Gamma- and X-Ray Interactions in Matter

Chapter 7

F.A. Attix, Introduction to Radiological Physics and Radiation Dosimetry

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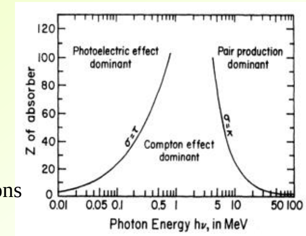
Photon interactions in matter

- Compton effect
- Photoelectric effect
- Pair production
- Rayleigh (coherent) scattering
- Photonuclear interactions



Kinematics
Interaction cross sections
Energy-transfer cross sections
Mass attenuation coefficients

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Compton interaction

- Inelastic photon scattering by an electron
- Main assumption: the electron struck by the incoming photon is *unbound* and *stationary*
  - The largest contribution from binding is under condition of high Z, low energy
  - Under these conditions photoelectric effect is dominant
- Consider two aspects: kinematics and cross sections

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A.H. Compton

- Arthur Holly Compton (September 10, 1892 – March 15, 1962)
- Received Nobel prize in physics 1927 for his discovery of the Compton effect
- Was a key figure in the Manhattan Project, and creation of first nuclear reactor, which went critical in December 1942

Born and buried in Wooster, OH

http://en.wikipedia.org/wiki/Arthur_Compton
http://www.findagrave.com/cgi-bin/fg.cgi?page=gr&GRid=22551

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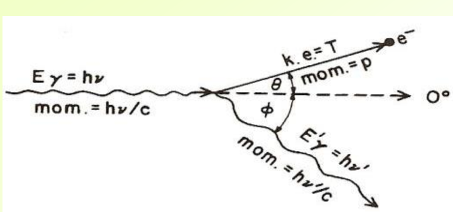
Compton interaction: Kinematics



- Inelastic collision
- After the collision the electron departs at angle θ, with kinetic energy *T* and momentum *p*
- The photon scatters at angle φ with a new, lower quantum energy *h*ν′ and momentum *h*ν′/*c*

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Compton interaction: Kinematics

- An earlier theory of γ-ray scattering by Thomson, based on observations only at low energies, predicted that the scattered photon should always have the same energy as the incident one, regardless of *h*ν or φ
- The failure of the Thomson theory to describe high-energy photon scattering necessitated the development of Compton's theory

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Compton interaction: Kinematics

- The collision kinetics is based upon conservation of both energy and momentum
- Energy conservation requires

$$T = h\upsilon - h\upsilon'$$

- Conservation of momentum along the (0°) direction

$$h\upsilon = h\upsilon' \cos\varphi + pc\cos\theta$$

- Conservation of momentum perpendicular to the direction of incidence:

$$h\upsilon' \sin\varphi = pc\sin\theta$$

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Compton interaction: Kinematics

- pc can be written in terms of T : $pc = \sqrt{T(T + 2m_0c^2)}$ where m_0 is the electron's rest mass
- We get a set of three simultaneous equations in these five parameters: $h\nu$, $h\nu'$, T, θ, and φ :

$$h\upsilon' = \frac{h\upsilon}{1 + \left(h\upsilon / m_0c^2\right)\left(1 - \cos\varphi\right)}$$

$$T = h\upsilon - h\upsilon'$$

$$\cot\theta = \left(1 + \frac{h\upsilon}{m_0c^2}\right)\tan\left(\frac{\varphi}{2}\right)$$

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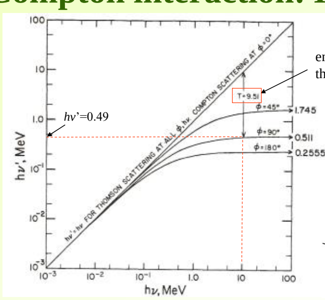
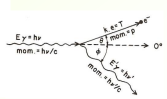
Compton interaction: Kinematics





When photon energy is lower than electron rest mass energy m_0c^2, relativistic effects do not contribute => pure elastic scattering

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Compton interaction: Kinematics

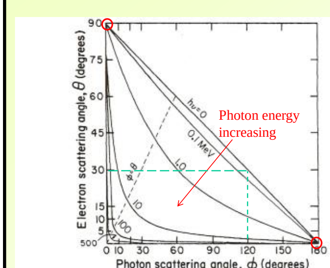
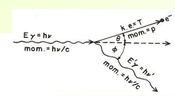


FIGURE 7.4. Relationship of the electron scattering angle θ to the photon scattering angle φ in the Compton effect, from Eq. (7.10). Curves are shown for the incident photon energies 0, 0.1, 1.0, 10, 100, and 500 MeV. The dashed line is the locus where θ = φ, when the electron and photon are scattered at equal angles on opposite sides of the incident photon's direction.



Dependence of θ on φ is a strong function of photon energy:

- Low photon energies $\theta \cong 90° - \varphi/2$
- High photon energies: electrons are mostly forward scattered

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Compton interaction: Cross sections
Interaction cross section

Cross section describes the probability of interaction

- Thomson: *elastic* scattering on a free electron, no energy is transferred to electron
- Differential cross section (per electron for a photon scattered at angle φ, per unit solid angle)

$$\frac{d_e\sigma_T}{d\Omega_\varphi} = \frac{r_0^2}{2}\left(1 + \cos^2\varphi\right)$$

max at $\varphi = 0, 180^o$
½ max at $\varphi = 90^o$

$r_0 = \frac{e^2}{m_0c^2}$ - classical radius of electron

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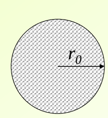
Compton interaction: Cross sections
Interaction cross section

Thomson: elastic scattering on free electron
- total cross section (integrated over all directions)

$$_e\sigma_T = \frac{8\pi r_0^2}{3} = 6.65 \cdot 10^{-25}\ \text{cm}^2/\text{electron}$$



$r_0 = \frac{e^2}{m_0c^2}$ - classical radius of electron

Works well for low photon energies, << m_0c^2
Overestimates for photon energies > 0.01MeV (factor of 2 for 0.4MeV)

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Compton interaction: Cross sections
Interaction cross section

- This cross section (can be thought of as an effective target area) is equal to the probability of a Thomson-scattering event occurring when a single photon passes through a layer containing one electron per cm^2
- It is also the fraction of a large number of incident photons that scatter in passing through the same layer, e.g., approximately 665 events for 10^{27} photons
- For such a small fraction of photons interacting in a layer of matter (by *all processes combined* remains less than about 0.05), the fraction may be assumed to be proportional to absorber thickness; for greater thicknesses the exponential relation must be used

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Compton interaction: Cross sections
Interaction cross section

- Klein-Nishina: Compton scattering on free electron but includes Dirac's quantum relativistic theory
- Differential cross section:

$$\frac{d_e\sigma_{K-N}}{d\Omega_\varphi} = \frac{r_0^2}{2}\left(\frac{h\nu'}{h\nu}\right)\left(\frac{h\nu}{h\nu'} + \frac{h\nu'}{h\nu} - \sin^2\varphi\right)$$

- For elastic scattering ($h\nu = h\nu'$) – reduces to Thomson's expression
- Needed at high photon energy

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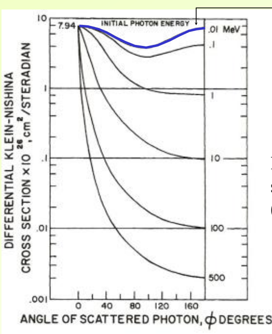
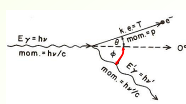
Compton interaction: Cross sections
Interaction cross section



Thomson's: $\frac{d_e\sigma_T}{d\Omega_\varphi} = \frac{r_0^2}{2}\left(1+\cos^2\varphi\right)$

$\frac{d_e\sigma_{K-N}}{d\Omega_\varphi}$ shows probability for a photon to be scattered at an angle φ

Photons with high energies tend to scatter in the forward direction ($\varphi \sim 0$)



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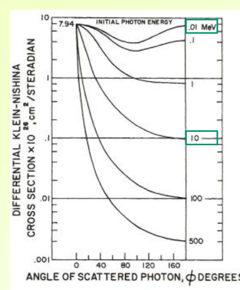
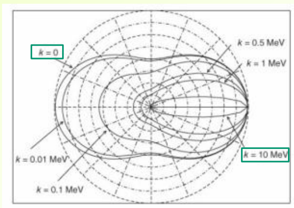
Compton interaction: Cross sections
Interaction cross section





K–N Compton cross section differential in solid angle: (a) data plotted in Cartesian coordinates; (b) the same data plotted in polar coordinates. When the photon energy increases, the scattered photon distribution becomes strongly forward peaked

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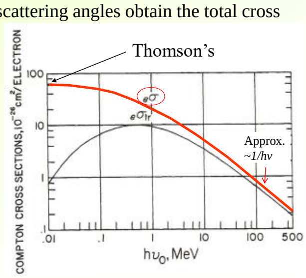
Compton interaction: Cross sections
Interaction cross section

Klein-Nishina: Compton scattering on free electron (includes Dirac's quantum relativistic theory)

Integrating over all photon scattering angles obtain the total cross section per electron

$${}_e\sigma_{K-N} = 2\pi r_0^2\{\ldots\}$$

$\{\ldots\}$ - depends on incident photon energy: higher energy => lower interaction probability; expressed through parameter $\alpha = h\nu/m_0c^2$



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Compton interaction: Cross sections
Energy-transfer cross section

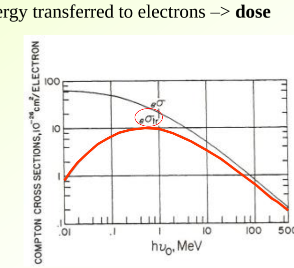
Total cross section –> fraction of energy diverted into Compton interactions –> fraction of energy transferred to electrons –> **dose**

Energy transfer cross section

$${}_e\sigma_{tr} = {}_e\sigma \cdot \frac{T}{h\nu}$$

Average kinetic energy of recoiling electrons:

$$\overline{T} = h\nu \cdot \frac{{}_e\sigma_{tr}}{{}_e\sigma}$$



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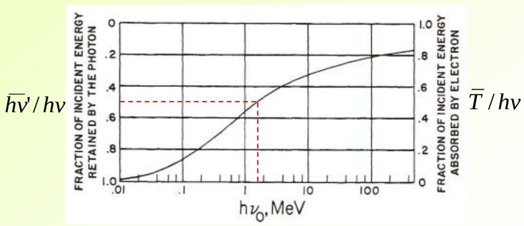
Compton interaction: Cross sections
Energy-transfer cross section



For $h\nu$ = 1.6 MeV – half of the photon energy is transferred to the electron ($\overline{T}$ = 0.8 MeV)

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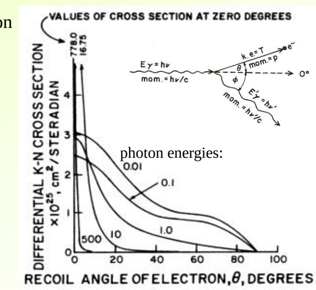
Compton interaction: Cross sections
Other cross sections

Differential K-N cross section for electron scattering at angle θ, per unit solid angle, per electron

$$\frac{d_e\sigma_{K-N}}{d\Omega_\theta}$$

For high photon energies electrons are preferentially forward scattered (θ = 0)



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Compton interaction: Cross sections
Other cross sections

Probability that a single photon will have Compton interaction in a layer of 1 e/cm² and transfers energy between T and $T+dT$

$$\frac{d_e\sigma_{K-N}}{dT} \text{ in } cm^2 MeV^{-1} e^{-1}$$

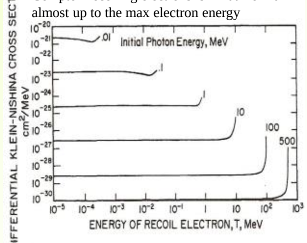
Energy distribution of electrons, averaged over all θ

For head-on collisions:

T_{max} ~ $h\nu$ - 0.2555 MeV

Stronger effect for high $h\nu$

The distribution of kinetic energies given to the Compton recoiling electrons is ~ flat from 0 almost up to the max electron energy



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Compton interaction: Mass attenuation coefficient

Cross section per electron (no Z dependence due to free electron assumption) $\quad {}_e\sigma \propto Z^0$

Cross section per atom $\quad {}_a\sigma \propto Z \cdot {}_e\sigma$

Cross section per unit mass (mass attenuation coefficient) $\quad \frac{\sigma}{\rho} = \frac{N_A Z}{A} \cdot {}_e\sigma$

N_A – Avogadro's constant; Z – number of electrons per atom; A – number of grams per mole of material; ρ – density in g/cm³

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Photoelectric effect: Kinematics

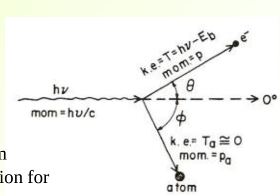
Most important at low photon energies

- Interaction with atomic-shell electrons tightly bound with potential energy $E_b < h\nu$
- Photon is completely absorbed
- Kinetic energy to electron:

$$T = h\nu - E_b$$

independent of scattering angle

- Atom acquires some momentum
- No universal analytical expression for cross sections



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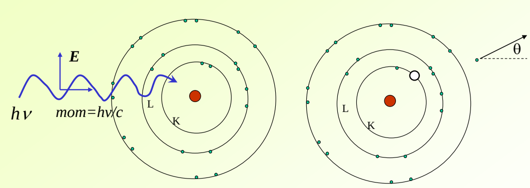
Photoelectric effect basics



- Photon transfers its momentum $h\nu/c$ plus some transversal momentum due to the perpendicular electric field in the electromagnetic wave
- Final state = free electron + hole in the atomic shell

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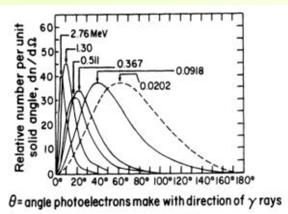
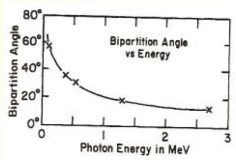
Photoelectric effect: Directional distribution

For higher photon energies electrons tend to scatter in forward direction (θ =0 is forbidden since it is perpendicular to the vector E)

Directional distribution



Half of all electrons is ejected within a forward cone of half angle equal to bipartition angle



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Photoelectric effect: Cross sections
Interaction cross section

Total interaction cross section per atom, in cm²/atom

$$_{a}\tau \cong k\frac{Z^{n}}{(h\nu)^{m}}$$

$k = Const$

m, n – energy dependent

$n \cong 4,\ m \cong 3$ at $h\nu = 0.1\,\text{MeV}$ $\qquad \tau \propto \frac{Z^4}{(h\nu)^3}$

Mass attenuation coefficient

$$\frac{\tau}{\rho} \propto \left(\frac{Z}{h\nu}\right)^3$$

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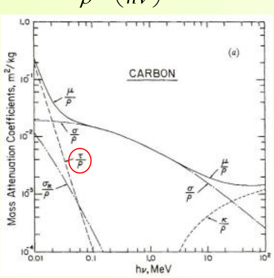
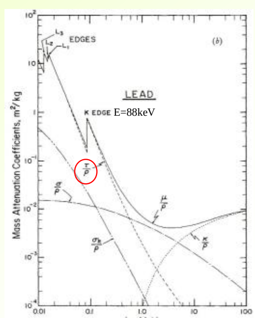
Photoelectric effect: Mass attenuation coefficient

$$\frac{\tau}{\rho} \cong \left(\frac{Z}{h\nu}\right)^3$$





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Photoelectric effect: Cross sections
Energy-transfer cross section

Fraction of energy transferred to all electrons

$$\frac{T}{h\nu} = \frac{h\nu - E_b}{h\nu}$$

Vacancy created by a photon in the inner shell has to be filled
If through Auger process - additional contribution to kerma

Final result:

$$\frac{\tau_{tr}}{\rho} = \frac{\tau}{\rho}\left[\frac{h\nu - P_K Y_K \cdot h\bar{\nu}_K - (1-P_K)P_L Y_L \cdot h\bar{\nu}_L}{h\nu}\right]$$

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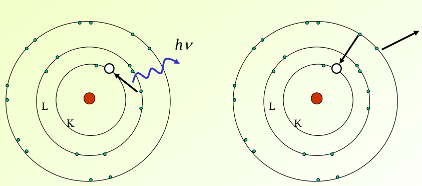
Photoelectric effect: Atom relaxation



Excited atom relaxes its energy by
- fluorescence (emission of photons) or
- Auger process (emission of electron)

when the higher energy shell electrons move downward

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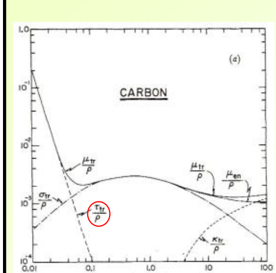
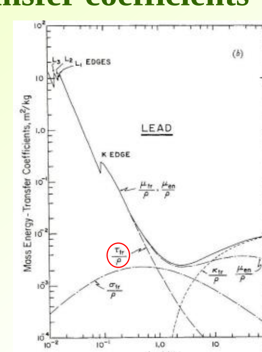
Mass energy-transfer coefficients





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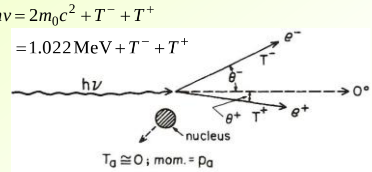
Pair production

- Photon is absorbed giving rise to electron and positron
- Occurs predominantly in Coulomb force field
  - usually near atomic nucleus
  - sometimes in a field of atomic electron
- Minimum photon energy required $2m_0c^2 = 1.022$ MeV

$$h\nu = 2m_0c^2 + T^- + T^+$$

$$= 1.022\,\text{MeV} + T^- + T^+$$



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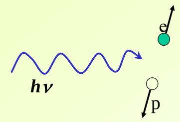
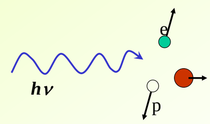
Pair production: Third body is needed



- There exists a reference frame where the total momentum of electron and positron is zero
- But photon momentum is always hν/c
- Third body needed for momentum conservation: electron or nucleus



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Pair production in Nuclear Coulomb Force Field

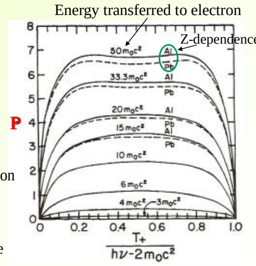
Total cross section per atom

$${}_a\kappa = \sigma_0 Z^2 \overline{P}$$

P – function of $h\nu$ and Z (at higher E)

$$\sigma_0 = \frac{r_0^2}{137} = \frac{1}{137}\left(\frac{e^2}{m_0c^2}\right)^2 = 5.80\text{x}10^{-28}\ \text{cm}^2/\text{electron}$$

Energy transferred to electron



Fraction of energy transferred to positron

Nuclear attraction and repulsion tend to give the positron slightly more energy than the electron, the difference being less than 0.0075Z MeV

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Pair production in Electron Coulomb Force Field

Triplet production – higher threshold $4m_0c^2 = 2.044$ MeV required for conservation of momentum

Ratio of cross section for all electrons of the atom to nuclear cross section of the same atom is small:

$$\frac{\kappa(electron)}{\kappa(nucleus)} \cong \frac{1}{CZ}$$

C – parameter depending on energy, close to 1
Example: for Pb (Z=82) triplet cross section is 1% of that of PP; rising to 5-10% for Z=10

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Pair production: Cross sections

Total cross section for pair production per unit mass:

$$\left(\frac{\kappa}{\rho}\right)_{pair} = \left(\frac{\kappa}{\rho}\right)_{nuclear} + \left(\frac{\kappa}{\rho}\right)_{electron}$$

Pair production energy transfer coefficient:

$$\frac{\kappa_{tr}}{\rho} = \frac{\kappa}{\rho}\left(\frac{h\nu - 2m_0c^2}{h\nu}\right)$$

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Rayleigh (coherent) scattering

- Photon is scattered by combined action of whole atom
- Photons do not lose energy, redirected through only a small angle
- No charged particles receive energy, no excitation produced
  - => No contribution to kerma or dose

Atomic cross section: $\frac{\sigma_R}{\rho} \propto \frac{Z}{(h\nu)^2}$

Typical ratios of Rayleigh to total attenuation coefficient σ_R / μ

Element	$h\nu$ = 0.01 MeV	0.1 MeV	1.0 MeV
C	0.07	0.02	0
Cu	0.006	0.08	0.007
Pb	0.03	0.03	0.03

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Photonuclear Interactions

- Photonuclear interactions (photodisintegration) are direct interactions of energetic photons and absorber atom nuclei
- The threshold energy is ~8 MeV for all nuclides with two exceptions: deuteron at 2.22 MeV and beryllium-9 at 1.67 MeV
- Cross sections for photonuclear reactions exhibit a broad "giant resonance" peak at ~23 MeV for low Z and at ~12 MeV for high Z absorbers. The peak's FWHM typically ranges from ~3 MeV to ~9 MeV
  - Contribution to the total atomic cross section is still only a few percent

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Photonuclear Interactions

- After being excited by a photon, nucleus emits a proton (γ,p) or, more likely, a neutron (γ,n)
- (γ,p) events contribute to kerma and dose
  - Relative amount less that 5% of pair production
  - Usually not included in dosimetry consideration
- (γ,n) events are important for shielding design, especially for higher energies
  - Typical RT threshold >10MeV, resulting in slight neutron contamination of the photon beam
  - Increases by order of magnitude from 10 to 25MeV

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Photonuclear Interactions

- High Z absorbers found in the linac head serve as the main site of photonuclear interactions for typical treatment energies; a smaller portion of interactions occurs in tissue/phantom
- Example: The dose from photon-induced nuclear particles (neutrons, protons, and alpha particles) generated by high-energy photon beams from medical linacs (Siemens 18 MV, Varian 15 MV, and Varian 18 MV)

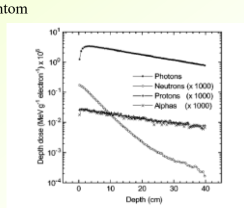

FIG. 3. Absolute dose components (photons, neutrons, protons, and alphas) for the Siemens 18 MV photon beam (10 cm × 10 cm) irradiating a tissue-equivalent phantom (TEP).

O. Chibani, C.-M. C Ma, Med. Phys. 30 1990 (2003)

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Photonuclear Interactions: further reading

- NCRP Report No. 79, ''Neutron contamination from medical electron accelerators,'' National Council on radiation Protection and Measurements, Bethesda, MD, 1984
- NCRP Report No. 144, Radiation Protection for Particle Accelerator Facilities, National Council on radiation Protection and Measurements, Bethesda, MD, 2003
- O. Chibani, C.-M. C Ma, Photonuclear dose calculations for high-energy photon beams from Siemens and Varian linacs, Med. Phys. 30 1990 (2003)

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Total coefficients for attenuation, energy transfer and absorption

Total mass attenuation coefficient for photon interactions - add probabilities for photoelectric effect, Compton effect, pair production and Rayleigh scattering

$$\frac{\mu}{\rho}=\frac{\tau}{\rho}+\frac{\sigma}{\rho}+\frac{\kappa}{\rho}+\frac{\sigma_R}{\rho}$$

Total mass energy-transfer coefficient:

$$\frac{\mu_{tr}}{\rho}=\frac{\tau_{tr}}{\rho}+\frac{\sigma_{tr}}{\rho}+\frac{\kappa_{tr}}{\rho}$$
$$=\frac{\tau}{\rho}\left[\frac{h\nu-p_K Y_K h\bar{\nu}_K}{h\nu}\right]+\frac{\sigma}{\rho}\left[\frac{T}{h\nu}\right]+\frac{\kappa}{\rho}\left[\frac{h\nu-2m_0c^2}{h\nu}\right]$$

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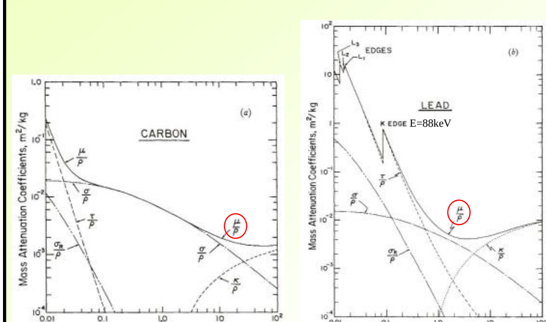
Mass attenuation coefficients



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Mass energy-absorption coefficient

$$\frac{\mu_{en}}{\rho} = \frac{\mu_{tr}}{\rho}(1-g)$$

g - average fraction of secondary electron energy lost in radiative interactions
For low Z and high $h\nu$, g --> 0

Appendix D

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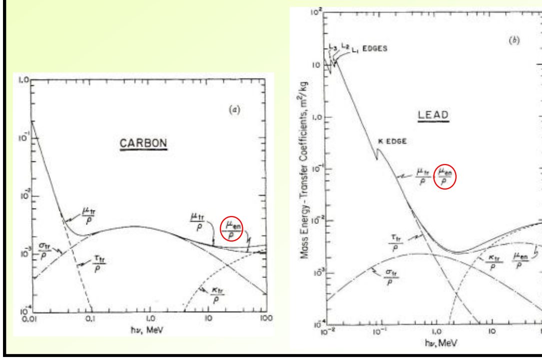
Mass energy-transfer coefficients



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Summary

- Compton effect
- Photoelectric effect
- Pair production

} the most important

- Rayleigh (coherent) scattering – no energy transferred to the medium
- Photonuclear interactions – relevant at high energies

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