Charged-Particle Interactions in Matter

Chapter 8

F. A. Attix, Introduction to Radiological Physics and Radiation Dosimetry

Introduction

• Charged particles have surrounding Coulomb field
• Always interact with electrons or nuclei of atoms in matter
• In each interaction typically only a small amount of particle’s kinetic energy is lost ("continuous slowing-down approximation" – CSDA)
• Typically undergo very large number of interactions, therefore can be roughly characterized by a common path length in a specific medium (range)

Charged-particle interactions in matter

Impact parameter $b$

• "Soft" collisions ($b \gg a$)
• Hard ("Knock-on") collisions ($b \sim a$)
• Coulomb interactions with nuclear field ($b \ll a$)
• Nuclear interactions by heavy charged particles

Types of charged-particle interactions in matter

• "Soft" collisions ($b \gg a$)
  – The influence of the particle’s Coulomb force field affects the atom as a whole
  – Atom can be excited to a higher energy level, or ionized by ejection of a valence electron
  – Atom receives a small amount of energy (~eV)
  – The most probable type of interactions; accounts for about half of energy transferred to the medium

Types of charged-particle interactions in matter

• Hard ("Knock-on") collisions ($b \sim a$)
  – Interaction with a single atomic electron (treated as free), which gets ejected with a considerable kinetic energy
  – Interaction probability is different for different particles
  – Ejected $\delta$-ray dissipates energy along its track
  – Characteristic x-ray or Auger electron is also produced

Types of charged-particle interactions in matter

• Coulomb interactions with nuclear field ($b \ll a$)
  – Most important for electrons
    – In all but 2-3% of cases electron is deflected through almost elastic scattering, losing almost no energy
    – In 2-3% of cases electron loses almost all of its energy through inelastic radiative (bremsstrahlung) interaction
  – Important for high Z materials, high energies (MeV)
• For antimatter only: in-flight annihilations
  – Two photons are produced
### Types of charged-particle interactions in matter

- **Nuclear interactions by heavy charged particles**
  - A heavy charged particle with kinetic energy ~ 100 MeV and \( b \simeq a \) may interact inelastically with the nucleus.
  - One or more individual nucleons may be driven out of the nucleus in an *intranuclear cascade* process.
  - The highly excited nucleus decays by emission of so-called *evaporation* particles (mostly nucleons of relatively low energy) and \( \gamma \)-rays.
  - Dose may not be deposited locally, the effect is \(<1\%\).

### Stopping Power

\[
\left( \frac{dT}{dx} \right)_{Y,T,Z}
\]

- The expectation value of the rate of energy loss per unit of path length \( x \)
  - Charged particle of type \( Y \)
  - Having kinetic energy \( T \)
  - Traveling in a medium of atomic number \( Z \)
- Units: MeV/cm or J/m

### Mass Stopping Power

\[
\left( \frac{dT}{\rho dx} \right)_{Y,T,Z}
\]

- \( \rho \) - density of the absorbing medium
- Units: \( \frac{MeV \times cm^2}{g} \) or \( \frac{J \times m^2}{kg} \)
- May be subdivided into two terms:
  - collision - contributes to local energy deposition
  - radiative - energy is carried away by photons

### Mass Collision Stopping Power

\[
\left( \frac{dT}{\rho dx} \right)_{c} = \int_{T_{\text{min}}}^{H} TQ_{c}dT + \int_{H}^{T_{\text{max}}} TQ_{h}dT
\]

1. \( T \) is the energy transferred to the atom or electron
2. \( H \) is the somewhat arbitrary energy boundary between soft and hard collisions, in terms of \( T \)
3. \( T_{\text{max}} \) is the maximum energy that can be transferred in a head-on collision with an atomic electron (unbound)
   - For a heavy particle with kinetic energy \( T \) < than its \( M_0c^2 \)
     \[ T_{\text{max}} = 2m_0c^2 \left( \frac{\beta}{1-\beta^2} \right) \approx 1.022 \left( \frac{\beta}{\sqrt{1-\beta^2}} \right) MeV, \beta = v/c \]
   - For positrons incident, \( T_{\text{max}} \approx T \) if annihilation does not occur
   - For electrons \( T_{\text{max}} = T/2 \)

### Mass Collision Stopping Power

\[
\left( \frac{dT}{\rho dx} \right)_{c} = \int_{T_{\text{min}}}^{H} TQ_{c}dT + \int_{H}^{T_{\text{max}}} TQ_{h}dT
\]

4. \( T_{\text{max}} \) is related to \( T_{\text{max}} \) by

\[
\frac{T_{\text{max}}}{T_{\text{max}}} = \left( \frac{2m_0c^2 \beta^2}{I} \right)^2 = \left( \frac{1.022 \times 10^6 \text{ eV} \beta^2}{I} \right)^2
\]

where \( I \) is the *mean excitation potential* of the atom

5. \( Q_{c} \) and \( Q_{h} \) are the respective differential mass collision coefficients for soft and hard collisions, typically in units of \( \text{cm}^2/g \text{ MeV} \) or \( \text{m}^2/\text{kg J} \)
For either electrons or heavy particles (z - elem. charges)
• No dependence on particle mass
• Combines both soft and hard collision contributions

- Depends on Z
The so-called collision term depends on

-Appendices B.1 and B.2 list some I-values

The form of the hard collision term may be written as

\[
\left( \frac{dT}{d\phi} \right)_H = \frac{2Cm_c\beta^2}{\beta^2} \ln \left( \frac{2m_c\beta^2 H}{F^2(1-\beta^2)} \right) - \beta^2
\]

here \( C = \pi(N_e Z/A)\rho_c^2 = 0.150Z/A \) cm²/γ; in which \( N_e Z/A \) is the number of electrons per gram of the stopping medium, and \( \rho_c = e^2m_c\rho_c = 2.818 \times 10^{-13} \) cm is the classical electron radius

- For either electrons or heavy particles (z - elem. charges)
- Based on Born approximation: particle velocity is much greater than that of the atomic electrons (\( v = \beta c > u \))
- Verified with cyclotron-accelerated protons

The mean excitation potential \( I \) is the geometric-mean value of all the ionization and excitation potentials of an atom of the absorbing medium

- In general \( I \) for elements cannot be calculated
- Must instead be derived from stopping-power or range measurements
  - Experiments with cyclotron-accelerated protons, due to their availability with high \( \beta \)-values and the relatively small effect of scattering as they pass through layers of material
- Appendices B.1 and B.2 list some I-values

**Mass Collision Stopping Power for Heavy Particles**

\[
\left( \frac{dT}{d\phi} \right)_H = \frac{3071}{A} \left[ \frac{Z^2}{I} \right] \ln \left( \frac{I}{I_c} \right) - \beta^2 - \ln I
\]

- Combines both soft and hard collision contributions
- Depends on \( Z \) - stopping medium, \( z \) - particle charge, particle velocity through \( \beta = v/c \) (not valid for very low \( \beta \))
- The term –ln \( I \) provides even stronger variation with \( Z \) (the combined effect results in \( (dT)/dx \), for Pb less than that for C by \( \approx40-60 \% \) within the \( \beta \)-range 0.85-0.1)
- No dependence on particle mass

**Soft-Collision Term**

- The mean excitation potential \( I \) is the geometric-mean value of all the ionization and excitation potentials of an atom of the absorbing medium
- In general \( I \) for elements cannot be calculated
- Must instead be derived from stopping-power or range measurements
  - Experiments with cyclotron-accelerated protons, due to their availability with high \( \beta \)-values and the relatively small effect of scattering as they pass through layers of material
- Appendices B.1 and B.2 list some I-values

**Hard-Collision Term**

- The form of the hard-collision term depends on whether the charged particle is an electron, positron, or heavy particle
- For heavy particles, having masses much greater than that of an electron, and assuming that \( H \ll T_{max} \), the hard-collision term may be written as

\[
\left( \frac{dT}{d\phi} \right)_H = k \left[ \ln \left( \frac{T_{max}}{H} \right) - \beta^2 \right]
\]

\( k \) is the geometric-mean value of all the ionization and excitation potentials of an atom of the absorbing medium

**Shell Correction**

- When the velocity of the passing particle ceases to be much greater than that of the atomic electrons in the stopping medium, the mass-collision stopping power is over-estimated
- Since K-shell electrons have the highest velocities, they are the first to be affected by insufficient particle velocity, the slower L-shell electrons are next, and so on
- The so-called “shell correction” is intended to account for the resulting error in the stopping-power equation
- The correction term \( C/Z \) is the same for all charged particles of the same \( \beta \), and is a function of the medium
Mass Collision Stopping Power for Electrons and Positrons

\[
\left( \frac{dT}{dx} \right) = k \left[ \ln \left( \frac{\tau^2 (\tau + 2)}{2 (\tau / \mu c^2)^2} \right) + F(\tau) - \frac{2C}{Z} \right]
\]

\( \tau = T/m_c^2 \)

• Combines both soft and hard collision contributions
• \( F(\tau) \) term – depends on \( \beta \) and \( \tau \)
• Includes two corrections:
  - shell correction \( 2C/Z \)
  - correction for polarization effect \( \delta \)

Polarization Effect

• Atoms near the particle track get polarized, decreasing the Coulomb force field and corresponding interaction
• Introduce density-effect correction influencing soft collisions
• The correction term, \( \delta \), is a function of the composition and density of the stopping medium, and of the parameter
  \[ Z = \log_{10}(p / m_c c) = \log_{10}(\beta / \sqrt{1 - \beta^2}) \]
  for the particle, in which \( p \) is its relativistic momentum \( mv \), and \( m_0 \) is its rest mass
• Mass collision stopping power decreases in condensed media
• Relevant in measurements with ion chambers at energies > 2 MeV

Mass Radiative Stopping Power

• Only electrons and positrons are light enough to generate significant bremsstrahlung (1/m² dependence for particles of equal velocities)
• The rate of bremsstrahlung production by electrons or positrons is expressed by the mass radiative stopping power (in units of MeV cm²/g)
  \[
  \left( \frac{dT}{dx} \right) = \sigma_0 \frac{N_Z^2}{A} (T + m_c^2) B
  \]
  here the constant \( \sigma_0 = 1/137(e^2/m_c c^2)^2 = 5.80 \times 10^{-28} \text{ cm}^2/\text{atom} \), \( T \) is the particle kinetic energy in MeV, and \( B \) is a slowly varying function of \( Z \) and \( T \)

Appendix E contains tables of electron stopping powers, ranges, radiation yields, and density-effect corrections \( \delta \)

The steep rise in collision stopping power for \( \beta < m_0 c^2 \) is not shown, but the minimum at \( \beta \approx 3 m_0 c^2 \) is evident, as is the continuing rise at still higher energy.
Mass Stopping Powers vs. Energy and Z

- Relatively independent of Z

Restricted Stopping Power

- Energy cutoff allows to account for escaping delta-rays
- Linear Energy Transfer

\[ L_a (\text{keV/\mu m}) = \frac{\rho}{10} \left( \frac{dE}{dX} \right) (\text{MeV cm}^2/\text{g}) \]  

Linear energy transfer is of greatest relevance in radiobiology and microdosimetry. If the cutoff energy \( \Delta \) is increased to equal \( T_{\text{max}} \) for electrons, \( T \) for positrons, and Eq. (8.4) for heavy particles, then

\[ \left( \frac{dE}{dX} \right) = \frac{\Delta}{dX} \]  

and

\[ L_a = L_{\infty} \]  

Range

- The range \( R \) of a charged particle of a given type and energy in a given medium is the expectation value of the pathlength \( p \) that it follows until it comes to rest (discounting thermal motion)
- The projected range \( <r> \) of a charged particle of a given type and initial energy in a given medium is the expectation value of the farthest depth of penetration \( t_f \) of the particle in its initial direction
- Both are non-stochastic quantities

Range

\[ R_{\text{CSDA}} = \frac{T_0}{\rho d} \int_0^t \left( \frac{dE}{dX} \right)^{-1} dT \] 

- \( T_0 \) - starting energy of the particle
- Units: g/cm²
- Appendix E

CSDA Range: Protons

Greater for higher Z due to decrease in stopping power
CSDA Range: Other Heavy Particles

For particles with the same velocity: 
\[ T = M^2 \sqrt{\frac{1}{\gamma_0^2 - 1}} \]

- Kinetic energy of a particle \( \approx \) to its rest mass
- Stopping power for singly charged particle is independent of mass
- Consequently, the range \( \approx \) to its rest mass
- Can calculate the range for a heavy particle based on CSDA range values for protons at energy \( T_0^p = T_0^p M_0^p / M_0 \)

\[ R_{\text{CSDA}} = \frac{R_{\text{CSDA}}}{M_0^p Z^2} \]

Projected Range

\[ N_0 \rightarrow \cdots \rightarrow N_i \]

- Count the number of particles that penetrate a slab of increasing thickness
- \( N_0 \) number of incident mono-energetic particles in a beam perpendicular to the slab

\[ \phi = \int_{z_0}^{z} \frac{dN(z)}{dz} \, dz = -\frac{1}{N_0} \int_{z_0}^{z} \phi \, dz \quad (8.25) \]

Electron Range

- Electrons typically undergo multiple scatterings
- Range straggling and energy straggling due to stochastic variations in rates of energy loss
- Makes range less useful characteristic, except for low-Z materials, where range is comparable to max penetration depth \( t_{\text{max}} \)
- For high-Z range increases, \( t \) is almost independent

Calculation of Absorbed Dose

Parallel beam of charged particles of kinetic energy \( T_0 \) perpendicularly incident on a foil Z

Assumptions:
- Collision stopping power is constant and depends on \( T_0 \)
- Scattering is negligible
- Effect of delta rays is negligible
Calculation of Absorbed Dose

Energy lost in collision interactions (energy imparted)

\[ E = \Phi \left( \frac{d\bar{T}}{d\bar{x}} \right) \rho \ell \] (8.26)

Absorbed dose

\[ D = \int \left( \frac{d\bar{T}}{d\bar{x}} \right) \rho \ell \, d\bar{x} \] (MeV/\ell)

mass per unit area of foil

\[ = 1.602 \times 10^{-10} \int \left( \frac{d\bar{T}}{d\bar{x}} \right) \rho \ell \, d\bar{x} \] Gy (8.27)

Dose in the foil is independent of its thickness

Dose from Heavy Particles

Based on range can find the residual kinetic energy of exiting particle

\[ \Delta T = T_0 - T_{ex} \]

\[ E = \Phi \Delta T \]

\[ D = 1.602 \times 10^{-10} \Phi \Delta T \cos \theta \rho \ell \]

If beam is not perpendicular – accounts for angle

Dose in Gray

Dose from Electrons

- Need to account for path lengthening due to scatter
- Need to account for bremsstrahlung production, consider radiation yield
- Energy spent in collisions:

\[ \Delta T = (T_0 - T_{ex}) \]

- Average dose:

\[ D = 1.602 \times 10^{-10} \Phi \Delta T \rho \ell \]

Electron Backscattering

The Bragg Curve

Dose vs. Depth for Electron Beams

- No Bragg peak
- Diffused peak at ~half of \( t_{max} \)

FIGURE 8.1.2. Electron foil of incident energy \( T_0 \) served away by backscattered electrons. Primary electrons are predominantly incident, with individual kinetic energy \( T_0 \) on infinitely thick \( \ell = \infty \) layers of the indicated scattering materials. After Wright and Trump (1962).

FIGURE 8.1.3a. Dose vs. depth for 187-MeV protons in water, showing Bragg peak. The dashed curve demonstrates the effect of passing the beam through optimally designed, variable-thickness absorbers such as oscillating wedges. (After Kafatos, 1984. Reproduced with permission from Strahlentherapie.)

Calculation of Absorbed Dose at Depth

\[ D_x = 1.602 \times 10^{-10} \int_0^{T_{max}} \Phi_x(T) \left( \frac{dT}{\rho dx} \right)_{\epsilon_{\infty}} dT \] (8.39)

- At any point \( P \) at depth \( x \) in a medium \( w \) for known fluence spectrum
- For \( x \) below particle range

\[ D_x = 1.602 \times 10^{-10} \Phi_x \left( \frac{dT}{\rho dx} \right)_{\epsilon_{\infty}} \] (8.40)

Summary

- Types of charged particle interactions
- Stopping power
- Range
- Calculation of absorbed dose