Chapter 7
Measurement of Radiation: Dosimetry

Radiation Dosimetry I

Text: H.E Johns and J.R. Cunningham, The physics of radiology, 4th ed.

http://www.utoledo.edu/med/depts/radther

Measurement of radiation

- Description of radiation beam
- Kerma, dose, and electronic equilibrium
- Calculation of the absorbed dose
  - Bragg-Grey cavity theory
  - Practical ion chambers
  - Determination of absorbed dose for energies above 3 MeV
  - Dosimetry of radio-nuclides

Description of radiation beam

- Fluence

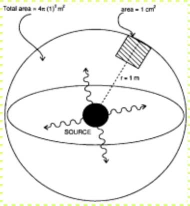
$$\Phi = \frac{dN}{da}$$

- Energy fluence

$$\Psi = \frac{dN \cdot hv}{da}$$

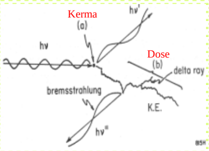
- Fluence rate

$$\phi = \frac{d\Phi}{dt}$$



Energy transfer

- Photon interaction involves two stages: (a) energy is transferred to charged particles and (b) charged particles transfer energy directly through excitations and ionizations
- The initial interaction can be described by kerma (kinetic energy released in medium):



$$K = \frac{d\bar{E}_{tr}}{dm}$$

$\bar{E}_{tr}$ - the average energy transferred to electrons in one interaction

- Kerma is related to photon fluence

$$K = \Phi \left(\frac{\mu}{\rho} \right) \cdot \bar{E}_{tr}$$

Absorbed dose

- Absorbed dose originates in the second interaction stage, describing the energy retained by the medium

$$D = \frac{d\bar{E}_{ab}}{dm}$$

$\bar{E}_{ab}$ - the average energy imparted to the medium in one interaction

- Units: 1Gy (gray)=1 J/kg
  - Older unit: 1 rad=10^{-2} Gy=1 cGy
- This absorbed energy causes ionizations along the charged particle track

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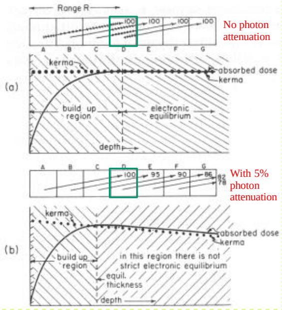
Electronic equilibrium

- Transfer of energy to charged particles (kerma) does not take place at the same location as the absorption of energy deposited by charged particles (dose)
- Kerma can be directly related to the fluence, but dose can be calculated only in the assumption of the *electronic equilibrium*: in any volume as many electrons are stopped as set in motion
- Under this condition dose is equal to kerma

Electronic equilibrium



No photon attenuation

With 5% photon attenuation

- In reality dose deposition at any point is the result of kerma *upstream*
- In case of electronic equilibrium:

$$D = \Phi\left(\frac{\mu}{\rho}\right)\cdot \bar{E}_{ab} = K(1-g)$$

- *g*- fraction of energy lost to bremsstrahlung
- Typically approximate electronic equilibrium is assumed

Electronic equilibrium

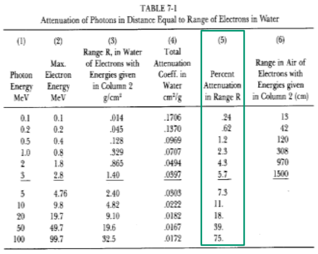


TABLE 7-1
Attenuation of Photons in Distance Equal to Range of Electrons in Water

(1) Photon Energy MeV	(2) Max. Electron Energy MeV	(3) Range R, in Water of Electrons with Energies given in Column 2 g/cm²	(4) Total Attenuation Coeff. in Water cm²/g	(5) Percent Attenuation in Range R	(6) Range in Air of Electrons with Energies given in Column 2 (cm)
0.1	0.1	.014	.1706	.24	13
0.2	0.2	.045	.1370	.62	42
0.5	0.4	.128	.0969	1.2	120
1.0	0.8	.329	.0707	2.3	308
2	1.8	.865	.0494	4.3	970
3	2.8	1.40	.0397	5.7	1500
5	4.76	2.40	.0303	7.3	
10	9.8	4.82	.0222	11.	
20	19.7	9.10	.0182	18.	
50	49.7	19.6	.0167	39.	
100	99.7	32.5	.0172	75.	

- For approximate electronic equilibrium, percentage of photons attenuated within the electron range *R* should be very small
- Becomes >5% for photon energies above 3MeV (column 5)

Example 1

- Calculate the kerma given the photon flux 10^{16}/m^2, photon energy 10 MeV, linear attenuation coefficient 0.028 cm^2/g and energy transfer attenuation coefficient 0.022 cm^2/g.

A. 5 J/kg
B. 15 J/kg
C. 25 J/kg
[✓ D.] 35 J/kg

$$K = \Phi(\mu/\rho)\cdot \bar{E}_{tr} = \Phi(\mu/\rho)\cdot \frac{\mu_{tr}/\rho}{\mu/\rho}E$$

$$K = 10^{16}\frac{1}{m^2}\cdot 0.022\frac{10^{-4}m^2}{10^{-3}kg}\cdot 10\text{MeV} =$$

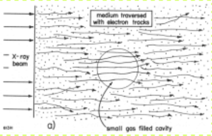
$$2.2\times 10^{14} MeV/kg =$$

$$2.2\times 10^{14}\times 1.6\times 10^{-13} J/kg = 35 J/kg$$

Bragg-Gray cavity theory

- Most dose measurements are based on a measurement of charge produced through gas ionization:

$$D_{gas} = \frac{Q}{m_{gas}}W$$



Cavity does not perturb electron tracks

- *W* – is the average energy required to cause one ionization in the gas
- In air *W*=33.85 eV/ion pair

Bragg-Gray cavity theory

- Need dose to the surrounding medium (“wall”)
- Relate the dose in gas to the dose in the “wall” through the ratio of mean stopping powers in gas and wall
- Bragg-Gray formula relates ionization in the gas cavity to absorbed dose in the medium

$$\boxed{D_{wall} = D_{gasl}\cdot \bar{\bar{S}}^{\,wall}_{gas} = \frac{Q}{m_{gas}}W\cdot \bar{\bar{S}}^{\,wall}_{gas}}$$

- $\bar{\bar{S}}^{\,wall}_{gas}$ designates stopping power averaging over both photon and electron spectra

Bragg-Gray cavity theory

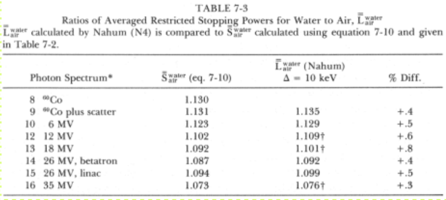


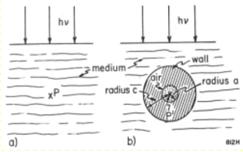
TABLE 7-3
Ratios of Averaged Restricted Stopping Powers for Water to Air, $\bar{\bar{L}}^{water}_{air}$
$\bar{\bar{L}}^{water}_{air}$ calculated by Nahum (N4) is compared to $\bar{\bar{S}}^{water}_{air}$ calculated using equation 7-10 and given in Table 7-2.

Photon Spectrum*	$\bar{\bar{S}}^{water}_{air}$ (eq. 7-10)	$\bar{\bar{L}}^{water}_{air}$ (Nahum) Δ = 10 keV	% Diff.
8 ^{60}Co	1.130		
9 ^{60}Co plus scatter	1.131	1.135	+.4
10 6 MV	1.123	1.129	+.5
12 12 MV	1.102	1.109†	+.6
13 18 MV	1.092	1.101†	+.8
14 26 MV, betatron	1.087	1.092	+.4
15 26 MV, linac	1.094	1.099	+.5
16 35 MV	1.073	1.076†	+.3

- Using restricted stopping powers gives more accurate result
- Since the ratio is not very sensitive to the choice of Δ, $\bar{\bar{S}}^{\,wall}_{gas}$ is used
- Cavity is assumed small and not affecting the beam spectrum

Absolute ion chamber

- An ionization chamber made of a known material and having a cavity of a known volume
- Have three materials involved
- The wall thickness has to be greater than the range of electrons to separate the wall from the medium
- From measurement can find the dose to the wall



- Knowing the ratio of average energy absorption coefficients in wall and medium $\left(\frac{\bar{\mu}_{ab}}{\rho}\right)_{wall}^{med}$ arrive at:

$$D_{med} = D_{wall}\left(\frac{\bar{\mu}_{ab}}{\rho}\right)_{wall}^{med}$$

Determination of absorbed dose

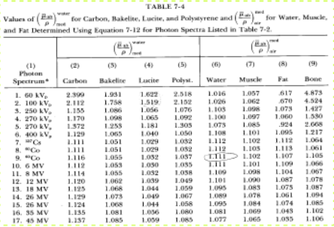


TABLE 7-4

Values of $\left(\frac{\bar{\mu}_{ab}}{\rho}\right)_{med}^{water}$ for Carbon, Bakelite, Lucite, and Polystyrene and $\left(\frac{\bar{\mu}_{ab}}{\rho}\right)_{air}^{med}$ for Water, Muscle, and Fat Determined Using Equation 7-12 for Photon Spectra Listed in Table 7-2.

	$\left(\frac{\bar{\mu}_{ab}}{\rho}\right)_{med}^{water}$				$\left(\frac{\bar{\mu}_{ab}}{\rho}\right)_{air}^{med}$			
(1) Photon Spectrum*	(2) Carbon	(3) Bakelite	(4) Lucite	(5) Polyst.	(6) Water	(7) Muscle	(8) Fat	(9) Bone
1. 60 kV_p	2.399	1.931	1.622	2.518	1.016	1.057	.617	4.873
2. 100 kV_p	2.112	1.758	1.519	2.152	1.026	1.062	.670	4.524
3. 250 kV_p	1.155	1.086	1.056	1.076	1.103	1.098	1.073	1.427
4. 270 kV_p	1.170	1.098	1.065	1.092	1.100	1.097	1.060	1.530
5. 270 kV_p	1.372	1.253	1.181	1.303	1.073	1.085	.924	2.668
6. 400 kV_p	1.129	1.065	1.040	1.050	1.108	1.101	1.095	1.217
7. ^{137}Cs	1.111	1.051	1.029	1.032	1.112	1.102	1.112	1.064
8. ^{60}Co	1.111	1.051	1.029	1.032	1.112	1.103	1.113	1.061
9. ^{60}Co	1.116	1.055	1.032	1.037	1.111	1.102	1.107	1.105
10. 6 MV	1.112	1.053	1.030	1.035	1.111	1.101	1.109	1.066
11. 8 MV	1.114	1.055	1.032	1.038	1.109	1.098	1.104	1.067
12. 12 MV	1.120	1.062	1.039	1.049	1.101	1.090	1.087	1.078
13. 18 MV	1.125	1.068	1.044	1.059	1.095	1.083	1.075	1.087
14. 26 MV	1.129	1.073	1.049	1.067	1.089	1.078	1.061	1.094
15. 26 MV	1.124	1.068	1.044	1.058	1.095	1.084	1.074	1.085
16. 35 MV	1.135	1.081	1.056	1.080	1.081	1.069	1.043	1.102
17. 45 MV	1.137	1.085	1.059	1.085	1.077	1.065	1.035	1.106

$$\boxed{D_{med} = W\frac{Q}{m}\bar{\bar{S}}_{air}^{wall}\left(\frac{\bar{\mu}_{ab}}{\rho}\right)_{wall}^{med}}$$

Correction factor

- Both the air cavity and wall introduce perturbations to the beam
- In order to account for the finite size of both the air cavity and wall, need to introduce attenuation correction factor k_c

$$D_{med} = W\frac{Q}{m}\bar{\bar{S}}_{air}^{wall}\left(\frac{\bar{\mu}_{ab}}{\rho}\right)_{wall}^{med} \cdot k_c$$

Correction factor

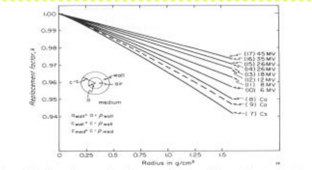


Figure 7-5. Attenuation correction factors as a function of the equivalent radius in cm of water. The insert shows how the three equivalent radii (a_{wall}, c_{wall}, c_{med}) are calculated. The numbers affixed to the graph correspond to the spectra of Table 7-2.

- Correction factor k_c includes properties of wall and medium through replacement factors

$$k_c = \frac{k(a_{wall}) \cdot k(c_{med})}{k(c_{wall})}$$

- Values of replacement factors *k*, and correction factor k_c are determined approximately

Effect of temperature and pressure

- Since the volume of an ion chamber is fixed, need to correct for change in gas mass due to change in temperature and pressure
- Correction factor relative to conditions of 0°C and 101.3 kPa (760 mm Hg):

$$k_{TP} = \left(\frac{273.2 + t}{273.2}\right)\left(\frac{101.3}{p}\right)$$

- If the instrument is calibrated for 22°C – adjust the temperature in denominator

Example 2

- Find the ratio of barometer readings taken at heights X and X+500 meters. Molar mass of Earth's air is 0.029 kg/mol, universal gas constant R=8.31 N·m/(mol·K).

A. 0.98
B. 0.96
[✓ C.] 0.94
D. 0.92

Barometric formula:

$$P = P_0 e^{-\frac{\mu g h}{RT}}$$

$$P / P_{+500} = e^{-\frac{\mu g \Delta h}{RT}} = e^{-\frac{0.029 \times 9.8 \times 500}{8.31 \times 295}} =$$

$$e^{-0.058} \approx 1 - 0.058 = 0.94$$

Exposure

- Exposure is a measure of the ability of radiation to ionize the air; defined as

$$X = \frac{dQ}{dm}$$

- Defined only for photons, and only for energies below 3 MeV
- Roentgen is defined as: $1R = 2.58 \times 10^{-4} C/kg$ of air
  - 1 C/kg=3876 R
  - Equivalent ionization: 3.335×10^{-10} C/cm^3 of air

Standard air chamber

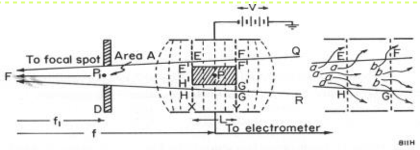


Figure 7-6. Schematic diagram of the standard ionization chamber.

- Exposure can only be measured directly by standard air chamber
- It has to be large since the sensitive volume is defined by the range of electrons set in motion: 3MeV photons produce electron tracks up to several meters long

Example 3

- Air kerma is 5 mGy. What is the exposure?

A. 0.3 R

[✓ B. 0.6 R]

C. 0.9 R

D. 1.2 R

$$K = \frac{Q}{m}W; \quad X = \frac{Q}{m}$$

$$X = K/W = 5 \times 10^{-3}\frac{J}{kg} \Big/ \left(33.4\frac{J}{C}\right) =$$

$$0.15 \times 10^{-3}\frac{C}{kg} = 0.15 \times 10^{-3} \times 3.9 \times 10^{3} R = 0.58R$$

Practical ion chambers

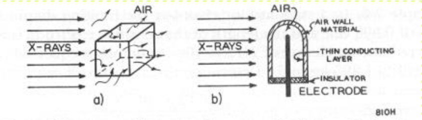


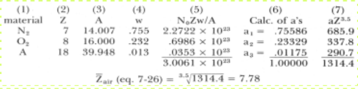
Figure 7-7. Diagram illustrating the nature of air "air wall" chamber.

- Assume that even for a volume of air small compared to the range of electrons the ionization is produced by electrons within the volume
- Adding air-equivalent solid wall and two electrodes – obtain a practical device for measurement of exposure
- It has to be calibrated against the standard chamber to produce energy dependent calibration factor N_x: get $X = MN_x$

Effective atomic number

- Air-equivalent material has to have appropriate Z to represent photoelectric effect interaction coefficient at low energies (30 to 80 keV)
- The effective atomic number of a mixture with a_n fractional numbers of electrons/g of materials Z_n (typically m=3.5):

$$\bar{Z} = \sqrt[m]{a_1 Z_1^m + a_2 Z_2^m + \ldots + a_n Z_n^m}$$



(1) material	(2) Z	(3) A	(4) w	(5) N_0Zw/A	(6) Calc. of a's	(7) $aZ^{3.5}$
N_2	7	14.007	.755	2.2722×10^{23}	$a_1 =$ .75586	685.9
O_2	8	16.000	.232	$.6986 \times 10^{23}$	$a_2 =$ .23329	337.8
A	18	39.948	.013	$.0353 \times 10^{23}$	$a_3 =$ .01175	290.7
				3.0061×10^{23}	1.00000	1314.4

$\bar{Z}_{air}$ (eq. 7-26) $= \sqrt[3.5]{1314.4} = 7.78$

- Other equations exist in literature

Effective atomic number

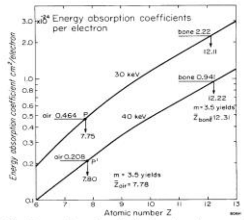


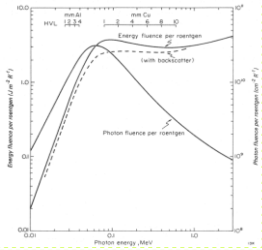
Figure 7-10. Plot of energy absorption coefficient in cm² per electron as a function of atomic number using data from Plechaty (P5) for 30 and 40 keV photons. The coefficients for air and bone are also shown, allowing one to determine an effective atomic number for air and bone.

- For high energies only electron density is important since Compton interaction is dominant

Absorbed dose determination above 3 MeV

- Ion chamber is still used as the basis for measurements
- A set of correction factors is employed to convert the raw measurement to the dose
- AAPM task group protocols for clinical dosimetry of high-energy photon and electron beams:
  - Older TG-21 (1983) is based on exposure (or air-kerma) standard and calibration factor (N_x)
  - New TG-51 (1999, updated protocol in 2014) is based on an absorbed-dose to water standard and calibration factor ($N_{D,w}$)
  - Parameters are published for ion chambers from different manufacturers

Fluence and exposure



- Energy fluence per roentgen:

$$\frac{\Psi}{X}=\frac{0.00873}{(\mu_{ab}/\rho)}\frac{\text{J}}{\text{kg R}}$$

- Fluence per roentgen

$$\frac{\Phi}{X}=\frac{0.00873}{hv(\mu_{ab}/\rho)}\frac{\text{J}}{\text{kg R}}$$

$1R$ is equivalent to $2.58\times10^{-4}\frac{C}{kg}\times33.85\frac{J}{C}=0.00873\,J/kg=0.873\text{cGy in air}$

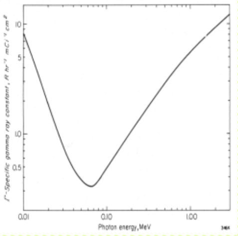
Exposure rate from γ-emitters

- The exposure rate constant Γ is the exposure rate in R/hr at a point 1 meter away from a source having activity of 1 Ci
- From the inverse square law the exposure rate at any point distance d away from a source with activity A:

$$\frac{X}{t}=\frac{\Gamma\cdot A}{d^2}$$

- Units of $\Gamma=\left[\frac{\text{R m}^2}{\text{hr Ci}}\right]$

Exposure rate from γ-emitters



- Exposure rate constant in air for a source emitting 1 photon of energy *hv* per disintegration:

$$\Gamma=194.5hv(\mu_{ab}/\rho)_{air}\,\text{Rm}^2\text{hr}^{-1}\text{Ci}^{-1}$$

Example 4

- Four 30 mCi 1-125 seeds are arranged at the corners of a 1 cm square. Neglecting tissue attenuation, the exposure rate in tissue at the center of the square is: (Exposure rate constant = 1.46 Rcm2/mCi-hr)

A. 3 15 R/hr
B. 376 R/h
C. 264 R/hr
D. 192 R/hr
[✓ E.] 350 R/hr

$$\frac{X}{t}=\frac{4\Gamma\cdot A}{d^2}=\frac{4\times1.46\times30}{(\sqrt{2\cdot0.5^2})^2}=$$

$$350.4\text{ R/hr}$$

Example 5

- The exposure rate constant for a radionuclide is 12.9 R cm^2/(mCi h). How many half-value layers (HVLs) of shielding are required to reduce the exposure rate from a 19.5 mCi source at 2 m to less than 1 mR/h?

A. 1
B. 2
[✓ C.] 3
D. 4
E. 6

$$\frac{X}{t}=\frac{\Gamma\cdot A}{d^2}=\frac{12.9\cdot19.5}{200^2}=$$

$$0.00629\text{ R/h}\approx6.3\text{ mR/h}$$

$$\frac{6.3}{2^n}<1\;\Rightarrow\;n>\frac{\ln 6.3}{\ln 2}=2.65\;\Rightarrow\;n=3$$

Chapter 8
The Quality of X-Rays
(Half-Value Layer)

Radiation Dosimetry I

Text: H.E Johns and J.R. Cunningham, The physics of radiology, 4th ed.

http://www.utoledo.edu/med/depts/radther

Effects of filters on x-ray beam

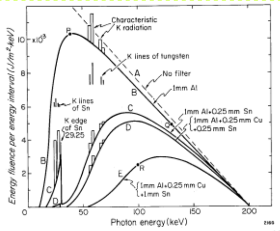


Figure 8-1. Graph showing how the spectral distribution of radiation generated by 200 keV electrons bombarding a thick W target changes with filtration. Dashed line A, unfiltered beam. Curves B, C, D, and E are obtained from A by calculating the attenuation produced by the indicated layers of A1, Cu, and Sn.

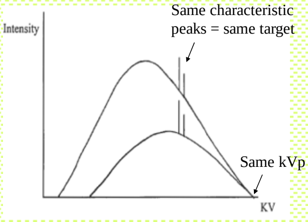
Composite filters: highest Z material closest to x-ray tube (to remove characteristic x-rays)

Example 6

- In the graph below, the two X-ray spectra shown have the same

1. Filtration
2. Target material
3. HVL
4. KVp

A. 1,2,3,4
B. 1,3
[✓ C.] 2,4
D. 3,4
E. 4 only

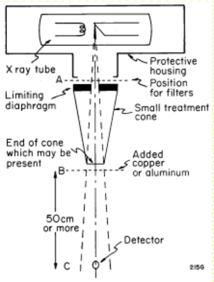


Same characteristic peaks = same target

Same kVp

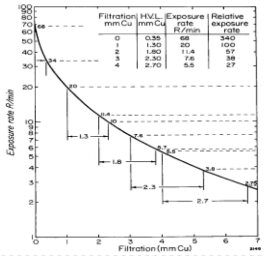
Different intensity – different filtration and HVL

Measurement of half-value layer



- Measure exposure rate for a series of attenuators placed in the beam
- Setup to measure primary beam only (away from walls, floor, etc.)
- Attenuators are placed at Pt.B
- Machine output is kept constant

Measurement of half-value layer



Filtration mm Cu	H.V.L. mm Cu	Exposure rate R/min	Relative exposure rate
0	0.35	68	340
1	1.30	20	100
2	1.80	11.4	57
3	2.30	7.6	38
4	2.70	5.5	27

kV beams are often labeled by their HVL in mm of filtration material (Cu, Al, etc.)

Figure 8-3. Experimentally determined attenuation curve for 200 kV radiation, showing how the HVL may be determined for a number of filtrations.

Measurement of half-value layer

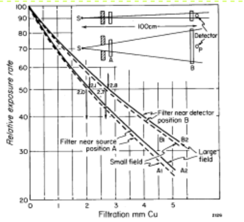


Figure 8-4. Diagram to illustrate how different apparent half-value layers may be obtained for the same beam by using different arrangements of field size and attenuator position in making the measurements.

Scatter getting to the detector will significantly affect the resulting HVL

Measurement of half-value layer

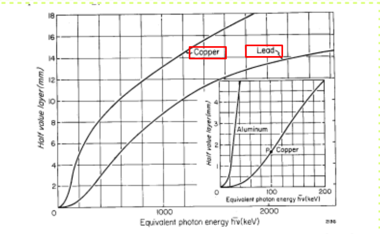


Figure 8-6. Graph showing the relation between half-value layer and equivalent photon energy, calculated from the total attenuation coefficients given in the appendix.

Types of spectral distribution

- Fluence or energy fluence as a function of energy
- Exposure distribution

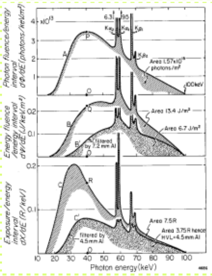


Figure 8-8. Curve A: Photon fluence per energy interval at 1 meter from a 100 kV constant potential x ray source. Measurements were made with a Ge(Li) detector placed 10 meters from the source behind a 0.51 mm pinhole in lead. The $K\alpha_1$ and $K\alpha_2$ at 59.321 and 57.984 keV are off scale. The $K\beta_1$ is really a mixture of $K\beta_3$ and $K\beta_1$ at 67.143 keV, while the $K\beta_2$ is a mixture of several lines with an average of 69.192 keV. Curve B: Same basic data converted to energy fluence per keV interval. Curve B′ is obtained from B by attenuation with 7.2 mm Al. Curve C: Same basic data as curve A but presented as an exposure distribution in roentgens per keV interval. Curve C′ contains half the area of curve C showing that the HVL = 4.5 mm Al.

MV spectra

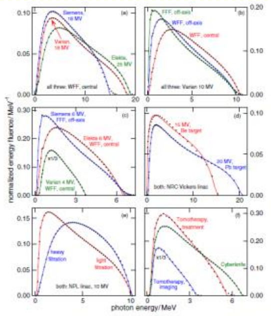


Figure 3. Example fits (solid lines) of the proposed function (function 13 in table 2) to 16 Monte Carlo spectra (dots) from the benchmark set of 65 spectra. The fits are typical of others. The terms 'central', 'off-axis', 'WFF' and 'FFF' refer to, respectively, a central-axis spectrum, an off-axis spectrum, with flattening filter and flattening-filter free. For graph clarity, the zero energy is offset, only every other original Monte Carlo point is shown except around the 511 keV peak, and in panels c and f two spectra are scaled down by a factor of 3. Spectra are normalized to unit energy fluence. See section 3 for the significance of the examples in each panel.

- Bremsstrahlung spectra
- Labeled by the energy of electron beam striking the target
- HVL is not typically used except for shielding calculations

Ali, Rogers, Functional forms for photon spectra of clinical linacs, Phys. Med. Biol. **57** (2012) 31–50

Conversion of exposure to absorbed dose

- Depends on a photon energy for mono-energetic beam after unit conversion

$$D_{med} = X \cdot f_{med}$$

$$f_{med} = \left(0.00873\frac{Gy}{R}\right)\left(\frac{\mu_{ab}}{\rho}\right)_{med} \Big/ \left(\frac{\mu_{ab}}{\rho}\right)_{air}$$

- Depends on the spectral distribution of the beam for poly-energetic beam (need to find average)

$$2.58 \times 10^{-4}\frac{C}{kg} \times 33.85\frac{J}{C} = 0.00873\frac{J}{kg} \quad \text{for air}$$

Conversion of exposure to absorbed dose

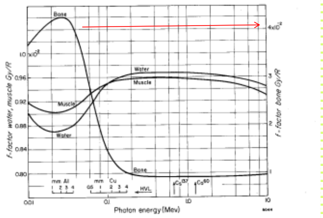


Figure 8-10. The f-factor as a function of photon energy for water, muscle, and bone taken from data in Table A-7. The auxiliary scale relates the HVL in Al and Cu to the energy scale.

Conversion of exposure to absorbed dose

TABLE 8-2

Values of $\overline{f}$ for Water and Bone

Calculated from tables similar to Table 8-1 for HVL = 2.2 mm Cu (Values are in rad/R)

Depth (cm)	0 cm²	25 cm²	50 cm²	100 cm²	200 cm²	400 cm²
Water						
0	.946	.942	.941	.940	.939	.938
5	.949	.940	.938	.934	.932	.931
10	.950	.945	.942	.940	.938	.937
15	.952	.937	.934	.931	.928	.925
Bone						
0	1.756	1.920	1.961	1.999	2.038	2.068
5	1.643	1.989	2.083	2.238	2.312	2.361
10	1.590	1.803	1.903	2.005	2.093	2.142
15	1.505	2.114	2.244	2.368	2.482	2.592

- Presence of heterogeneities leads to scatter
- Since scatter is lower energy – affects dose to higher Z materials (bone) more

Summary

- Measurement of radiation
  - Description of radiation beam
  - Kerma, dose, and electronic equilibrium
  - Calculation of the absorbed dose
  - Bragg-Grey cavity theory
  - Practical ion chambers
- The quality of X-rays and half-value layer
- Conversion of exposure to absorbed dose