# Cavity Theory

Chapter 10

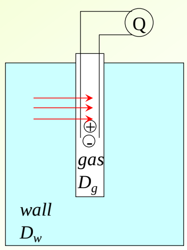
F.A. Attix, Introduction to Radiological Physics and Radiation Dosimetry

# Outline

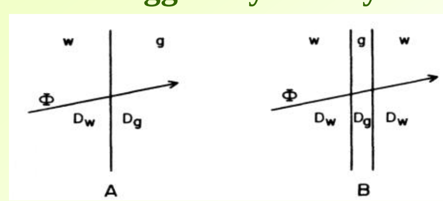
- Problem statement
- Bragg-Gray cavity theory
- Spencer cavity theory
- Burlin cavity theory
- Dose near interfaces between dissimilar media
- Summary

# Cavity Theory: Problem Statement

- Homogeneous medium, wall ($w$)
- Probe - cavity - thin layer of gas ($g$)
- Charged particles crossing $w$-$g$ interface
- Objective: find a relation between the dose in a probe to that in the medium
- Basis for dosimetry




# Bragg-Gray Theory




(*A*) A fluence Φ of charged particles crossing an interface between media *w* and *g*

(*B*) A fluence Φ of charged particles passing through a thin layer of medium *g* sandwiched between regions contain medium *w*

Fluence Φ is assumed to be continuous across all interfaces; it is related to the dose

# Bragg-Gray Theory

- Charged particles of fluence Φ, kinetic energy $T$
- $D_g$, $D_w$ - absorbed doses on each side of the boundary
- $\left(\frac{dT}{\rho dx}\right)_c$ - mass collision stopping power, evaluated at energy T
- Assuming Φ continuous across the interface

$$D_g = \Phi\left[\left(\frac{dT}{\rho dx}\right)_{c,g}\right]_T$$

$$D_w = \Phi\left[\left(\frac{dT}{\rho dx}\right)_{c,w}\right]_T$$

$$\frac{D_w}{D_g} = \frac{(dT/\rho dx)_{c,w}}{(dT/\rho dx)_{c,g}}$$

# Bragg-Gray Theory

- Two conditions:
  - Thickness of $g$ layer is much smaller than the range of charged particles (medium $g$ is close to $w$ in atomic number)
  - The absorbed dose in the cavity is deposited entirely by the charged particles ***crossing*** it
- Additional assumptions:
  - Existence of CPE
  - Absence of bremsstrahlung generation
  - No backscattering

## Bragg-Gray Theory

For differential energy distribution $\Phi_T$, average mass collision stopping power

$$_m\bar{S}_w \equiv \frac{\int_0^{T_{\max}} \Phi_T \left(\frac{dT}{\rho dx}\right)_{c,w} dT}{\int_0^{T_{\max}} \Phi_T dT} = \frac{1}{\Phi}\int_0^{T_{\max}} \Phi_T \left(\frac{dT}{\rho dx}\right)_{c,w} dT = \frac{D_w}{\Phi}$$

$$_m\bar{S}_g \equiv \frac{\int_0^{T_{\max}} \Phi_T \left(\frac{dT}{\rho dx}\right)_{c,g} dT}{\int_0^{T_{\max}} \Phi_T dT} = \frac{1}{\Phi}\int_0^{T_{\max}} \Phi_T \left(\frac{dT}{\rho dx}\right)_{c,g} dT = \frac{D_g}{\Phi}$$

$$\frac{D_w}{D_g} = \frac{_m\bar{S}_w}{_m\bar{S}_g} \equiv {_m\bar{S}_g^w}$$

## Bragg-Gray Theory

For charge $Q$ (of either sign) produced in gas of mass $m$ by radiation, the dose in gas

$$D_g = \frac{Q}{m}\left(\frac{\bar{W}}{e}\right)_g$$

Then the B-G relation expressed in terms of cavity ionization:

$$D_w = \frac{Q}{m}\left(\frac{\bar{W}}{e}\right)_g \cdot {_m\bar{S}_g^w}$$

This equation allows calculating the absorbed dose in the medium immediately surrounding a B-G cavity, based on the charge produced in the cavity gas, provided that the parameters are known

## Bragg-Gray Theory

- As long as $_m\bar{S}_g^w$ is evaluated for the charged-particle spectrum $\Phi_T$ that crosses the cavity, the B-G relation requires neither CPE nor a homogeneous field of radiation
- The charged-particle fluence $\Phi_T$ must be the same in the cavity and in the medium $w$
- If CPE does exist in the neighborhood of a point of interest in the medium $w$, then the insertion of a B-G cavity at the point may be assumed not to perturb the “equilibrium spectrum” of charged particles existing there

## Bragg-Gray Theory

- First Bragg-Grey Corollary: two different gases filling the cavity

$$\frac{Q_2}{Q_1} = \frac{\rho_2 V}{\rho_1 V} \cdot \frac{(\bar{W}/e)_1}{(\bar{W}/e)_2} \cdot {_m\bar{S}_{g_1}^{g_2}}$$

- Second Bragg-Grey Corollary: two chambers (walls) of different volume and material

$$\frac{Q_2}{Q_1} = \frac{V_2}{V_1} \cdot \frac{(\bar{\mu}_{\text{en}}/\rho)_{w_2}}{(\bar{\mu}_{\text{en}}/\rho)_{w_1}} \cdot \frac{_m\bar{S}_g^{w_1}}{_m\bar{S}_g^{w_2}}$$

## Spencer derivation of B-G

- Consider a small cavity filled with medium $g$, surrounded by a homogeneous medium $w$ that contains a homogeneous source emitting $N$ identical charged particles per gram, each with kinetic energy $T_0$ (MeV)
- The cavity is assumed to be far enough from the outer limits of $w$ that CPE exists
- Both B-G conditions are assumed to be satisfied by the cavity, and bremsstrahlung generation is assumed to be absent

## Spencer derivation of B-G

- The absorbed dose at any point in the undisturbed medium $w$ where CPE exists:

$$D_w \overset{\text{CPE}}{=} K_w = NT_0 \quad \text{(MeV/g)}$$

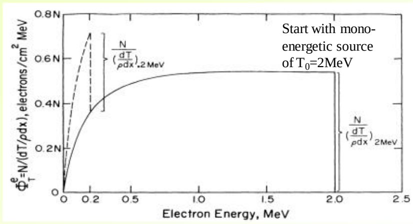
- For an equilibrium charged-particle fluence spectrum $\Phi^e_T$ the absorbed dose

$$D_w = \int_0^{T_0} \Phi_T^e \left(\frac{dT}{\rho dx}\right)_w dT$$

- Then the spectrum

$$\Phi_T^e = \frac{N}{(dT/\rho dx)_w}$$

## Spencer derivation of B-G




- Example of an equilibrium fluence spectrum, $\Phi^e_T = N/(dT/\rho dx)$, of primary electrons under CPE conditions in water, assuming the continuous-slowing-down approximation

## Spencer derivation of B-G

- Assuming the same spectrum on both sides, ratio of the dose in the cavity to that of the medium *w*

$$\frac{D_g}{D_w} = \frac{1}{T_0}\int_0^{T_0} \frac{(dT/\rho dx)_g}{(dT/\rho dx)_w} dT = {}_m\bar{S}_w^g$$

- Can be generalized to accommodate bremsstrahlung generation by electrons

$$\frac{D_g}{D_w} = \frac{1}{T_0[1-Y_w(T_0)]}\int_0^{T_0} \frac{(dT/\rho dx)_{c,g}}{(dT/\rho dx)_{c,w}} dT = {}_m\bar{S}_w^g$$

## Averaging of Stopping Powers

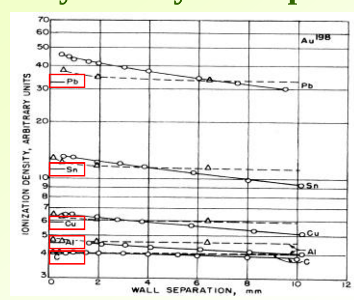
- Extend a single starting energy $T_0$ to a distribution *T* (spectrum): stopping power has to be integrated over the distribution T

$$\frac{D_g}{D_w} = \frac{\int_0^{T_{max}} N_{T_0} \int_0^{T_0} \frac{(dT/\rho dx)_{c,g}}{(dT/\rho dx)_w} dT}{\int_0^{T_{max}} N_{T_0} T_0 [1-Y_w(Y_0)] dT_0} = {}_m\bar{\bar{S}}_w^g$$

Averaging over each $T_0$, and over T for each $T_0$

- Ratio of collision stopping powers is a slowly varying function, therefore average energy $\bar{T}$ may be used

## B-G Cavity Theory vs. Experiment




Comparison of measured ionization densities (solid curves) in flat air-filled ion chambers having various wall materials and adjustable gap widths, with Bragg-Gray theory (tick marks at left) and Spencer theory (dashed curves), for $^{198}$Au γ rays

## B-G Cavity Theory vs. Experiment

- Experiments had shown deviations from B-G theory
- Dependence on cavity size and Z
- For walls of high atomic number δ-ray production becomes an issue
  - δ-rays cross the cavity with the rest of electrons
  - they change spectrum, enhancing low energy part
- Spencer theory

## Spencer Cavity Theory

- Goals: to account for δ-rays and cavity size effect
- Starts with two B-G conditions (narrow *g* region and dose produced by crossing particles) and two additional assumptions (existence of CPE and absence of bremsstrahlung generation)
- Introduces mean energy Δ, needed to cross the cavity
- Based on their energy *T*, electrons in a spectrum are divided into 'fast' ($T \geq \Delta$) and 'slow' ($T < \Delta$) groups

## Spencer Cavity Theory

- For monoenergetic electron beam with $T_0$ emitting N particles per gram through a homogeneous medium *w* the absorbed dose is expressed in terms of restricted stopping power

$$D_w \overset{\text{CPE}}{=} NT_0 = \int_{\Delta}^{T_0} \Phi_T^{e,\delta} \cdot {}_m S_w (T,\Delta)\, dT$$

- The equilibrium spectrum including δ-rays

$$\Phi_T^{e,\delta} = \frac{NR(T_0,T)}{(dT/\rho dx)_w}$$

- $R(T,T_0)$ – ratio of differential electron fluence, including δ-rays to that of primary electrons alone

## Spencer Cavity Theory

Approximate Values of $R(T_0,T) = \Phi^{e,\delta}_T/\Phi^e_T$, the Ratio of the Differential Electron Fluences with and without δ-rays

| | $R(T_0, T)$ | | | | |
|---|---|---|---|---|---|
| $T/T_0$ | C | Al | Cu | Sn | Pb |
| 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 0.50 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 0.25 | 1.05 | 1.05 | 1.06 | 1.06 | 1.07 |
| 0.125 | 1.21 | 1.23 | 1.25 | 1.27 | 1.29 |
| 0.062 | 1.60 | 1.66 | 1.73 | 1.79 | 1.85 |
| 0.031 | 2.4 | 2.6 | 2.8 | 2.9 | 3.1 |
| 0.016 | 4.4 | 4.7 | 5.2 | 5.5 | 6.0 |
| 0.008 | 8.5 | 9.4 | 10.5 | 11.3 | 12.3 |
| 0.004 | 17 | 19 | 22 | 24 | — |

Spectrum enhanced many-fold at low electron energies

## Spencer Cavity Theory

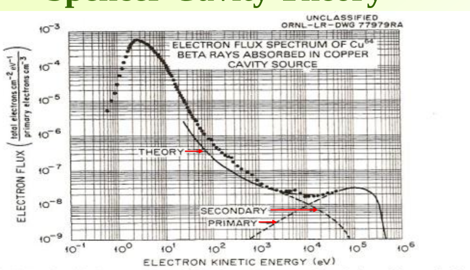



**FIGURE 10.4.** Equilibrium spectrum of $^{64}$Cu β rays in copper. The "primary" curve is the equilibrium spectrum of primary $\beta^-$- and $\beta^+$-particles emitted by the distributed source. The "secondary" curve, extending into the solid curve labeled "theory", is the δ-ray contribution calculated by means of the factor $R(T_0, T)$. The solid curve combines the primaries and calculated secondaries. The points were measured with an electrostatic spectrometer. (After McConnell et al., 1964. Reproduced with permission from H. H. Hubbell, Jr. and the Oak Ridge National Laboratory.)

## Spencer Cavity Theory

- Taking into account adjustment for electron spectrum, dose to the wall

$$D_w \overset{\text{CPE}}{=} NT_0 = N\int_{\Delta}^{T_0} \frac{R(T_0,T)}{(dT/\rho dx)_w} \cdot {}_m S_w (T,\Delta)\, dT$$

- Δ regulates the cavity size; for Δ=0 $R(T_0,T) = \frac{(dT/\rho dx)_w}{{}_m S_w(T,\Delta)}$

- Dose to the cavity

$$D_g = N\int_{\Delta}^{T_0} \frac{R(T_0,T)}{(dT/\rho dx)_w} \cdot {}_m S_g(T,\Delta)\, dT$$

## Spencer Cavity Theory

- Ratio of doses in cavity and wall

$$\frac{D_g}{D_w} = \frac{\int_{\Delta}^{T_0} \frac{R(T_0,T)}{(dT/\rho dx)_w} \cdot {}_m S_g(T,\Delta) dT}{\int_{\Delta}^{T_0} \frac{R(T_0,T)}{(dT/\rho dx)_w} \cdot {}_m S_w(T,\Delta) dT}$$

- Works well for small cavities (electron range is much larger than the cavity size)
- If actual spectrum of crossing charged particles is known, can replace the ratio term

## Spencer Cavity Theory

cavity size ↓

| | | | $D_g/D_w$ | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | Spencer | | | | | | |
| Wall Medium | $T_0$ (keV) | Δ (keV) = Range (mm) = | 2.5 0.015 | 5.1 0.051 | 10.2 0.19 | 20.4 0.64 | 40.9 2.2 | 81.8 7.2 | Bragg-Gray |
| C | 1308 | | 1.001 | 1.002 | 1.003 | 1.004 | 1.004 | 1.005 | 1.005 |
| | 654 | | 0.990 | 0.991 | 0.992 | 0.992 | 0.993 | 0.994 | 0.994 |
| | 327 | | 0.985 | 0.986 | 0.987 | 0.988 | 0.988 | 0.989 | 0.989 |
| Al | 1308 | | 1.162 | 1.151 | 1.141 | 1.134 | 1.128 | 1.123 | 1.117 |
| | 654 | | 1.169 | 1.155 | 1.145 | 1.137 | 1.131 | 1.126 | 1.125 |
| | 327 | | 1.175 | 1.161 | 1.151 | 1.143 | 1.136 | 1.130 | 1.134 |
| Cu | 1308 | | 1.456 | 1.412 | 1.381 | 1.359 | 1.340 | 1.327 | 1.312 |
| | 654 | | 1.468 | 1.421 | 1.388 | 1.363 | 1.345 | 1.329 | 1.327 |
| | 327 | | 1.485 | 1.436 | 1.400 | 1.375 | 1.354 | 1.337 | 1.353 |
| Sn | 1308 | | 1.786 | 1.694 | 1.634 | 1.592 | 1.559 | 1.535 | 1.508 |
| | 654 | | 1.822 | 1.723 | 1.659 | 1.613 | 1.580 | 1.551 | 1.547 |
| | 327 | | 1.861 | 1.756 | 1.687 | 1.640 | 1.602 | 1.571 | 1.595 |
| Pb | 1308 | | — | 2.054 | 1.940 | 1.865 | 1.811 | 1.770 | 1.730 |
| | 654 | | — | 2.104 | 1.985 | 1.904 | 1.848 | 1.801 | 1.796 |
| | 327 | | — | 2.161 | 2.030 | 1.946 | 1.881 | 1.832 | 1.876 |

Values of $D_g/D_w$ Calculated for Air Cavities by Spencer from Spencer Cavity Theory, vs. Bragg-Gray Theory

Better agreement between Spencer and B-G for large cavity sizes

## Spencer Cavity Theory

- The Spencer cavity theory gives somewhat better agreement with experimental observations for small cavities than does simple B-G theory, by taking account of δ-ray production and relating the dose integral to the cavity size
- However, it still relies on the B-G conditions, and therefore fails to the extent that they are violated
- In particular, in the case of cavities that are large (i.e., comparable to the range of the secondary charged particles generated by indirectly ionizing radiation), neither B-G condition is satisfied

## Burlin Cavity Theory

- γ-ray cavity theory, for intermediate cavity size

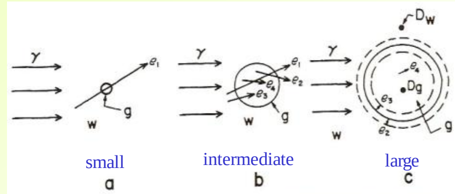



FIGURE 10.5. The cavity-size transition in Burlin theory (see text).

$e_1$ – crossers $e_3$ – stoppers
$e_2$ – starters $e_4$ – insiders

## Burlin Cavity Theory

To arrive at a usefully simple theory Burlin made the following assumptions, either explicitly or implicitly:

1. The media *w* and *g* are homogeneous.
2. A homogeneous γ-ray field exists everywhere throughout *w* and *g*. (This means that no γ-ray attenuation correction)
3. Charged-particle equilibrium exists at all points that are farther than the max electron range from the cavity boundary
4. The equilibrium spectra of secondary electrons generated in *w* and *g* are the same
5. The fluence of electrons entering from the wall is attenuated exponentially as it passes through the medium *g*, without changing its spectral distribution.
6. The fluence of electrons that originate in the cavity builds up to its equilibrium value exponentially as a function of distance into the cavity, according to the same attenuation coefficient β that applies to the incoming electrons

## Burlin Cavity Theory

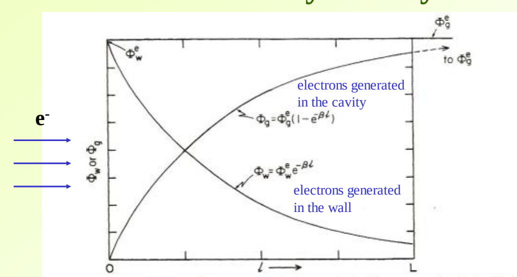



FIGURE 10.6. Illustration of the exponential-decay and -buildup assumption in the Burlin cavity theory. The equilibrium wall fluence of electrons, $\Phi_w^e$, is shown decaying exponentially as they progress into a homogeneous cavity for which the wall *w* and cavity *g* media are assumed to be identical. The electrons under consideration are only those flowing from left to right. The buildup of the cavity-generated electron fluence $\Phi_g$ follows a complementary exponential, asymptotically approaching its equilibrium value $\Phi_g^e = \Phi_w^e$.

## Burlin Cavity Theory

- Cavity relation accounts for 2 sources of electrons depositing dose

$$\frac{\bar{D}_g}{D_w} = d \cdot {}_m\bar{S}_w^g + (1-d)\left(\frac{\bar{\mu}_{en}}{\rho}\right)_w^g$$

- Parameter is related to the cavity size, expressed as

$$d \equiv \frac{\bar{\Phi}_w}{\Phi_w^e} = \frac{\int_0^L \Phi_w^e e^{-\beta l} dl}{\int_0^L \Phi_w^e dl} = \frac{1-e^{-\beta L}}{\beta L}$$

- *l* is the distance (cm) of any point in the cavity from the wall, along a mean chord of length *L*

## Burlin Cavity Theory

- Parameter d~1 for small, d~0 for large cavities
- The corresponding relation for 1 – *d*, representing the average value of $\Phi_g/\Phi_g^e$ throughout the cavity:

$$1-d \equiv \frac{\bar{\Phi}_g}{\Phi_g^e} = \frac{\int_0^L \Phi_g^e\left(1-e^{-\beta l}\right) dl}{\int_0^L \Phi_g^e dl} = \frac{\beta L + e^{-\beta L} - 1}{\beta L}$$

## Burlin Cavity Theory

- For the nonhomogeneous case where $g \neq w$, $(\Phi^e_w \neq \Phi^e_g)$
- Moreover, if the β-value of the cavity medium for the wall electrons is not the same as for the cavity-generated electrons, due to a difference in spectral distributions, then in general

$$\frac{\bar{\Phi}_g}{\Phi_g^e} \equiv d' \neq (1-d)$$

and hence

$$d' + d \neq 1$$

- The Burlin theory ignores this possible source of error in adopting assumptions 5 and 6

## Burlin Cavity Theory

- Theory works well for wide range of cavity sizes and materials
- Parameter β estimated for air-filled cavity

$$\beta = \frac{16\rho}{(T_{\max} - 0.036)^{1.4}}$$

$$e^{-\beta t_{\max}} = 0.01$$

- $T_{max}$ – max starting energy, $t_{\max}$ – max electron penetration depth

## Burlin Cavity Theory Verification

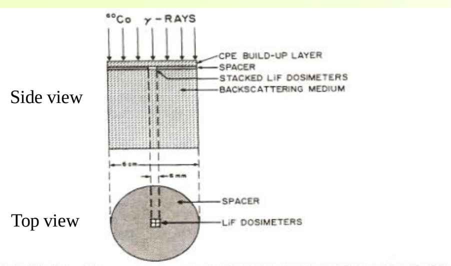



**FIGURE 10.7.** $^{60}$Co γ-ray experiment to test the Burlin theory as applied to LiF TLD chips, each 0.38 × 3.18 × 3.18 mm³, ρ = 2.64 g/cm³, stacked four per layer in 1, 2, 3, 5, and 7 layers. The CPE buildup layer and backscattering medium were both made of the same wall material, either LiF, polystyrene, Al, Cu, or Pb. The spacer was adjusted to equal the TLD stack thickness, and for the results presented here was made of LiF to produce a semi-infinite one-dimensional cavity. (After Ogunleye, et al., 1980. Reproduced with permission of The Institute of Physics, U.K.)

## Burlin Cavity Theory Verification

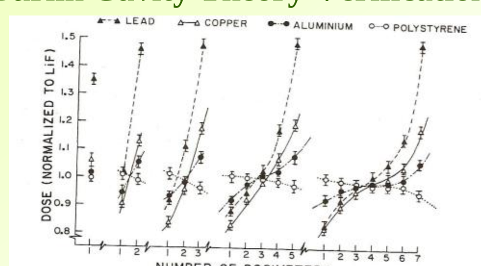



**FIGURE 10.8.** Relative absorbed dose in individual TLD layers in the experiment described in Fig. 10.7. $^{60}$Co γ-rays pass from left to right in relation to this graph. (After Ogunleye et al., 1980. Reproduced with permission of The Institute of Physics, U.K.)

Dose distribution across the cavity of increasing size

## Burlin Cavity Theory Verification

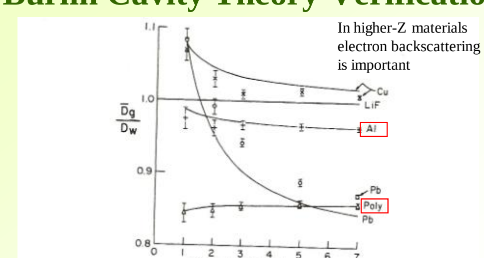



**FIGURE 10.9.** Comparison of the Burlin theory (solid curves) with the experiment referred to in Figs. 10.7 and 10.8. The application of the theory in this case, as described in the text, differs from that of Ogunleye et al. (1980).

Good agreement for polystyrene and aluminum wall media

## The Fano Theorem

- In practice the requirement for small cavity is ignored by matching atomic numbers of wall and cavity materials
- Theorem statement:

In an infinite medium of given atomic composition exposed to a uniform field of indirectly ionizing radiation, the field of secondary radiation is also uniform and independent of the density of the medium, as well as of density variations from point to point

- Proof employs radiation transport equations

## Other Cavity Theories

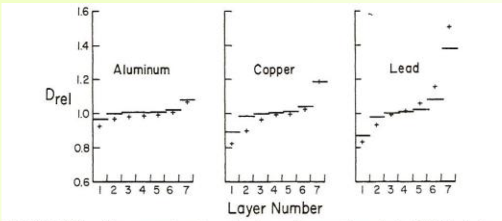



FIGURE 10.10. The average dose to a given layer of a seven-layer stack of LiF dosimeters divided by the equilibrium LiF dose, as calculated using the Kearsley model (bars) and as measured by Ogunleye et al. (+) for Al, Cu, and Pb walls. (After Kearsley, 1984b. Reproduced with permission from E. E. Kearsley and The Institute of Physics, U.K.)

Kearsly theory – modification of Burlin's theory, accounts for electron scattering; predicts dose distribution across the cavity

## Other Cavity Theories

- Luo Zheng-Ming (1980) has developed a cavity theory based on application of electron transport equation in the cavity and surrounding medium. It is very detailed and provides good agreement with experiment
- The effort to develop new and more complicated cavity theories may be diminishing due to strong competition with Monte Carlo methods
- Simple theories will always be useful for approximate solution and estimates

## Dose near interfaces between dissimilar media

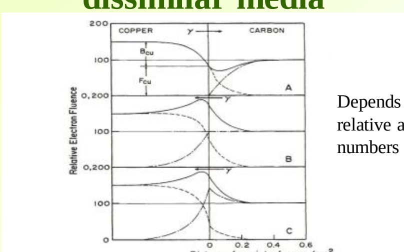



FIGURE 10.11 Variation of electron fluence with distance from a copper–carbon interface irradiated perpendicularly by $^{60}Co$ γ rays (Dutreix and Bernard, 1966). Solid curves: Ionization measured in a thin, flat air layer as it is gradually traversed through the interface by addition and removal of copper and carbon foils at the air-gap walls. Dashed curves: Electrons arising in copper. Dash–dotted curves: Electrons arising in carbon.

The arrows indicate the photon direction in each case: left to right in A, right to left in B and C. $F_{Cu}$ is the fraction of the equilibrium fluence of electrons that flow with a component in the γ-ray direction in the copper; $B_{Cu}$ is the backscattered component. In the carbon the backscattered component is small, and is assumed to be negligible here. (Reproduced with permission from J. Dutreix and The British Journal of Radiology.)

Depends on relative atomic numbers

## Dose near interfaces between dissimilar media

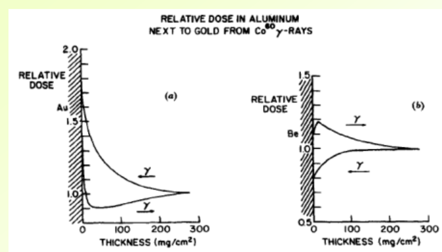
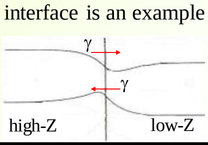



FIGURE 10.12. Variation of dose and electron fluence in aluminum as a function of distance from an interface with (a) gold, (b) beryllium. Arrows indicate the direction of $^{60}Co$ γ rays. (After Wall and Burke, personal communication, 1970.)

## Dose near interfaces between dissimilar media

- A *minimum* is observed just beyond the interface when the photons go from a higher-Z to a lower-Z medium
- A *maximum* is observed just beyond the interface when the photons go from a lower-Z to a higher-Z medium
- Tissue-bone interface is an example




## Summary

- Bragg-Gray theory – works best for small cavities, media of similar atomic numbers
- Spencer theory – includes delta rays, cavity size effect
- Burlin theory – for a range of cavity sizes, no electron scattering included
- Cavity theories create a basis for dosimetry