) decay or electron capture
  3) β^- (negatron) decay
  4) isomeric transition

Alpha disintegrations

- Emission of helium nucleus, mainly from heavy nuclei



$$^{226}_{88}\text{Ra} \xrightarrow[\text{1620 years}]{} {}^{222}_{86}\text{Rn} + {}^{4}_{2}\text{He}$$

4.78 MeV (94.5%)

4.60 MeV + 0.18 γ (5.5%)

Figure 3-4. Energy level diagram for radium showing the 2 modes of disintegration into radon. (H3)

Beta disintegrations

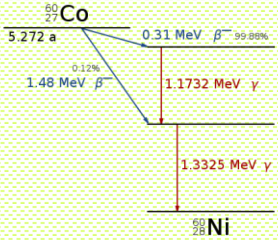
- Ejection of positive or negative electron
- Positron (β^+) or negatron (β^-) emitters
- Proton-rich nucleus – emits positron, neutron-rich emits electron

β^- emission : $n \rightarrow p + \beta^- + \tilde{\nu}$ Z increases by 1

β^+ emission : $p \rightarrow n + \beta^+ + \nu$ Z decreases by 1

In beta-minus emission an antineutrino is emitted instead of neutrino. They have opposite helicity (projection of spin on direction of momentum)

Beta disintegrations



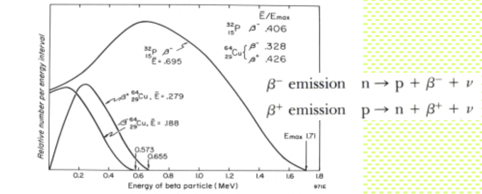
Co-60 decay scheme:

Energy is released in transitions from excited levels of Ni-60 to the ground state via emission of gamma rays (average γ-ray energy 1.25MeV)

Image from https://www.boundless.com

Beta disintegrations

- The ejected beta particle shares its energy with neutrino (anti-neutrino), and therefore may have energy in a continuous spectrum up to E_{max}



β^- emission $n \rightarrow p + \beta^- + \nu$

β^+ emission $p \rightarrow n + \beta^+ + \nu$

Figure 3-6. β^- ray spectra for P-32 and β^+ and β^- spectra for Cu-64. The graphs show the relative number of beta particles per energy interval as a function of energy. The maxima and mean energies of the spectra are given.

Beta disintegrations

- In the production of isotopes in a nuclear reactor a neutron is usually added to a stable nucleus, resulting in β^- emitters
- Example: to produce ^{60}Co a neutron is added to ^{59}Co
- Particle accelerators (more expensive) produce β^+ emitters
- Since $2m_0c^2$=1.022 MeV energy is required to produce electron-positron pair – this is the minimum difference between the initial and final energy (parent-daughter) for β^+ emitters

Electron capture

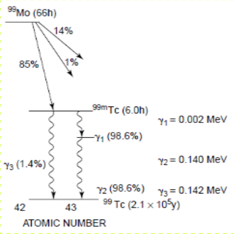
$$^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Q$$

- Orbital electron can be captured by nucleus

p + e (usually K electron) → n + ν

- This process is competing with positron emission in proton-rich nuclei
- The only process when the energy difference between the initial and final state <1.022 MeV

Isomeric transitions



Simplified radioactive decay scheme for ^{99}Mo

Hendee, Ritenour, Medical imaging physics, 4th edition, fig. 3-7

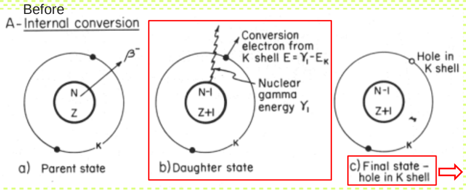
- Isomeric transitions: metastable to stable state, no change in A or Z
- Always preceded by another transition, leaving nucleus in excited state
- Energy is released by
  - Emission of a photon
  - Internal conversion
- Metastable state: an excited state with the lifetime $>10^{-6}$ s

Isomeric transitions: Photon emission

- Technetium is predominantly an artificially produced radioactive metal. Example: $^{99}_{42}$Mo (t_h=66.7 hours) produced in nuclear reactors by β^- decay to $^{99m}_{43}$Tc
- All isotopes are radioactive, most common: $^{99}_{43}$Tc (t_h= 210,000 years) and $^{99m}_{43}$Tc (t_h= 6 hours)
- ^{99m}Tc is widely used
  - as a tracer for medical diagnosis (γ of 140 keV is detected with gamma-camera)
  - functional scans of brain, bone, liver, spleen, kidney, thyroid
  - for blood flow studies

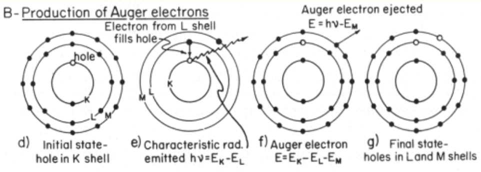
Isomeric transitions: Internal conversion

- Nucleus relaxes to the ground state
- Alternatively to a γ-ray emission an orbital electron may be ejected with the energy $E= \gamma - E_b$



Auger electron production

- A hole in K, L, or M shell has to be filled
- The relative probability of the emission of characteristic radiation to the emission of the Auger electron is called fluorescence yield
- Fluorescence yield is higher for larger Z



Charts of isotopes

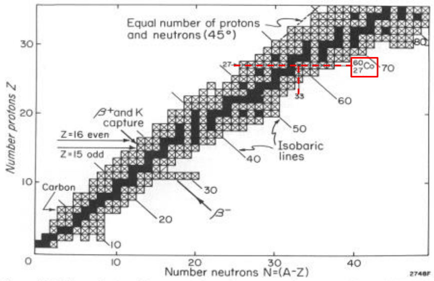
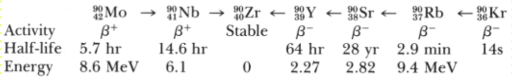


Figure 3-3. Chart showing the proportions of protons and neutrons in nuclei. Stable nuclei are represented by solid squares, radioactive nuclei by crosses. Nuclei with equal numbers of neutrons and protons lie along the line N = Z. Isotopes appear along horizontal lines and isobars along lines at 45°.

Decay series

- Almost all the radioactive isotopes of the *lighter elements* achieve stability by keeping their mass number constant, through beta decay or K capture
- Decay is usually along an isobaric line (constant mass number) and often involves a number of successive transitions before stability is reached



	$^{90}_{42}$Mo →	$^{90}_{41}$Nb →	$^{90}_{40}$Zr	← $^{90}_{39}$Y	← $^{90}_{38}$Sr	← $^{90}_{37}$Rb	← $^{90}_{36}$Kr
Activity	β^+	β^+	Stable	β^-	β^-	β^-	β^-
Half-life	5.7 hr	14.6 hr		64 hr	28 yr	2.9 min	14s
Energy	8.6 MeV	6.1	0	2.27	2.82	9.4 MeV	

Isobaric line for mass number 90

Nuclear transformations

	ΔA	ΔZ	Energy deposition
α disintegration	-4	-2	Total energy of α particle
β^- decay	0	+1	E_{mean}~1/3 E_{max}
β^+ decay	0	-1	E_{mean}~1/3 E_{max} and E_γ=1.02 MeV
Electron capture	0	-1	Most energy carried away by ν; $E_{Fluorescence}$ and/or E_{Auger}
Isomeric transitions	0	0	
γ ray	0	0	E_γ available
Internal conversion	0	0	$E_{electron}=E_\gamma-E_b$ $E_{Fluorescence}$ and/or E_{Auger}

Particle tracks

Figure 3-5. Cloud chamber photograph of alpha, beta, and gamma ray tracks. (J.B. Hoag: *Electron and Nuclear Physics*, courtesy D. Van Nostrand Co.) (H4)

- Ionizations can be made visible in a cloud (bubble) chamber: the air is expanded just after the particle passage, making vapor condensate in the places of ionization
- Photons transfer energy to the secondary electrons, which then ionize the vapor

Example 3

- The mass number (A) changes only for _____ decay:

[✓ A.] alpha
B. beta minus
C. beta plus
D. electron capture
E. isomeric

Example 4

$${}^{99}_{42}\text{Mo} \rightarrow {}^{99m}_{43}\text{Tc} \rightarrow {}^{99}_{43}\text{Tc}$$

I II

- In the above decay of molybdenum, the modes of decay labeled (I) and (II) are, respectively:

[✓ A.] beta minus, isomeric transition
B. isomeric transition, beta minus
C. beta plus, isomeric transition
D. beta minus, beta minus
E. electron capture, beta minus

Growth of radioactive daughter

- Radioactive daughter is formed at the rate at which the parent decays (λ_1), and itself decays with a different rate (λ_2)

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$$

Solution: $$N_2 = \frac{\lambda_1}{\lambda_2 - \lambda_1}(N_1)_0\left[e^{-\lambda_1 t} - e^{-\lambda_2 t}\right]$$

Growth of radioactive daughter

- Express the same solution for activity

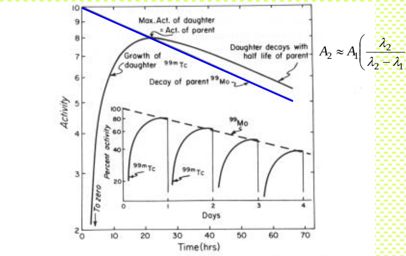
$$A_2 = A_1 \frac{\lambda_2}{\lambda_2 - \lambda_1}\left[1 - e^{-(\lambda_2 - \lambda_1)t}\right]$$

- For a special case of $\lambda_2 >> \lambda_1$ can simplify to

$$A_2 \approx A_1\left[1 - e^{-\lambda_2 t}\right]$$

- After some time – reach a state of parent-daughter equilibrium, where $A_1 = A_2$

Growth of radioactive daughter



$$A_2 \approx A_1\left(\frac{\lambda_2}{\lambda_2 - \lambda_1}\right)$$

Figure 3-11. Decay of parent ^{99}Mo and growth of daughter ^{99m}Tc with time, assuming that initially we have a pure source of parent (λ_1 = 0.01039 h $^{-1}$, t_{h1} = 66.7 h; λ_2 = 0.1149 h $^{-1}$, t_{h2} = 6.03 h). The insert shows the same data when the daughter is "milked" from the parent once each day. (This diagram illustrates the principles involved. The actual situation is somewhat more complicated because only about 86% of the parent ^{99}Mo nuclei decay to the daughter ^{99m}Tc. This means that the activity of the ^{99m}Tc is at all times about 14% less than the values shown.)

Example 5

- After _____ hours, ^{99m}Tc will be approximately in radioactive equilibrium with its parent element ^{99}Mo. (Half-lives are: 67 hours for ^{99}Mo and 6 hours for ^{99m}Tc.)

A. 6
[✓ B.] 24
C. 48
D. 54
E. 268

$$A_2 = A_1 \frac{\lambda_2}{\lambda_2 - \lambda_1}\left[1 - e^{-(\lambda_2 - \lambda_1)t}\right]$$

$$A_2 = A_1 \quad \Rightarrow \quad 1 - e^{-(\lambda_2 - \lambda_1)t} = 1 - \frac{\lambda_1}{\lambda_2}$$

$$t = \frac{1}{\lambda_1 - \lambda_2}\ln\frac{\lambda_1}{\lambda_2}; \quad \lambda_1 = \frac{0.693}{t_{h1}}; \quad \lambda_2 = \frac{0.693}{t_{h2}}$$

$$t \approx \frac{1}{0.7/6 - 0.7/70}\ln\frac{67}{6} \approx 10x2.4 = 24\,\text{h}$$

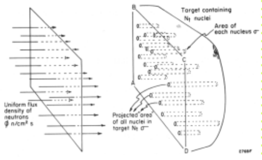
Activation of isotopes

- Atomic nuclei can be activated by neutron bombardment with the probability dependent on the interaction cross section σ
- Number of activated atoms:

$$\Delta N = N_t \sigma \phi \Delta t$$

$\sigma \sim 10^{-24}$ cm^2

Figure 3-12. Diagram to illustrate the activation of target containing N_t nuclei bombarded by a neutron flux density ϕ. The cross section for activation is σ.



Activation of isotopes

- More practically relevant quantity is the activity of the irradiated sample $\Delta A = \lambda \Delta N = \lambda N_t \sigma \phi \Delta t$

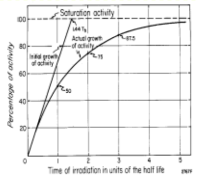


Figure 3-13. Graph showing the growth of activity as a function of irradiation time. The time is expressed in half-lives and the activity as a percentage of the saturation activity.

- If the bombardment time Δt >> t_h, activity reaches its maximum

$$A_s = N_t \sigma \phi$$

- Accounting for decay during irradiation:

$$A = A_s\left(1 - e^{-0.693t/t_h}\right)$$

Example 6

What is the activity of a 20 g sample of ^{59}Co irradiated in a neutron flux of 10^{14} neutrons/(cm^{2}s) for 6 years? Cross section σ=36 barns, t_h=5.3 years.

First find the number of atoms in the target

$$N_t = \frac{m}{M/N_A} = \frac{20}{59/(6.02\times10^{23})} = 2.04\times10^{23}$$

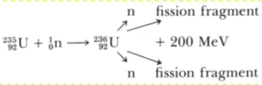
The activity at a time *t*:

$$A = \phi N_t \sigma\left(1 - e^{-0.693t/t_h}\right) =$$

$$10^{14}\cdot 2.04\times10^{23}\cdot 36\times10^{-24}\left(1 - e^{-0.6936/5.3}\right) = 4.0\times10^{14}\mathrm{Bq}$$

Nuclear fission and fusion

- Nuclear fission – a nucleus is split into lighter fragments with release of very large energy. Example:



$${}^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \longrightarrow {}^{236}_{92}\mathrm{U} \rightarrow \begin{array}{l} \mathrm{n}\quad \text{fission fragment} \\ \quad + 200\ \mathrm{MeV} \\ \mathrm{n}\quad \text{fission fragment} \end{array}$$

- Nuclear fusion – two lighter atoms are fused into a heavier one also with release of energy. Example:

${}^{2}_{1}\mathrm{H} + {}^{2}_{1}\mathrm{H} \rightarrow {}^{3}_{2}\mathrm{He} + \mathrm{n} + \text{Energy}$ ~3.3 MeV

Summary

- Terms: activity, half life, average life
- Nuclear disintegration schemes
- Parent-daughter relationships
- Activation of isotopes