Exponential Attenuation

Chapter 3

F.A. Attix, Introduction to Radiological Physics and Radiation Dosimetry

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Outline

- Simple exponential attenuation and plural modes of absorption
- Narrow-beam vs. broad-beam attenuation
- Spectral effects
- The build-up factor
- The reciprocity theorem
- Summary

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Introduction

- Uncharged particles (photons and neutrons)
  - lose their energy in relatively few large interactions
  - have a significant probability of passing through matter without interactions
  - no limiting range
- Charged particles
  - typically undergo many small collisions, losing their kinetic energy gradually
  - must always lose some or all of their energy
  - range defined by kinetic energy

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Simple exponential attenuation

- The concept is relevant primarily to *uncharged ionizing radiation*
- Consider a monoenergetic parallel beam of a very large number N_0 of uncharged particles incident perpendicularly on a flat plate of material of thickness L
- Assume <u>ideal</u> case where each particle either is completely absorbed in a single interaction, producing no secondary radiation, or passes straight through the entire plate unchanged in energy or direction

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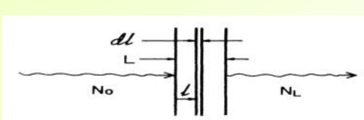
Simple exponential attenuation



- Define ($\mu \cdot 1$) to be the probability that an individual particle interacts in a unit thickness of material traversed
- If N particles are incident upon dl, the change dN in the number N due to absorption is

$$dN = -\mu N\, dl$$

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Simple exponential attenuation

- To find the total change in the number of particles due to absorption in a medium of thickness L:

$$\int_{N_0}^{N_L} \frac{dN}{N} = -\int_0^L \mu dl$$

$$\frac{N_L}{N_0} = e^{-\mu L}$$

- The law of exponential attenuation applies to the *ideal case* of no scattering or secondary radiation in the medium (or scattered particles are not counted in N_L)

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Simple exponential attenuation

- The equation can be replaced by the infinite series

$$\frac{N_L}{N_0} = e^{-\mu L} = 1 - \mu L + \frac{(\mu L)^2}{2!} - \frac{(\mu L)^3}{3!} + \ldots$$

- If the thickness L is small or absorption is low, $\mu L << 1$

$$\frac{N_L}{N_0} = e^{-\mu L} \approx 1 - \mu L$$

- For example, for $\mu L<0.05$ this approximation is valid within ~0.1%

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Simple exponential attenuation

- The quantity μ is the *linear attenuation coefficient*, typically given in units of cm^{-1} or m^{-1}, and *dl* is correspondingly in cm or m
- Also in use is the *mass attenuation coefficient*, μ/ρ, where ρ is the density of attenuating medium; units are cm^2/g or m^2/kg
- The quantity $1/\mu$ (cm or m) is known as the *mean free path* or *relaxation length* of the primary particles. It is the average distance a single particle travels through an attenuating medium before interacting

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Plural modes of absorption

- If more than one absorption process is present, then we can write that the total linear attenuation coefficient μ is equal to the sum of its parts:

$$\mu = \mu_1 + \mu_2 + \cdots$$

where μ_1 is called the *partial linear attenuation coefficient* for process 1, and likewise for the other processes
- Again we assume that each event by each process is totally absorbing, producing no scattered or secondary particles

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Plural modes of absorption

- The law of exponential attenuation

$$\frac{N_L}{N_0} = e^{-(\mu_1 + \mu_2 + \cdots)L}$$

or

$$N_L = N_0 (e^{-\mu_1 L})(e^{-\mu_2 L}) \cdots$$

which demonstrates that the number N_L of particles penetrating through the slab *L* depends on the total effect of all the partial attenuation coefficients

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Plural modes of absorption

- The total number of interactions by all types of processes is given by

$$\Delta N = N_0 - N_L = N_0 - N_0 e^{-\mu L}$$

and the number of interactions by a single process *x* alone is

$$\Delta N_x = (N_0 - N_L)\frac{\mu_x}{\mu} = N_0 (1 - e^{-\mu L})\frac{\mu_x}{\mu}$$

where μ_x/μ is the fraction of the interactions that go by process *x*. Note that you need to know total μ

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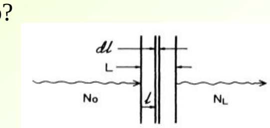
Example 3.1

- Let $\mu_1 = 0.02\ cm^{-1}$ and $\mu_2 = 0.04\ cm^{-1}$ be the partial linear attenuation coefficients in the slab. Let $L = 5$ cm, and $N_0 = 10^6$ particles. How many particles N_L are transmitted, and how many are absorbed by each process in the slab?



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Example 3.1

- The number of transmitted particles

$$N_L = N_0 e^{-(\mu_1+\mu_2)L} = 10^6 \times e^{-(0.02+0.04)5} = 7.408\times 10^5$$

- The number of absorbed particles

$$N_0 - N_L = (10^6 - 7.408\times 10^5) = 2.592\times 10^5$$

- The number of absorbed by processes 1or 2

$$\Delta N_1 = (N_0 - N_L)\frac{\mu_1}{\mu} = 2.592\times 10^5 \times \frac{0.02}{0.06} = 8.64\times 10^4$$

$$\Delta N_2 = (N_0 - N_L)\frac{\mu_2}{\mu} = 2.592\times 10^5 \times \frac{0.04}{0.06} = 1.728\times 10^5$$

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Example 3.1

- If we do not take into account the total μ

$$\Delta N_1 \neq N_0 - N_0 e^{-\mu_1 L} = 10^6 \text{ -}10^6 e^{-0.02x5} = 9.52\times 10^4$$

Error ~ 10%

$$\Delta N_2 \neq N_0 - N_0 e^{-\mu_2 L} = 10^6 \text{ -}10^6 e^{-0.04x5} = 1.813\times 10^4$$

Error ~ 5%

- The result for ΔN_2 has lower error due to μ_2 being closer to μ

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"Narrow-beam" attenuation

- Exponential attenuation will be observed for a monoenergetic beam of identical uncharged particles that are "ideal" - absorbed without producing scattered or secondary radiation
- Real beams of particles interact with matter by processes that may generate either charged or uncharged secondary radiations, as well as scatter
- The total number of particles that exit from the slab is hence *greater* than just the surviving primaries
- What should be counted by a detector?

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"Narrow-beam" attenuation

- Secondary charged particles should not to be counted as uncharged particles
  - charged particles are usually much less penetrating, and thus tend to be absorbed in the attenuator
  - those that do escape can be prevented from entering the detector by enclosing it in a thick enough shield
- Energy given to charged particles is thus regarded as having been absorbed (it is not a part of the primary beam anymore)

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"Narrow-beam" attenuation

- The scattered and secondary uncharged particles can either be counted in N_L, or not
- If they are counted, the exponential attenuation equation becomes invalid in describing the variation of N_L vs. L: case of *broad-beam attenuation*
- If scattered or secondary uncharged radiation reaches the detector, but only the primaries are counted in N_L, the exponential attenuation equation is valid: case of broad-beam geometry but *narrow-beam attenuation*

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"Narrow-beam" attenuation

- Real attenuation coefficient μ must be numerically larger than the value of any corresponding *effective attenuation coefficient* μ´ that is observed under broad-beam attenuation conditions
- There are two general ways of achieving narrow-beam attenuation:
  - *Discrimination* against all scattered and secondary particles that reach the detector, on the basis of particle energy, penetrating ability, direction, coincidence, anticoincidence, time of arrival (for neutrons), etc.
  - *Narrow-beam geometry*, which prevents any scattered or secondary particles from reaching the detector

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Narrow-beam geometry

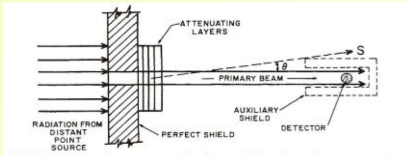

FIGURE 3.2. Narrow-beam geometry. The diameter of the primary photon or neutron beam is made just large enough to cover the detector uniformly. The detector is placed at a large enough distance from the attenuator that the number of scattered or secondary particles (*S*) that reach the detector is negligible in comparison with the number of primary rays.

- Achieving narrow-beam geometry (“good” geometry) is not difficult experimentally
- Used to obtain tabulated values of μ

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Narrow-beam geometry

- The shield is assumed to stop all radiation incident upon it except that passing through its aperture
- If it allows any leakage, it may be necessary to put a supplementary shield around the detector that allows entry of radiation at angles $\theta \approx 0°$
  - Lead is the usual shielding material for x- or γ-rays, especially where space is limited
  - Iron and hydrogenous materials are preferable for fast neutrons

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Broad-beam attenuation

- Any attenuation geometry other than narrow-beam geometry is called broad-beam geometry
- The concept of an ideal broad-beam geometry is more difficult to define, and is experimentally less accessible
- In *ideal broad-beam geometry* every scattered or secondary uncharged particle strikes the detector, but only if generated in the attenuator by a primary particle on its way to the detector, or by a secondary charged particle resulting from such a primary

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Ideal broad-beam attenuation

- The attenuator must be thin enough to allow the escape of
  - all the uncharged particles resulting from first interactions by the primaries
  - all the x-rays and annihilation γ-rays emitted by secondary charged particles that are generated by primaries in the attenuator
- Multiple scattering is excluded from this ideal case
- If we have ideal broad-beam geometry, and the detector that responds in proportion to the radiant energy of all the primary, scattered, and secondary uncharged radiation, then we have a case of *ideal broad-beam attenuation*

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Ideal broad-beam attenuation

- For this case we can write an exponential equation:

$$\frac{R_L}{R_0} = e^{-\mu_{en} L}$$

  - R_0 is the primary radiant energy incident on the detector
  - R_L is the radiant energy of uncharged particles striking the detector when the attenuator is in place
  - L is the attenuator thickness (must remain thin enough to allow escape of all scattered and secondary uncharged particles)
  - μ_{en} is the energy-absorption coefficient

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Ideal broad-beam attenuation

- μ_{en} is often used as an approximation to the effective attenuation coefficient μ' for thin absorbing layers in broad-beam attenuation
- It is referred to as the “straight-ahead approximation”: the scattered and secondary particles are supposed to continue straight ahead until they strike the detector
- The approximation is often not accurate even for thin absorbers, but the true μ' is often not known
- It is adequate in calculating photon attenuation in the wall of an ionization chamber made of low-*Z* material

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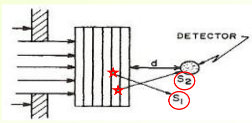
Broad-beam attenuation

- Practical broad-beam geometries usually are not ideal
  - Some of the scattered and secondary radiation that is supposed to reach the detector fails to arrive - this loss of radiation can be called *out-scattering* (particles S_1)
  - Similarly, *in-scattering is* defined as the arrival at the detector of scattered and secondary uncharged particles that are generated in the attenuator by primaries that are *not* aimed at the detector (particles S_2)



Ideal broad-beam geometry may be simulated if in-scattered particles compensate for out-scattered

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Types of geometries and attenuations

- Narrow-beam geometry: only primary strikes the detector, μ is observed for monoenergetic beams
- Narrow-beam attenuation: Only primaries are counted in N_L, μ is observed for monoenergetic beams
- Broad-beam geometry: at least some scattered and secondary radiation strikes the detector
- Broad-beam attenuation: scattered and secondary radiation is counted in N_L, $\mu' < \mu$ is observed
- Ideal broad-beam geometry: every scattered or secondary uncharged particle generated by primary strikes the detector
- Ideal broad-beam attenuation: ideal broad beam geometry and the detector response ~ to the radiant energy striking; $\mu' = \mu_{en}$

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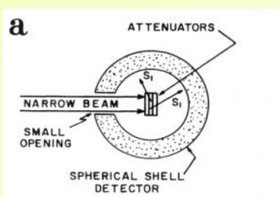
Broad-beam geometries



The narrow radiation beam enters through a small hole and impinges on the attenuating layers of material inside of a (hypothetical) spherical-shell detector

- Practically all scattered rays (S_1) originating in the attenuator will strike the detector, regardless of their direction (except ≈ 180°)
- A deep well-type detector could roughly approximate this geometry
- This setup approaches ideal broad-beam geometry

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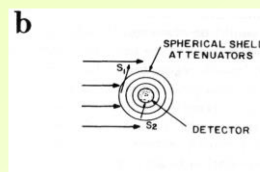
Broad-beam geometries



The attenuating material is arranged in spherical shells surrounding the detector; the beam is made large enough to irradiate the attenuators fully

- In this case the out-scattered rays such as S_1 generated in the attenuator upstream of the detector, but not striking it, tend to be compensated by in-scattered rays such as S_2 originating elsewhere in the attenuator
- This simulates ideal broad-beam geometry at least as closely as any arrangement that relies on such compensation

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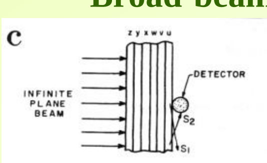
Broad-beam geometries



A plane beam that is infinitely wide compared to the effective maximum range of scattered and secondary radiation, and incident perpendicularly on similarly wide attenuating plates

- The detector is kept as close as possible to the attenuator to allow laterally out-scattered rays such as S_1 to be maximally replaced by in-scattered rays such as S_2
- In practice the detector is kept stationary, and the attenuating slabs are added in sequence of increasing thickness ($u \rightarrow z$)

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Broad-beam geometries

- For a diverging beam the observed attenuation would be exaggerated by a loss of intensity in proportion to the inverse square of the distance from the source
- The detector receives no back-scattered radiation, since there is no material behind it
- The irradiated attenuator subtends a solid angle at the detector of only about 2π radians, as compared to 4π radians for "b"
- The smaller the subtended solid angle, the poorer the "coupling" between the detector and the attenuator, and the less scattered radiation will reach the detector

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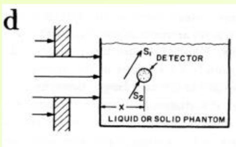
Broad-beam geometries



A detector that may be positioned at a variable depth *x* from the front surface of a large mass of solid or liquid medium, designed to simulate the attenuating properties of the human body (a *phantom)*

- Uncharged particle (usually photon) beams of various cross-sectional dimensions are directed perpendicularly on the phantom, and the detector response is measured vs. depth
- The resulting function, the "central-axis depth-dose" of the beam, for a specified SSD is used in radiotherapy treatment planning
- If the beam and tank were very wide, the attenuation function observed would be similar to that in geometry "c"

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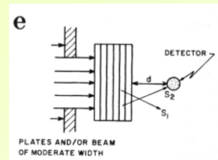
Broad-beam geometries



If a smaller beam size is used in geometries "c" and "d", out-scattered rays S_1 are less fully compensated by in-scattered rays (S_2), and the response of the detector to scattered radiation decreases relative to its response to primary radiation

- The effective attenuation coefficient μ′ observed at a given depth will be closer to μ
- This trend is even more accentuated by moving the detector a distance *d* away from the attenuators
- The larger the ratio *d/w* (for beam width *w* large enough to cover the detector), the closer the setup is to narrow-beam geometry

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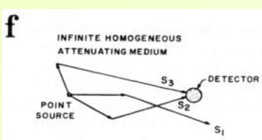
Broad-beam geometries



A point source and a detector are immersed in an infinite homogeneous medium (e.g., water) and separated by a variable distance

- The effect of attenuation can be separated from that of the inverse square law by comparing the detector response in the medium with that in vacuum for the same distance
- Note that out-scattered rays like S_1 are compensated by in-scattered rays like S_2, but additional backscattered rays such as S_3 may also strike the detector, especially when it is close to the source and the primary (source) energy is low

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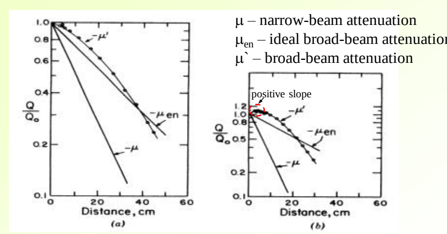
Broad-beam attenuation examples



Broad-beam attenuation of (a) ^{60}Co (1.25 MeV) and (b) ^{203}Hg (0.279 MeV) gamma rays as a function of distance from a point source in an infinite water medium (geometry "f")

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Spectral effects

- Energy dependence of detector response will affect observed attenuation in broad-beam geometries due to difference in sensitivity to primary vs. scatter
- For narrow-beam attenuation the spectral effect will be observed for poly-energetic beams
- In general, for the differential energy-fluence spectrum $\Psi'_L(E)$ (in J/m^2 keV) reaching the detector through attenuator thickness *L*, and the narrow-beam attenuation coefficient $\mu_{E,Z}$ - need to define a mean value:

$$\bar{\mu}_{\Psi,L} = \frac{\int_{E=0}^{E_{\max}} \Psi'_L(E)\mu_{E,Z}\, dE}{\int_{E=0}^{E_{\max}} \Psi'_L(E)\, dE}$$

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Spectral effects

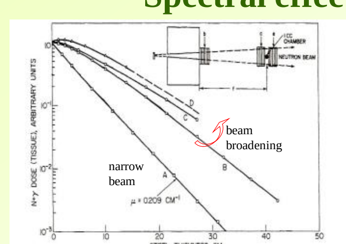

FIGURE 3.5. Narrow vs. broad-beam attenuation of a fast-neutron beam ($\bar{E} \cong 14$ MeV) in steel. (After Attix et al., 1976.) The detector was a 1-cm^3 ion chamber, located $f = 161$ cm from the shield. The beam at that distance was 3 cm in diameter for curve A, 13 × 13 cm^2 for curve B, and 28 × 28 cm^2 for curves C and D. The attenuators were located at position b for curve A, position c for curves B, C, and D, and also position e for curve D. Curve A is for narrow-beam geometry; curves B, C, and D for progressively broader-beam geometry. Reproduced with permission of *Physics in Medicine and Biology*.

- Scattered particles have lower energy and affect attenuation curves the most at shallower depth
- Even for narrow-beam attenuation the slope changes if the beam is poly-energetic (spectrum)

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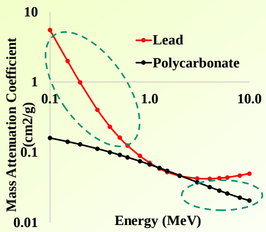
Spectral effects: detector response example



- Measuring transmission to restore the beam spectrum: weak dependence of μ on energy
- Variation in low energy range stronger for lead
- Better variation in higher energy range for polycarbonate material

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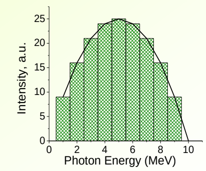
Unfolding a spectrum from transmission measurements

- Total signal at depth (thickness) n

$$S_n = \sum_{i=1}^{N} s_n^i f_i$$

s_n^i = signal from a source of energy in bin i, E_i

- Find response function f_i from a system of NxM linear equations
- Ill-conditioned problem: small change in signal in energy bin i leads to large change in the total signal



N – total number of points collected, M – number of energy bins (N=M typically)

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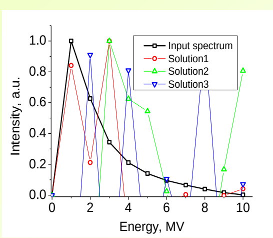
Spectral effects: detector response example



- Transmission measurements result in a set of equations representing an ill-conditioned problem
- Example of using direct matrix inversion

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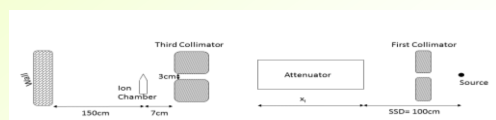
Spectral effects: detector response example

- Positives
  - Experimental set up is easy
  - Allows restoring spectra of YOUR machine
  - Can be implemented in a standard clinic
- Negatives
  - μ dependence on E at therapy energies very weak
  - Therefore ill-conditioned problem requiring involved data analysis



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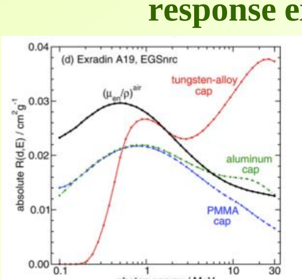
Spectral effects: detector response example



- Response of ion chamber varies with build up caps
- Adds sensitivity to transmission measurements
- More than just an attenuation coefficient of one material
- Use of MC to model the response function

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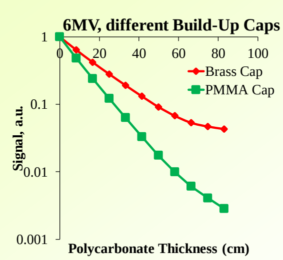
Spectral effects: detector response example



- Significant response difference of the ion chamber as beam is attenuated via low-Z material
- Helps extract spectral information if the detector response is known

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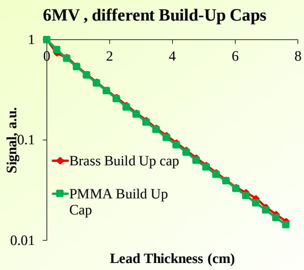
Spectral effects: detector response example



- Observed the response of the ion chamber did not vary much as beam was attenuated via high-Z material

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The buildup factor

- The concept of *buildup factor B* is useful in quantitative description of broad-beam attenuation
- It can be applied with respect to any specified geometry, attenuator, or physical quantity (number of particles, dose, etc.)
- The general definition can be written as

$$B = \frac{\text{quantity due to primary} + \text{scattered and seconday radiation}}{\text{quantity due to primary radiation alone}}$$

- B=1 for narrow-beam geometry
- B>1 and is depth-dependent for broad-beam geometry

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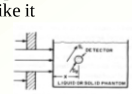
The buildup factor

- For energy fluence arriving at the detector behind a medium thickness *L*, and the narrow-beam attenuation coefficient μ

$$\frac{\Psi_L}{\Psi_0} = Be^{-\mu L}$$

- When *L*=0 (i.e., no attenuator between source and detector), *B* becomes equal to $B_0 \equiv \Psi_L/\Psi_0=1$ for most broad-beam geometries
- For geometry "d", when the detector is at the phantom surface (depth *L* = 0), backscattered rays will still strike it
  - Hence $\Psi_L > \Psi_0$ and so $B_0 > 1$ even for $L = 0$
  - In that case B_0 is called the *backscatter factor*



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The buildup factor

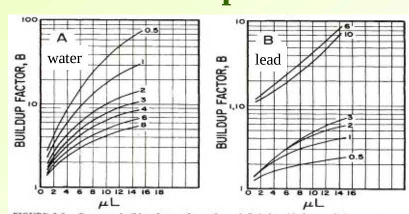

FIGURE 3.6. Exposure buildup factors for a plane, infinitely wide beam of photons perpendicularly incident on semi-infinite media of (*A*) water and (*B*) lead. Curves are labeled with photon energies in MeV. Abscissae indicate the depth in units of the mean free path 1/μ. (Goldstein, 1957.) Reproduced with the author's permission.

- Build-up factors for other quantities show similar behavior of increase with depth
- Rate of increase depends on material (Z) and photon energy

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The buildup factor

- An alternative concept to the buildup factor is the *mean effective attenuation coefficient*, which can be defined through energy fluence ratio as:

$$\frac{\Psi_L}{\Psi_0} = Be^{-\mu L} \equiv e^{-\bar{\mu}' L}$$

or, solving for $\bar{\mu}'$,

$$\bar{\mu}' \equiv \mu - \frac{\ln B}{L}$$

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The buildup factor

B	μL	$X_L/X_s = Be^{-\mu L}$	L (cm)	$\bar{\mu}'$ (cm^{-1})	$\bar{\mu}'/\mu$	$\bar{\mu}'/\mu_{en}$
3	1.7	0.548	24	0.025	0.35	0.81
6	4.0	0.110	57	0.039	0.55	1.26
10	6.3	1.84×10^{-2}	89	0.045	0.64	1.46
20	10.9	3.69×10^{-4}	154	0.051	0.72	1.65
30	14.6	1.37×10^{-5}	207	0.054	0.76	1.75

$\mu = 0.0706\ cm^{-1}$; $\mu_{en} = 0.0309\ cm^{-1}$

$$\bar{\mu}' < \mu' < \mu$$

- Comparison of exposure buildup factor *B* and mean effective attenuation coefficient for a plane beam of 1-MeV γ-rays in water
- Mean effective attenuation coefficient $\bar{\mu}$\` is not as strongly dependent on depth L as B

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The reciprocity theorem

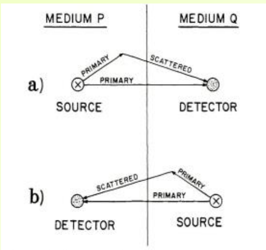

FIGURE 3.8. The reciprocity theorem in radiation transport (see text).

The simplest formulation of the *reciprocity theorem* for the attenuation of any kind of radiation:

Reversing the positions of a point detector and a point source within an infinite homogeneous medium does not change the amount of radiation detected

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The reciprocity theorem

- If *P* and *Q* are different with respect to their scattering and/or attenuating properties, *the transmission of primary rays still remains the same, left or right*
- However, the generation and/or transmission of a scattered ray may differ
  - If the scattered ray is absorbed more strongly in medium *Q* than in *P*, all else being equal, it is more likely to reach the detector in case *b* than in case *a*, since its path length in medium *Q* is longer in case *a*

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The reciprocity theorem

- The theorem is not exact in dissimilar media, except for primary rays
- It is still useful in calculating the attenuation of radiation in dissimilar or nonhomogeneous media, if
  - either the primary rays dominate
  - or the generation and propagation of scattered rays is not strongly dissimilar in the different media

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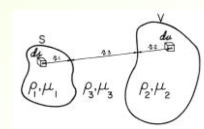
The reciprocity theorem

- Mayneord extended the reciprocity theorem to the case where the source and detector were both extended volumes:
  - The integral dose in a volume *V* due to a γ-ray source uniformly distributed throughout source volume *S* is equal to the integral dose that would occur in *S* if the same activity density per unit mass were distributed throughout *V*
- This can be exact with respect to the dose resulting from primary rays only, unless *V* and *S* are parts of an infinite homogeneous medium
- The theorem as stated is only true if the mass energy-absorption coefficients are the same for the materials in *S* and *V*



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The reciprocity theorem

- As a corollary to this theorem, one can state that:
  - If *S* and *V* contain identical, uniformly distributed total activities, they will each deliver to the other the same average absorbed dose
- Furthermore:
  - If all the activity in *S* is concentrated at an internal point *P*, then the dose at *P* due to the distributed source in *V* equals the average dose in *V* resulting from an equal source at *P*
- This latter statement can be taken a step further to say:
  - The dose at any internal point *P* in *S* due to a uniformly distributed source throughout *S* itself is equal to the average absorbed dose in *S* resulting from the same total source concentrated at *P*
- This relationship, though exact only in an infinite homogeneous medium, or for primary radiation, is nevertheless practically useful in calculation of internal dose due to distributed sources in the body (MIRD tables)

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Summary

- Simple exponential attenuation equation is valid only for mono-energetic beams in narrow-beam geometry
- Broad-beam geometry is more achievable in realistic experiments
- Broad-beam attenuation equation can be effectively used, taking into account energy-dependent detector response
- The build-up factor and mean effective attenuation coefficient are used for quantitative description of broad-beam attenuation
- The reciprocity theorem is practically useful in calculation of internal dose due to distributed sources in the body

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