Chapter 7
Measurement of Radiation: Dosimetry

Radiation Dosimetry I

Text: H.E Johns and J.R. Cunningham, The physics of radiology, 4th ed.

http://www.utoledo.edu/med/depts/radther

Measurement of radiation

- Description of radiation beam
- Calculation of the absorbed dose
  - Bragg-Grey cavity theory
  - Practical ion chambers
  - Determination of absorbed dose for energies above 3 MeV
  - Dosimetry of radio-nuclides

Description of radiation beam

- Fluence

$$\Phi = \frac{dN}{da}$$

- Energy fluence

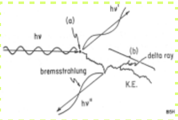
$$\Psi = \frac{dN \cdot hv}{da}$$

- Fluence rate

$$\phi = \frac{d\Phi}{dt}$$

Energy transfer

- Photon interaction involves two stages: (a) energy is transferred to charged particles and (b) charged particles transfer energy directly through excitations and ionizations
- The initial interaction can be described by kerma (kinetic energy released in medium):



$$K = \frac{d\overline{E}_{tr}}{dm}$$

- Kerma is related to photon fluence

$$K = \Phi\left(\frac{\mu}{\rho}\right)\cdot\overline{E}_{tr}$$

Absorbed dose

- Absorbed dose originates in the second interaction stage, describing the energy retained by the medium

$$D = \frac{d\overline{E}_{ab}}{dm}$$

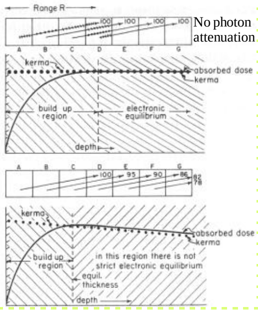
- Units: 1Gy (gray)=1 J/kg
  - Older unit: 1 rad=10^{-2} Gy=1 cGy
- This absorbed energy causes ionizations along the charged particle track

---

Electronic equilibrium

- Transfer of energy to charged particles (kerma) does not take place at the same location as the absorption of energy deposited by charged particles (dose)
- Kerma can be directly related to the fluence, but dose can be calculated only in the assumption of the *electronic equilibrium*: in any volume as many electrons are stopped as set in motion
- Under this condition dose is equal to kerma

Electronic equilibrium



- In reality dose deposition at any point is the result of kerma upstream
- In case of electronic equilibrium:

$$D = \Phi\left(\frac{\mu}{\rho}\right) \cdot \overline{E}_{ab} = K(1-g)$$

- *g*- fraction of energy lost to bremsstrahlung
- Typically approximate electronic equilibrium is assumed

Example 1

- Calculate the kerma given the photon flux $10^{16}/m^2$, photon energy 10 MeV, linear attenuation coefficient 0.028 cm^2/g and energy transfer attenuation coefficient 0.022 cm^2/g.

A. 5 J/kg
B. 15 J/kg
[✓ C. 25 J/kg]
D. 35 J/kg

$$K = \Phi(\mu/\rho) \cdot \overline{E}_{tr} = \Phi(\mu/\rho) \cdot \frac{\mu_{tr}/\rho}{\mu/\rho} E$$

$$K = 10^{16} \frac{1}{m^2} \cdot 0.022 \frac{10^{-4} m^2}{10^{-3} kg} \cdot 10\text{MeV} =$$

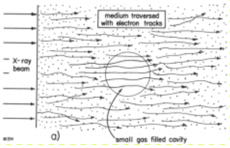
$$2.2 \times 10^{14} MeV/kg =$$

$$2.2 \times 10^{14} \times 1.6 \times 10^{-13} J/kg = 35 J/kg$$

Bragg-Gray cavity theory

- Most dose measurements are based on a measurement of charge produced through gas ionization:

$$D_{gas} = \frac{Q}{m_{gas}} W$$



- *W* – is the average energy required to cause one ionization in the gas
- In air *W*=33.85 eV/ion pair

Bragg-Gray cavity theory

- Can relate the dose in gas to the dose in the surrounding medium ("wall") through the ratio of mean stopping powers in gas and wall
- Bragg-Gray formula relates ionization in the gas cavity to absorbed dose in the medium

$$D_{wall} = \frac{Q}{m_{gas}} W \cdot \overline{\overline{S}}{}^{wall}_{gas}$$

- $\overline{\overline{S}}{}^{wall}_{gas}$ designate averaging over both photon and electron spectra

Bragg-Gray cavity theory

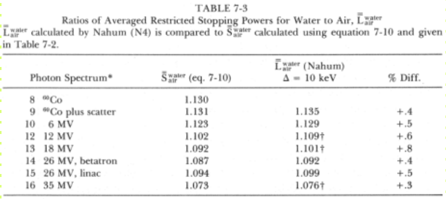


TABLE 7-3

Ratios of Averaged Restricted Stopping Powers for Water to Air, $\overline{\overline{L}}{}^{water}_{air}$

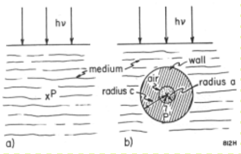
$\overline{\overline{L}}{}^{water}_{air}$ calculated by Nahum (N4) is compared to $\overline{\overline{S}}{}^{water}_{air}$ calculated using equation 7-10 and given in Table 7-2.

Photon Spectrum*		$\overline{\overline{S}}{}^{water}_{air}$ (eq. 7-10)	$\overline{\overline{L}}{}^{water}_{air}$ (Nahum) Δ = 10 keV	% Diff.
8	^{60}Co	1.130		
9	^{60}Co plus scatter	1.131	1.135	+.4
10	6 MV	1.123	1.129	+.5
12	12 MV	1.102	1.109†	+.6
13	18 MV	1.092	1.101†	+.8
14	26 MV, betatron	1.087	1.092	+.4
15	26 MV, linac	1.094	1.099	+.5
16	35 MV	1.073	1.076†	+.3

- Using restricted stopping powers gives more accurate result
- The ratio is not very sensitive to the choice of Δ
- The cavity is always assumed so small that it does not affect the beam spectrum

Absolute ion chamber

- An ionization chamber made of a known material and having a cavity of a known volume
- Have three materials involved
- The wall thickness has to be greater than the range of electrons to separate the wall from the medium
- From measurement can find the dose to the wall



- Knowing the ratio of average energy absorption coefficients in wall and medium $\left(\frac{\overline{\mu}_{ab}}{\rho}\right)^{med}_{wall}$ arrive at:

$$D_{med} = D_{wall} \left(\frac{\overline{\mu}_{ab}}{\rho}\right)^{med}_{wall}$$

Determination of absorbed dose

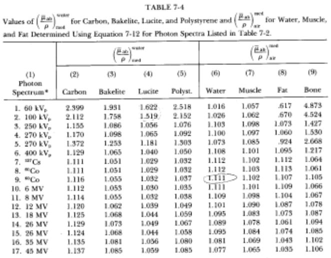


TABLE 7-4

Values of $\left(\frac{\bar{\mu}_{ab}}{\rho}\right)^{water}_{med}$ for Carbon, Bakelite, Lucite, and Polystyrene and $\left(\frac{\bar{\mu}_{ab}}{\rho}\right)^{med}_{air}$ for Water, Muscle, and Fat Determined Using Equation 7-12 for Photon Spectra Listed in Table 7-2.

	$\left(\frac{\bar{\mu}_{ab}}{\rho}\right)^{water}_{med}$				$\left(\frac{\bar{\mu}_{ab}}{\rho}\right)^{med}_{air}$			
(1) Photon Spectrum*	(2) Carbon	(3) Bakelite	(4) Lucite	(5) Polyst.	(6) Water	(7) Muscle	(8) Fat	(9) Bone
1. 60 kV$_p$	2.399	1.931	1.622	2.518	1.016	1.057	.617	4.873
2. 100 kV$_p$	2.112	1.758	1.519	2.152	1.026	1.062	.670	4.524
3. 250 kV$_p$	1.155	1.086	1.056	1.076	1.103	1.098	1.073	1.427
4. 270 kV$_p$	1.170	1.098	1.065	1.092	1.100	1.097	1.060	1.530
5. 270 kV$_p$	1.372	1.253	1.181	1.303	1.073	1.085	.924	2.668
6. 400 kV$_p$	1.129	1.065	1.040	1.050	1.108	1.101	1.095	1.217
7. ^{137}Cs	1.111	1.051	1.029	1.032	1.112	1.102	1.112	1.064
8. ^{60}Co	1.111	1.051	1.029	1.032	1.112	1.103	1.113	1.061
9. ^{60}Co	1.116	1.055	1.032	1.037	1.111	1.102	1.107	1.105
10. 6 MV	1.112	1.053	1.030	1.035	1.111	1.101	1.109	1.066
11. 8 MV	1.114	1.055	1.032	1.038	1.109	1.098	1.104	1.067
12. 12 MV	1.120	1.062	1.039	1.049	1.101	1.090	1.087	1.078
13. 18 MV	1.125	1.068	1.044	1.059	1.095	1.083	1.073	1.087
14. 26 MV	1.129	1.073	1.049	1.067	1.089	1.078	1.061	1.094
15. 26 MV	1.124	1.068	1.044	1.058	1.095	1.084	1.074	1.085
16. 35 MV	1.135	1.081	1.056	1.080	1.081	1.069	1.043	1.102
17. 45 MV	1.137	1.085	1.059	1.085	1.077	1.065	1.035	1.106

$$D_{med} = W\frac{Q}{m}\bar{\bar{S}}^{wall}_{air}\left(\frac{\bar{\mu}_{ab}}{\rho}\right)^{med}_{wall}\cdot k_c$$

Correction factor

- Both the air cavity and wall introduce perturbations to the beam
- In order to account for the finite size of both the air cavity and wall, need to introduce attenuation correction factor k_c
- Values of k_c are determined approximately

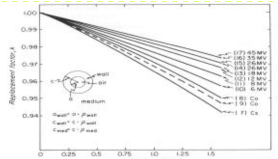


Figure 7-5. Attenuation correction factors as a function of the equivalent radius in cm of water. The insert shows how the three equivalent radii (a_{wall}, c_{wall}, c_{med}) are calculated. The numbers affixed to the graph correspond to the spectra of Table 7-2.

Effect of temperature and pressure

- Since the volume of an ion chamber is fixed, need to correct for change in gas mass due to change in temperature and pressure
- Correction factor relative to conditions of 0°C and 101.3 kPa (760 mm Hg):

$$k_{TP} = \left(\frac{273.2+t}{273.2}\right)\left(\frac{101.3}{p}\right)$$

- If the instrument is calibrated for 22°C – adjust the temperature in denominator

Example 2

- Find the ratio of barometer readings taken at heights X and X+500 meters. Molar mass of Earth's air is 0.029 kg/mol, universal gas constant R=8.31 N·m/(mol·K).

A. 0.98
B. 0.96
[✓ C. 0.94]
D. 0.92

Barometric formula:

$$P = P_0 e^{-\frac{\mu g h}{RT}}$$

$$P/P_{+500} = e^{-\frac{\mu g \Delta h}{RT}} = e^{-\frac{0.029\times 9.8\times 500}{8.31\times 295}} =$$

$$e^{-0.058} \approx 1-0.058 = 0.94$$

Exposure

- Exposure is a measure of the ability of radiation to ionize the air; defined as

$$X = \frac{dQ}{dm}$$

- Defined only for photons, and only for energies below 3 MeV
- Roentgen is defined as: $1R = 2.58\times10^{-4}C/kg$ of air
  - 1 C/kg=3876 R
  - Equivalent ionization: 3.335x10^{-10} C/cm^3 of air

Standard air chamber

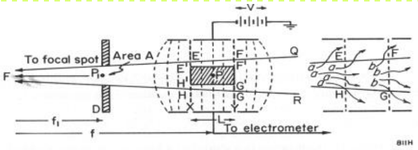


Figure 7-6. Schematic diagram of the standard ionization chamber.

- Exposure can only be measured directly by standard air chamber
- It has to be large since the sensitive volume is defined by the range of electrons set in motion: 3MeV photons produce electron tracks ~1.5 m long

Example 3

- Air kerma is 5 mGy. What is the exposure?

[✓ A.] 0.3 R
B. 0.6 R
C. 0.9 R
D. 1.2 R

$$K=\frac{Q}{m}W;\quad X=\frac{Q}{m}$$

$$X=K/W=5\times10^{-3}\frac{J}{kg}/\left(33.4\frac{J}{C}\right)=$$

$$0.15\times10^{-3}\frac{C}{kg}=0.15\times10^{-3}\times3.9\times10^{3}R=0.58R$$

Practical ion chambers

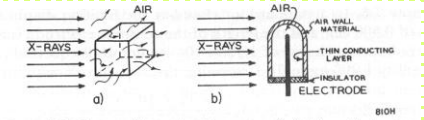


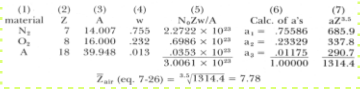
Figure 7-7. Diagram illustrating the nature of air "air wall" chamber.

- Assume that even for a volume of air small compared to the range of electrons the ionization is produced by electrons within the volume
- Adding air-equivalent wall and two electrodes – obtain a practical device for measurement of exposure
- It has to be calibrated against the standard chamber to produce energy dependent calibration factor N_x: get $X=MN_x$

Effective atomic number

- Air-equivalent material has to have appropriate Z to represent photoelectric effect interaction coefficient at low energies (30 to 80 keV)
- The effective atomic number of a mixture (typically take m=3.5):

$$\bar{Z}=\sqrt[m]{a_1Z_1^m+a_2Z_2^m+\ldots+a_nZ_n^m}$$



(1) material	(2) Z	(3) A	(4) w	(5) N_0Zw/A	(6) Calc. of a's	(7) $aZ^{3.5}$
N_2	7	14.007	.755	2.2722×10^{23}	$a_1=$ .75586	685.9
O_2	8	16.000	.232	$.6986\times10^{23}$	$a_2=$ .23329	337.8
A	18	39.948	.013	$.0353\times10^{23}$	$a_3=$ .01175	290.7
				3.0061×10^{23}	1.00000	1314.4

$\bar{Z}_{air}$ (eq. 7-26) $=\sqrt[3.5]{1314.4}=7.78$

Effective atomic number

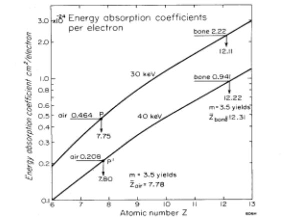


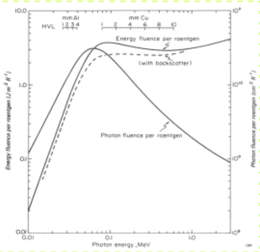
Figure 7-10. Plot of energy absorption coefficient in cm² per electron as a function of atomic number using data from Plechaty (P5) for 30 and 40 keV photons. The coefficients for air and bone are also shown, allowing one to determine an effective atomic number for air and bone.

- For high energies only electron density is important since Compton interaction is dominant

Absorbed dose determination above 3 MeV

- Ion chamber is still used as the basis for measurements
- A set of correction factors is employed to convert the raw measurement to the dose
- AAPM task group protocols for clinical dosimetry of high-energy photon and electron beams:
  - Older TG-21 (1983) is based on exposure (or air-kerma) standard and calibration factor (N_x)
  - New TG-51 (1999) is based on an absorbed-dose to water standard and calibration factor ($N_{D,w}$)
  - Parameters are published for ion chambers from different manufacturers

Fluence and exposure



- Energy fluence per roentgen:

$$\frac{\Psi}{X}=\frac{0.00873}{(\mu_{ab}/\rho)}\frac{\text{J}}{\text{kg R}}$$

- Fluence per roentgen

$$\frac{\Phi}{X}=\frac{0.00873}{h\nu(\mu_{ab}/\rho)}\frac{\text{J}}{\text{kg R}}$$

$1R$ is equivalent to $2.58\times10^{-4}\frac{C}{kg}\times33.85\frac{J}{C}=0.00873J/kg=0.873$cGy in air

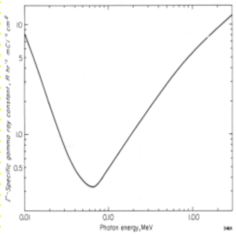
Exposure rate from γ-emitters

- The exposure rate constant is the exposure rate in R/hr at a point 1 meter away from a source having activity of 1 Ci
- From the inverse square law the exposure rate at any point distance d away from a source with activity A:

$$\frac{X}{t} = \frac{\Gamma \cdot A}{d^2}$$

- Units of $\Gamma = \left[\frac{\text{R m}^2}{\text{hr Ci}}\right]$

Exposure rate from γ-emitters



- Exposure rate constant in air for a source emitting 1 photon of energy *hv* per disintegration:

$$\Gamma = 194.5 h\nu \left(\mu_{ab}/\rho\right)_{air} \text{Rm}^2\text{hr}^{-1}\text{Ci}^{-1}$$

Example 4

- Four 30 mCi 1-125 seeds are arranged at the corners of a 1 cm square. Neglecting tissue attenuation, the exposure rate in tissue at the center of the square is: (Exposure rate constant = 1.46 Rcm2/mCi-hr)

A. 3 15 R/hr
B. 376 R/h
C. 264 R/hr
D. 192 R/hr
[✓ E.] 350 R/hr

$$\frac{X}{t} = \frac{4\Gamma \cdot A}{d^2} = \frac{4 \times 1.46 \times 30}{(\sqrt{2 \cdot 0.5^2})^2} =$$

$$350.4 \text{ R/hr}$$

Example 5

- The exposure rate constant for a radionuclide is 12.9 R cm²/(mCi h). How many half-value layers (HVLs) of shielding are required to reduce the exposure rate from a 19.5 mCi source at 2 m to less than 1 mR/h?

A. 1
B. 2
[✓ C.] 3
D. 4
E. 6

$$\frac{X}{t} = \frac{\Gamma \cdot A}{d^2} = \frac{12.9 \cdot 19.5}{200^2} =$$

$$0.00629 \text{R/h} \approx 6.3 \text{mR/h}$$

$$\frac{6.3}{2^n} < 1 \;\Rightarrow\; n > \frac{\ln 6.3}{\ln 2} = 2.65 \;\Rightarrow\; n = 3$$