Absorbed Dose in Radioactive Media

Chapter 5

F.A. Attix, Introduction to Radiological Physics and Radiation Dosimetry

Outline

• General dose calculation considerations, absorbed fraction
• Radioactive disintegration processes and associated dose deposition
  – Alpha disintegration
  – Beta disintegration
  – Electron-capture transitions
  – Internal conversion
• Summary

Introduction

• We are interested in calculating the absorbed dose in radioactive media, applicable to cases of
  – Dose within a radioactive organ
  – Dose in one organ due to radioactive source in another organ
• If conditions of CPE or RE are satisfied, dose calculation is straightforward
• Intermediate situation is more difficult but can be handled at least in approximations

Radiation equilibrium

a. The atomic composition of the medium is homogeneous
b. The density of the medium is homogeneous
c. The radioactive source is uniformly distributed
d. No external electric or magnetic fields are present

Charged-particle equilibrium

• Each charged particle of a given type and energy leaving the volume is replaced by an identical particle of the same energy entering the volume
• Existence of RE is sufficient condition for CPE
• Even if RE does not exist CPE may still exist (for a very large or a very small volume)

Limiting cases

• Emitted radiation typically includes both photons (longer range) and charged particles (shorter range)
• Assume the conditions for RE are satisfied
  • Consider two limited cases based on the size of the radioactive object
  •
**Limiting cases: small object**

- A radioactive object V having a mean radius not much greater than the maximum charged-particle range \(d\)
- CPE is well approximated at any internal point \(P\) that is at least a distance \(d\) from the boundary of V
- If \(d \ll l/\mu\) for the \(\gamma\)-rays, the absorbed dose \(D\) at \(P\) approximately equals to the energy per unit mass of medium that is given to the charged particles in radioactive decay (less their radiative losses)
- The photons escape from the object and are assumed not to be scattered back by its surroundings

**Limiting cases: large object**

- A radioactive object V with mean radius \(\gg l/\mu\) for the most penetrating \(\gamma\)-rays
- RE is well approximated at any internal point \(P\) that is far enough from the boundary of V so \(\gamma\)-ray penetration through that distance is negligible
- The dose at \(P\) will then equal the sum of the energy per unit mass of medium that is given to charged particles plus \(\gamma\)-rays in radioactive decay

**Absorbed fraction**

- An intermediate-size radioactive object V
- Dose at \(P\) will then equal the sum of the energy per unit mass of medium that is given to charged particles plus dose from some \(\gamma\)-rays
- To estimate the dose from \(\gamma\)-rays define absorbed fraction:

\[
AF = \frac{\gamma\text{-ray radiant energy absorbed in target volume}}{\gamma\text{-ray radiant energy emitted by source}}
\]

**Intermediate-size radioactive object**

- Consider volume V filled by a homogeneous medium and a uniformly distributed \(\gamma\)-source
- The volume may be surrounded by
  - Case 1: infinite homogeneous medium identical to V, but non-radioactive (an organ in the body)
  - Case 2: infinite vacuum (object in air)

**Case 1**

- Reciprocity theorem: energy spent in \(dv\) due to the source in \(dv'\):

\[
E_{dv',dv} = E_{dv,dv'} \quad \text{and} \quad E_{dv,V} = E_{V,dv}
\]

- No source in \(dv''\)
- Define \(\bar{\gamma}_{dv,V}\) the part of that energy that is spent in \(V\) (source \(dv\) and target \(V\))

\[
\bar{\gamma}_{dv,V} ' = \text{energy spent} = \text{energy imparted}'
\]
Case 1 continued

- The absorbed fraction with respect to source $dv$ and target $V$ is:
  \[ AF_{dv,V} = \frac{\bar{\delta}_{dv,V}}{R_{dv}} \]

- Estimates reduction in absorbed dose relative to RE condition

- For very small radioactive objects ($V \rightarrow dv$) this absorbed fraction approaches zero; for an infinite radioactive medium it equals unity

\[ AF_{dv,V} \approx 1 - e^{-\mu \bar{r}} \]

Case 1 example

- Dose calculations published in MIRD (Medical Internal Radiation Dosimetry) reports
- The larger the radius, the lower the $g$-ray energy – the closer to RE condition ($AF=1$)

Case 2

- To obtain a crude estimate of the dose at some point $P$ within a uniformly $\gamma$-active homogeneous object, it may suffice to obtain the average distance $\bar{r}$ from the point to the surface of the object by
  \[ \bar{r} = \frac{1}{4\pi} \int_{0}^{2\pi} \int_{0}^{\pi} r \sin \theta \ d\theta \ d\beta \]
- Then one may employ $\mu \bar{r}$ in the straight-ahead approximation to obtain
  \[ AF_{dv,V} \approx 1 - e^{-\mu \bar{r}} \]

Case 1 continued

- Reduction in the absorbed dose due to $\gamma$-rays energy escaping from $V$

- Can estimate $AF$ using mean effective attenuation coefficient $\bar{\mu}$ for $\gamma$-rays energy fluence through a distance $r$ in the medium
  \[ AF_{dv,V} = \frac{1}{4\pi} \int_{0}^{\pi} \int_{0}^{2\pi} (1 - e^{-\bar{\mu} r}) \sin \theta \ d\theta \ d\beta \]

- For poly-energetic sources have to find an average value of the absorbed fraction

Case 2

- More difficult to calculate
- The reciprocity theorem is only approximate due to the lack of backscattering
- Energy dependence (max ~80keV)
- Dose is lower than in Case 1 (ratios >1)

MIRD tables

- $AF$ is incorporated in “S-value” tabulated for each radionuclide and source-target configuration used in nuclear medicine
- Dose absorbed in any organ (target) due to a source in some other organ is calculated based on cumulative activity, $D=A\times S$
- $S$ values are typically calculated by Monte Carlo technique
- Straightforward approach, but accuracy is limited
The accuracy is continuously improved through revisions and updates. Example: kidneys represent a frequent source of radiopharmaceutical uptake in both diagnostic and therapeutic nuclear medicine.

Radioactive disintegration processes
- Radioactive nuclei undergo transformations to a more stable state through expulsion of energetic particles:
  - $\alpha$-particle: $\Delta Z = -2$, $\Delta A = -4$
  - $\beta^-$-particle: $\Delta Z = +1$, $\Delta A = 0$
  - $\beta^+$-particle: $\Delta Z = -1$, $\Delta A = 0$
  - $\gamma$-ray: $\Delta Z = 0$, $\Delta A = 0$
- The total mass, energy, momentum, and electric charge are conserved.
- Energy equivalent of the rest mass:
  - $1 \text{ amu}=1/12 \text{ of the mass of } ^{12}\text{C nucleus}=931.50 \text{ MeV}$
  - $1 \text{ electron mass}=0.51100 \text{ MeV}$

Alpha disintegration
- Occurs mainly in heavy nuclei.
- Example: decay of radium to radon, presented by the mass-energy balance equation:
  \[ ^{226}\text{Ra} \quad \xrightarrow{\text{parent}} \quad ^{222}\text{Rn} + ^{4}\text{He} + 4.78 \text{ MeV} \]
- Each of the elemental terms represents the rest mass of a neutral atom of that element.
- Energy term represents kinetic energy released.

Absorbed dose from $\alpha$-disintegration
- Take into account both branches through the average branching ratios.
- For radium average kinetic energy given to charged particles per disintegration:
  \[ E_\alpha = 0.946 \times 4.78 \text{ MeV} + 0.054 \times 4.6 \text{ MeV} = 4.77 \text{ MeV} \]
- For CPE condition in a small (1 cm) radium-activated object: $D=4.77 \times n \text{ MeV/g}$, where $n$ = number of disintegrations per gram of the matter.
- For RE condition (large object): $D=4.78 \times n \text{ MeV/g}$, includes $\gamma$-ray energy.
Beta disintegration

- Nuclei having excess of neutrons typically emit an electron, \( \beta^- \) particle; atomic number \( Z \) is increased by 1
- Nuclei having excess of protons typically emit positron, \( \beta^+ \) particle; atomic number \( Z \) is decreased by 1
- Nucleus is left in an exited state, emitting one or more \( \gamma \)-rays
- Example:

\[
{^{32}_{15}P} \rightarrow {^{32}_{16}S}^+ + {^0_0}\nu + 1.71\text{MeV}
\]

\( \text{kinetic energy} \)

- Average kinetic energy of the \( \beta^- \) or \( \beta^+ \) particle is typically 0.3-0.4 \( E_{\text{max}} \); \( 1/3 \) \( E_{\text{max}} \) is often used for purposes of roughly estimating absorbed dose
- Neutrino is not included in dose estimates due to almost zero rest mass energy and no charge

Absorbed dose from \( \beta^- \) disintegration

- Under CPE condition \( D = nE_{\text{avg}} \) in MeV/g for \( n \) disintegrations per gram of medium
- Any additional contributions to energy deposition due to \( \gamma \)-rays must be included for RE condition
- Radiative losses by \( \beta^- \) rays, such as bremsstrahlung and in-flight annihilation, are ignored

Example 5.1

- Uniformly distributed \( \beta^- \) and \( \gamma \)-ray source
- The rest-mass loss is spent
  - half in 1-MeV \( \gamma \)-ray production and
  - half in \( \beta^- \)-decay, for which \( E_{\text{max}} = 5 \) MeV and \( E_{\gamma} = 2 \) MeV
- The point of interest \( P \) is located 5 cm inside the boundary of the object, at an average distance \( r = 20 \) cm from the boundary.
- \( \mu_{\gamma} = 0.0306 \text{ cm}^{-1} \) and \( \mu = 0.0699 \text{ cm}^{-1} \) for the \( \gamma \)-rays
- A total energy of \( 10^2 \text{ J} \) converted from rest mass in each kg of the object
- Estimate the absorbed dose at \( P \)
Example 5.1

- For $\beta$-ray $E_{\text{max}}=5$ MeV corresponds to maximum particle range (Appendix E) $d \approx 2.6 \text{ cm} < 1/\mu=14.3 \text{ cm}$
- CPE exists at P, therefore dose due to $\beta$-rays:

$$D_\beta = E_{\text{avg}} \times \frac{1}{2} \times 10^{-7} \text{ J/kg} = 10^{-7} \text{ J/kg}$$

- For $\gamma$-ray 20 cm is not $> \frac{1}{1/\mu}=14.3 \text{ cm}$
- RE does not exist at P, therefore have to use absorbed fraction:

$$D_\text{int} = (2.3+10) \times 10^{-7} \text{ J/kg} = 1.23 \times 10^{-7} \text{ J/kg}$$

Example 5.2

- $E_{\text{max}}=1.71$ MeV corresponds to maximum particle ($\beta^-$) range of $\approx 0.8 \text{ cm} < 1 \text{ cm}$
- CPE condition
- Absorbed dose rate: $\dot{D} = \dot{N} \times E_{\text{avg}}$

$$\dot{D} = 6 \times 10^5 \text{ dis/g/sec} \times 0.694 \text{ MeV/dis}$$

$$= 4.164 \times 10^8 \text{ MeV/g/sec} \times 3600 \text{ s/hr} \times 1.602 \times 10^{-10} \text{ Gy/MeV/g}$$

$$= 2.4 \text{ Gy/h}$$

Electron-capture transitions

- Example: \(^{22}_{11}\text{Na} \rightarrow ^{22}_{10}\text{Ne} \) with half-life for both branches of 2.60 years
  - $\beta^+$ branch

\( ^{22}_{11}\text{Na} \rightarrow ^{22}_{10}\text{Ne} + e^- + \beta^+ + \gamma + 0.546 \text{ MeV (k,e.)} \)

2m$_0 c^2 = 1.022 \text{ MeV}$

- EC branch

\( ^{22}_{11}\text{Na} \rightarrow ^{22}_{10}\text{Ne} + 0^- + \gamma + E_\beta + 1.275 \text{ MeV (E}_\gamma) \)

$1.568 \text{ MeV}$

Electron-capture transitions

- Parent nucleus captures its own atomic electron from K-shell (-90% probability) or L-shell (-10% probability) and emits monoenergetic neutrino
- Resulting shell vacancy is filled with electron from a higher orbit, leading to emission of a fluorescence x-ray
- Process competing with $\beta^+$ disintegrations

Electron-capture transitions

- Binding energy for K-shell $E_0 \sim 1 \text{ keV}$
- For $\beta^+$ to occur the minimum atomic mass decrease of $2m_0$ between the parent and daughter nuclei is required to supply $\beta^+$ with kinetic energy; EC does not have this requirement
Absorbed dose for EC process

- Most of the energy is carried away by neutrino
- The only available energy for dose deposition comes from electron binding term $E_b$, which is very small compared to that of neutrino

Internal conversion

- An excited nucleus can impart its energy directly to its own atomic electron, which then escapes with the net kinetic energy of $h\nu - E_b$ ($h\nu$ is the excitation energy)
- No photon is emitted in this case
- Process competing with $\gamma$-ray emission
- Internal conversion coefficient is the ratio of $N_e/N_\gamma$

Internal conversion

- Example: $^{137}\text{Cs} \rightarrow ^{137}\text{Ba}$

Absorbed dose for internal conversion

- If IC occurs in competition with $\gamma$-ray emission, it results in increase in absorbed dose in small objects (CPE condition) due to release of electron locally depositing the energy $E_{Kc} = h\nu - E_b$
- In addition electron binding energy is contributed to the dose unless it escapes as a fluorescence x-ray

Absorbed dose for internal conversion

- If the fraction $p = 1 - AF$ of these fluorescence x-rays escape, then the energy contributed to dose per IC event under CPE condition

$$f_{Kc} = h\nu - p_k Y_k h\nu - p_l Y_l h\nu$$

- Using straight-ahead approximation $p \equiv e^{-\mu r}$
- Values of fluorescence yield $Y_{K,L}$ and the mean emitted x-ray energies $h\nu_{K,L}$ are tabulated

Fluorescence data

- Fluorescence yield for K and L shells
- Electron binding energies and mean fluorescence x-ray energies, K and L shells
Example 5.5

• A sphere of water 10 cm in diameter contains a uniform source of $^{137}$Cs undergoing $10^3$ dis/(g s). What is the absorbed dose at the center, in Gy, for a 10-day period, due only to the decay of $^{137m}$Ba? Use the mean-radius straight-ahead approximation.

For $\gamma$-ray of 0.662 MeV in water $\mu_w = 0.0327$ cm$^{-1}$

Example 5.5

• First check RE condition: $1/\mu = 30.6$ cm $>> r = 5$

• Find absorbed fraction

• Dose in 10 days $= 8.64 \times 10^5$ s is

$$AF = 1 - e^{-0.125 \cdot 10} = 0.151$$

$$D_y = 10^3 \frac{\text{dis}}{\text{g s}} \times 0.85 \frac{\gamma - \text{rays}}{\text{dis}} \times 0.662 \frac{\text{MeV}}{\gamma - \text{ray}} \times 1.602 \times 10^{-10} \frac{\text{Gy}}{\text{MeV/g}} \times 8.64 \times 10^5 \times AF$$

$$= 1.17 \times 10^{-2} \text{Gy}$$

Example 5.5

• Similarly, for the L+M+...-shell conversion process we need $Y_K = 0.90$, $h\nu = 6$ keV, and $\mu_w = 24$ cm$^{-1}$ for 6 keV. Then $p_L \equiv e^{-\mu_w r} = 0$ and the corresponding dose contribution

$$D^L_K = 10^3 \frac{\text{dis}}{\text{g s}} \times 0.07 \frac{\text{IC}(L)}{\text{dis}} \times (h\nu - p_L Y_L h\nu) \frac{\text{MeV}}{\text{IC}(L)} \times 1.602 \times 10^{-10} \frac{\text{Gy}}{\text{MeV/g}} \times 8.64 \times 10^5 \times AF$$

$$= 1.65 \times 10^{-3} \text{Gy}$$

• The total absorbed dose is $D_{tot} = D_y + D^L_K = 2.03 \times 10^{-7}$ Gy

Summary

• General approach to dose calculation within and outside of distributed radioactive source
  – Absorbed fraction
  – Radioactive disintegration processes and calculation of absorbed dose
    – Alpha disintegration
    – Beta disintegration
    – Electron-capture transitions
    – Internal conversion [Less important for dose deposition]

Radioactive decay

Chapter 6

F.A. Attix, Introduction to Radiological Physics and Radiation Dosimetry
Outline
- Decay constants
- Mean life and half life
- Parent-daughter relationships: removal of daughter products
- Radioactivation by nuclear interactions
- Exposure-rate constant
- Summary

Introduction
- Particles inside a nucleus are in constant motion
- Natural radioactivity: a particle can escape from a nucleus if it acquires enough energy
- Most lighter atoms with $Z \leq 82$ (lead) have at least one stable isotope
- All atoms with $Z > 82$ are radioactive and disintegrate until a stable isotope is formed
- Artificial radioactivity: nucleus can be made unstable upon bombardment with neutrons, high energy protons, etc.

Total decay constants
- Consider a large number $N$ of identical radioactive atoms
- The rate of change in $N$ at any time
  \[
  \frac{dN}{dt} = \lambda N
  \]
- We define $\lambda$ as the \textit{total radioactive decay} (or transformation) \textit{constant}, it has the dimensions reciprocal time (usually $s^{-1}$)
- The activity of the group is defined as $A = \lambda N$

Total decay constants
- Integrating the rate of change in number of atoms we find
  \[
  \frac{N}{N_0} = e^{-\lambda t}
  \]
- The ratio of activities at time $t$ to that at $t_0 = 0$
  \[
  \frac{\lambda N}{\lambda N_0} = e^{-\lambda t}
  \]

Partial decay constants
- If a nucleus has more than one possible mode of disintegration (i.e., to different daughter products), the total decay constant can be written as the sum of the partial decay constants $\lambda_i$:
  \[
  \lambda = \lambda_A + \lambda_B + \cdots
  \]
- The total activity is $N\lambda = N\lambda_A + N\lambda_B + \cdots$

Partial decay constants
- The \textit{partial activity} of the group of $N$ nuclei with respect to the $i$th mode of disintegration can be written
  \[
  \lambda_i N = \lambda_i N_0 e^{-\lambda t}
  \]
- Each partial activity $\lambda_i N$ decays at the rate determined by the total decay constant $\lambda$, since the stock of nuclei ($N$) available at time $t$ for each type of disintegration is the same for all types, and its depletion is the result of their combined activity
- The fractions $\lambda_i N_0 / N$ are constant
**Units of activity**

- The old unit of activity was the Curie (Ci), originally defined as the number of disintegrations per second occurring in a mass of 1 g of 226Ra.
- When the activity of 226Ra was measured more accurately the Curie was set equal to \(3.7 \times 10^{10}\) s\(^{-1}\).
- More recently it was decided by an international standards body to establish a new special unit for activity, the becquerel (Bq), equal to 1 s\(^{-1}\).

\[1\text{Ci} = 3.7 \times 10^{10}\text{Bq}\]

**Mean life and half life**

- The expectation value of the time needed for an initial population of \(N_0\) radioactive nuclei to decay to 1/e of their original number is called the mean life \(\tau = 1/\lambda\).

\[\frac{N}{N_0} = e^{-\lambda t} = \frac{1}{e} = 0.3679\]

- The half-life \(\tau_{1/2}\) is the expectation value of the time required for 1/2 of the initial number of nuclei to disintegrate, and hence for the activity to decrease by half

\[\frac{\Delta N}{N_0} = e^{-\lambda \tau_{1/2}} = 0.5; \quad \tau_{1/2} = \frac{0.6931}{\lambda} = 0.6391\tau\]

**Radioactive parent-daughter relationships**

- Consider an initially pure large population \((N_1)_0\) of parent nuclei, which start disintegrating with a decay constant \(\lambda_1\) at time \(t = 0\).
- The number of parent nuclei remaining at time \(t\) is \(N_1 = (N_1)_0 e^{-\lambda_1 t}\).
- Simultaneously the daughter will disintegrate with a decay constant of \(\lambda_2\) (2nd generation doing the decaying).
- The rate of removal of the \(N_1\) daughter nuclei which exist at time \(t_0\) will be \(-\lambda_2 N_2\).

**Radioactive parent-daughter relationships**

- Thus the net rate of accumulation of the daughter nuclei at time \(t\) is

\[
\frac{dN_2}{dt} = \lambda_2 N_1 - \lambda_2 N_2
\]

\[= \lambda_1 (N_1)_0 e^{-\lambda_1 t} - \lambda_2 N_2\]

- The activity of the daughter product at any time \(t\), assuming \(N_2 = 0\) at \(t = 0\), is

\[\lambda_2 N_2 = \lambda_1 (N_1)_0 \frac{\lambda_2}{\lambda_2 - \lambda_1} \left( e^{-\lambda_1 t} - e^{-\lambda_2 t} \right)\]

**Radioactive parent-daughter relationships**

- The ratio of daughter to parent activities vs. time:

\[
\frac{\lambda_2 N_2}{\lambda_1 N_1} = \frac{\lambda_2}{\lambda_2 - \lambda_1} \left( 1 - e^{-\lambda_1 t} \right)
\]

- If \(\lambda_1\) is composed of partial decay constants \(\lambda_{1A}\), \(\lambda_{1B}\), and so on, resulting from disintegrations of \(A\), \(B\), … types, then the ratio for a particular type \(A\) is

\[
\frac{\lambda_{1A} N_2}{\lambda_{1A} N_1} = \frac{\lambda_{1A}}{\lambda_{1A} - \lambda_1} \left( 1 - e^{-\lambda_1 t} \right)
\]

Here "2" describes 2nd generation of that A daughter

**Equilibria in parent-daughter activities**

- The activity of a daughter resulting from an initially pure population of parent nuclei will be zero both at \(t = 0\) and \(\infty\).

\[
\frac{d(\lambda_1 N_1)}{dt} = 0 = \left(-\lambda_1 e^{-\lambda_1 t} + \lambda_2 e^{-\lambda_2 t}\right)
\]

- We can find the time \(t_n\) when \(\lambda_2 N_2\) reaches a maximum

\[
\frac{d(\lambda_2 N_2)}{dt} = 0 = \left(-\lambda_2 e^{-\lambda_1 t} + \lambda_2 e^{-\lambda_2 t}\right)
\]

\[t_n = \frac{\ln(\lambda_2 / \lambda_1)}{\lambda_2 - \lambda_1}\]

- Occurs at the same time that the activities of the parent and daughter are equal (if only one daughter)
Daughter longer-lived than parent, \( \lambda_2 < \lambda_1 \)
- For a single daughter product the ratio of activities
  \[
  \frac{\lambda_2 N_2}{\lambda_1 N_1} = \frac{\lambda_2}{\lambda_1} - \frac{\lambda_2}{\lambda_1} e^{(\lambda_1-\lambda_2) t}
  \]
- This ratio increases continuously with \( t \) for all times
- Since \( \lambda_2 N_1 = \lambda_1 (N_1)e^{\lambda_1 t} \) one can construct the activity curves vs. time for the representative case of metastable tellurium-131 decaying to its only daughter iodine-131; and then to xenon-131:

\[
{^{131}}{\text{Te}} \xrightarrow{\beta} {^{131}}{\text{I}} \rightarrow {^{131}}{\text{Xe}}
\]

Daughter shorter-lived than parent, \( \lambda_2 > \lambda_1 \)
- For \( t \gg t_m \) the value of the daughter/parent activity ratio becomes a constant, achieving transient equilibrium:
  \[
  \frac{\lambda_2 N_2}{\lambda_1 N_1} = \frac{\lambda_2}{\lambda_1}
  \]
- If the decay scheme is branching (\( \lambda_2 = \lambda_{1A} + \lambda_{1B} + \ldots \))
  \[
  \frac{\lambda_2 N_2}{\lambda_1 N_1} = \frac{\lambda_{1A}}{\lambda_1} \frac{\lambda_2}{\lambda_1}
  \]

\[ n \text{ large for } 1 \approx \frac{\lambda_2}{\lambda_1} \]

Daughter shorter-lived than parent, \( \lambda_2 > \lambda_1 \)

Daughter longer-lived than parent, \( \lambda_2 < \lambda_1 \)

Only daughter much shorter-lived than parent, \( \lambda_2 \gg \lambda_1 \)
- For long times (\( t \gg \tau_2 \)) the ratio of activities
  \[
  \frac{\lambda_2 N_2}{\lambda_1 N_1} = \frac{\lambda_2}{\lambda_2 - \lambda_1} \approx 1
  \]
  the activity of the daughter very closely approximates that of the parent
- Such a special case of transient equilibrium, where the daughter and parent activities are practically equal, is called secular equilibrium (typically, with a long-lived parent “lasting through the ages”)

Example of transient equilibrium: \( ^{99}\text{Mo} \) to \( ^{99m}\text{Tc} \)
- Two branches: 86% decays to \( ^{99m}\text{Tc} \), 14% to other excited states of \( ^{99}\text{Tc} \)
**Only daughter much shorter-lived than parent, \( \lambda_2 >> \lambda_1 \)**

- An example of this is the relationship of \(^{226}\text{Ra}\) parent, decaying to \(^{222}\text{Rn}\) daughter, then to \(^{218}\text{Po}\):

\[
{^{226}\text{Ra}}_{88} \xrightarrow{\alpha, \nu} {^{222}\text{Rn}}_{88} \alpha \rightarrow {^{218}\text{Po}}_{84}
\]

- The ratio of activities

\[
\frac{\lambda_2 N_2}{\lambda_1 N_1} = \frac{0.18125}{0.18125 - 1.1845 \times 10^{-6}} = 1.000007 \equiv 1
\]

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**Removal of daughter products**

- For diagnostic or therapeutic applications of short-lived radioisotopes, it is useful to remove the daughter product from its relatively long-lived parent, which continues producing more daughter atoms for later removal and use.
- The greatest yield *per milking* will be obtained at time \( t_m \) since the previous milking, assuming complete removal of the daughter product each time.
- Waiting much longer than \( t_m \) results in loss of activity due to disintegrations of both parent and daughter.

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**Radioactivation by nuclear interactions**

- Stable nuclei may be transformed into radioactive species by bombardment with suitable particles, or photons of sufficiently high energy.
- Thermal neutrons are particularly effective for this purpose, as they are electrically neutral, hence not repelled from the nucleus by Coulomb forces, and are readily captured by many kinds of nuclei.
- Tables of isotopes list typical reactions which give rise to specific radionuclides.

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**Radioactivation by nuclear interactions**

- Let \( N_i \) be the number of target atoms present in the sample to be activated:

\[
N_i = \frac{N A m}{A}
\]

where \( N_A = \) Avogadro’s constant (atoms/mole)

\( A = \) gram-atomic weight (g/mole), and

\( m = \) mass (g) of target atoms only in the sample.
Radioactivation by nuclear interactions

- If $\varphi$ is the particle flux density ($s^{-1} \text{cm}^{-2}$) at the sample, and $\sigma$ is the interaction cross section ($\text{cm}^2/\text{atom}$) for the activation process, then the initial rate of production ($s^{-1}$) of activated atoms is
  \[
  \frac{dN_{\text{act}}}{dt} = \varphi N_t \sigma
  \]
- The initial rate of production of activity of the radioactive source being created (Bq $s^{-1}$) is given by
  \[
  \frac{dN_{\text{act}}}{dt} = \lambda \varphi N_t \sigma
  \]
  here $\lambda$ is the total radioactive decay constant of the new species.

Radioactivation by nuclear interactions

- If we may assume that $\varphi$ is constant and that $N_t$ is not appreciably depleted as a result of the activation process, then the rates of production given by these equations are also constant.
- As the population of active atoms increases, they decay at the rate $\lambda N_{\text{act}}$ ($s^{-1}$).
- Thus the net accumulation rate can be expressed as
  \[
  \frac{dN_{\text{act}}}{dt} = \varphi N_t \sigma - \lambda N_{\text{act}}
  \]

Radioactivation by nuclear interactions

- After an irradiation time $t \gg \tau = 1/\lambda$, the rate of decay equals the rate of production, reaching the equilibrium activity level $\lambda N_{\text{act}} = \varphi N_t \sigma$.
- Assuming $\lambda N_{\text{act}} = 0$ at $t = 0$, the activity in Bq at any time $t$ after the start of irradiation, can be expressed as
  \[
  \lambda N_{\text{act}} = (\lambda N_{\text{act}})(1 - e^{-\lambda t}) = \varphi N_t \sigma (1 - e^{-\lambda t})
  \]
- If no decay occurs during the irradiation period $t$ (which will be approximately correct if $t << \tau$)
  \[
  \lambda N_{\text{act}} \approx \lambda \varphi N_t \sigma t
  \]

Exposure-rate constant

- The exposure-rate constant $\Gamma_\delta$ of a radioactive nuclide emitting photons is the quotient of $P(dX/dt)_\delta$ by $A$, where $(dX/dt)_\delta$ is the exposure rate due to photons of energy greater than $\delta$, at a distance $l$ from a point source of this nuclide having an activity $A$:
  \[
  \Gamma_\delta = \frac{l^2}{A} \frac{dX}{dt}_\delta
  \]
- Units are R m$^2$ Ci$^{-1}$ h$^{-1}$ or R cm$^2$ mCi$^{-1}$ h$^{-1}$
- It includes contributions of characteristic x-rays and internal bremsstrahlung.

Exposure-rate constant

- The exposure-rate constant $\Gamma_\delta$ was defined by the ICRU to replace the earlier specific gamma-ray constant $\Gamma$, which only accounts for the exposure rate due to $\gamma$-rays.
- $\Gamma_\delta$ is greater than $\Gamma$ by 2% or less, with except for Ra-226 (12%) and I-125 (in which case $\Gamma$ is only about 3% of $\Gamma_\delta$ because K-fluorescence x-rays following electron capture constitute most of the photons emitted).
**Exposure-rate constant**

- We would like to calculate specific γ-ray constant $\Gamma$ at a given point source; the exposure-rate constant $\Gamma_X$ may be calculated in the same way by taking account of the additional x-ray photons (if any) emitted per disintegration.
- At a location $l$ meters (in vacuo) from a γ-ray point source having an activity $A$ Ci, the flux density of photons of the single energy $E_i$ is given by

$$\phi_i = 3.7 \times 10^8 A \ell \frac{k_i}{4\pi l^2} = 2.944 \times 10^8 \frac{A k_i}{l^2}$$

where $k_i$ is the number of photons of energy $E_i$ emitted per disintegration.

- A single disintegration from a 60-Co source would be accompanied by the emission of two photons, 1.17 and 1.33 MeV. The specific γ-ray constant for this source is defined as the exposure rate given in units of C/kg and is expressed in MeV s C/kg.

**Exposure-rate constant**

- Flux density can be converted to energy flux density, expressed in units of J/s m² (expressing $E_i$ in MeV):

$$\psi_i = 4.717 \times 10^{-4} \frac{A k_i E_i}{l^2} \quad (\text{J/s m}^2)$$

- And related to the exposure rate by recalling

$$\psi = \frac{d\psi}{dt} = d \left( \frac{dR}{dt} \right)$$

$$X = \psi \left( \frac{\mu_{en}}{\rho} \right) \left( \frac{E}{W} \right) = (K_i) \left( \frac{E}{W} \right) = (K_i) / 33.97$$

**Exposure-rate constant**

- Substituting the expression for the energy flux density, we obtain

$$\frac{dX}{dt} = -1.389 \times 10^4 \frac{A}{l^2} \sum_{i=1}^{E} k_i E_i \left( \frac{\mu_{en}}{\rho} \right)_{E_i} = \frac{C}{kg s}$$

$$\text{This can be converted into R/h, remembering that } 1 \text{ R} = 2.58 \times 10^4 \text{ C/kg and } 3600 \text{ s} = 1 \text{ h:}$$

$$\frac{dX}{dt} = 193.8 \frac{A}{l^2} \sum_{i=1}^{E} \frac{k_i E_i \left( \frac{\mu_{en}}{\rho} \right)_{E_i}}{R/h}$$

**Exposure-rate constant**

- Applying this to, for example, 60-Co, we note first that each disintegration is accompanied by the emission of two photons, 1.17 and 1.33 MeV.
- Thus the value of $k_i$ is unity at both energies.
- Using the mass energy absorption coefficient values for air at these energies, we find

$$\Gamma = 193.8 (1.17 \times 0.00270 + 1.33 \times 0.00262) = 1.29 \text{ R m}^2/\text{Ci h}$$

which is close to the value given in the table.
Exposure-rate constant

• The exposure rate (R/hr) at a distance \( l \) meters from a point source of \( A \) curies is given by

\[
\frac{dX}{dt} = \frac{\Gamma A}{l^2}
\]

where \( \Gamma \) is given for the source in R m\(^2\)/Ci h, and attenuation and scattering by the surrounding medium are assumed to be negligible

Summary

• Decay constants
• Mean life and half life
• Parent-daughter relationships; removal of daughter products
• Radioactivation by nuclear interactions
• Exposure-rate constant