Charged-Particle Interactions in Matter

Chapter 8

F.A. Attix, Introduction to Radiological Physics and Radiation Dosimetry

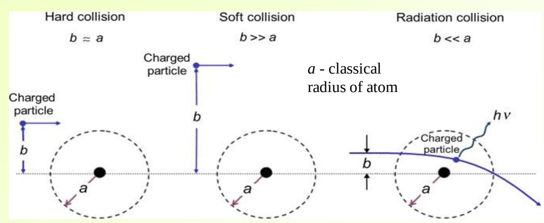
Outline

- Impact parameter and types of charged particle interactions
- Stopping power
- Range
- Calculation of absorbed dose
- Approach typically depends on particle mass (heavy vs. electrons)

Introduction

- Charged particles have surrounding Coulomb field
- Always interact with electrons or nuclei of atoms in matter
- In each interaction typically only a small amount of particle's kinetic energy is lost ("continuous slowing-down approximation" – CSDA)
- Typically undergo very large number of interactions, therefore can be roughly characterized by a common path length in a specific medium (range)

Charged-particle interactions in matter



Impact parameter *b*
- "Soft" collisions ($b>>a$)
- Hard ("Knock-on" collisions (b~a)
- Coulomb interactions with nuclear field ($b<<a$)
- Nuclear interactions by heavy charged particles

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Types of charged-particle interactions in matter

- "Soft" collisions ($b>>a$)
  - The influence of the particle's Coulomb force field affects the atom as a whole
  - Atom can be excited to a higher energy level, or ionized by ejection of a valence electron
  - Atom receives a small amount of energy (~eV)
  - The *most probable* type of interactions; accounts for about half of energy transferred to the medium

Types of charged-particle interactions in matter

- Hard ("Knock-on") collisions (b~a)
  - Interaction with a single atomic electron (treated as free), which gets ejected with a considerable K.E.
  - Ejected δ*-ray* dissipates energy along its track
  - Interaction probability is different for different particles (much lower than for soft collisions, but fraction of energy spent is comparable)
  - Characteristic x-ray or Auger electron is also produced

Types of charged-particle interactions in matter

- Coulomb interactions with nuclear field ($b<<a$)
  - Most important for electrons
    - In *all but* 2-3% of cases electron is *deflected* through almost elastic scattering, losing almost no energy
    - In 2-3% of cases electron loses almost all of its energy through inelastic *radiative* (bremsstrahlung) interaction
  - Important for high Z materials, high energies (MeV)
- For antimatter only: in-flight annihilations
  - Two photons are produced

Types of charged-particle interactions in matter

- Nuclear interactions by **heavy charged** particles
  - A heavy charged particle with kinetic energy ~ 100 MeV and $b<a$ may interact inelastically with the nucleus
  - One or more individual nucleons may be driven out of the nucleus in an *intranuclear cascade* process
  - The highly excited nucleus decays by emission of so-called *evaporation* particles (mostly nucleons of relatively low energy) and γ-rays
  - Dose may not be deposited locally, the effect is <1-2%

Stopping Power

$$\left(\frac{dT}{dx}\right)_{Y,T,Z}$$

- The expectation value of the rate of energy loss per unit of path length x
  - Charged particle of type Y
  - Having kinetic energy T
  - Traveling in a medium of atomic number Z
- Units: MeV/cm or J/m

Mass Stopping Power

$$\left(\frac{dT}{\rho dx}\right)_{Y,T,Z}$$

- ρ - density of the absorbing medium
- Units: $\frac{\mathrm{MeV}\times\mathrm{cm}^2}{\mathrm{g}}$ or $\frac{\mathrm{J}\times\mathrm{m}^2}{\mathrm{kg}}$
- May be subdivided into two terms:
  - collision - contributes to local energy deposition
  - radiative - energy is carried away by photons

Mass Collision Stopping Power

- Only collision stopping power contributes to the energy deposition (dose to medium)
- Can be further subdivided into soft and hard collision contributions

$$\left(\frac{dT}{\rho dx}\right)_c = \left(\frac{dT_s}{\rho dx}\right)_c + \left(\frac{dT_h}{\rho dx}\right)_c$$

- Separately calculated for electrons and heavy particles

Mass Collision Stopping Power

$$\left(\frac{dT}{\rho dx}\right)_c = \int_{T'_{\min}}^{H} T'Q_c^s dT' + \int_{H}^{T'_{\max}} T'Q_c^h dT'$$

1. T' is the energy transferred to the atom or electron
2. H is the somewhat arbitrary energy boundary between soft and hard collisions, in terms of T'
3. $T'_{\max}$ is the maximum energy that can be transferred in a head-on collision with an atomic electron (unbound)
   - For a heavy particle with kinetic energy < than its M_0c^2

     $$T'_{\max} \approx 2m_0c^2\left(\frac{\beta^2}{1-\beta^2}\right) = 1.022\left(\frac{\beta^2}{1-\beta^2}\right)\mathrm{MeV}, \beta = \mathrm{v/c}$$
   - For positrons incident, $T'_{\max} = T$ if annihilation does not occur
   - For electrons $T'_{\max} \equiv T/2$

Mass Collision Stopping Power

$$\left(\frac{dT}{\rho dx}\right)_c = \int_{T'_{\min}}^{H} T' Q_c^s \, dT' + \int_{H}^{T'_{\max}} T' Q_c^h \, dT'$$

4. $T'_{\max}$ is related to $T'_{\min}$ by

$$\frac{T'_{\max}}{T'_{\min}} \approx \left(\frac{2m_0c^2\beta^2}{I}\right)^2 = \left(\frac{(1.022\times10^6 \text{ eV})\beta^2}{I}\right)^2$$

where I is the *mean excitation potential* of the atom

5. Q^s_c and Q^h_c are the respective differential mass collision coefficients for soft and hard collisions, typically in units of $\text{cm}^2/\text{g MeV}$ or $\text{m}^2/\text{kg J}$

Soft-Collision Term

$$\left(\frac{dT_s}{\rho dx}\right)_c = \frac{2Cm_0c^2z^2}{\beta^2}\left[\ln\left(\frac{2m_0c^2\beta^2H}{I^2(1-\beta^2)}\right) - \beta^2\right]$$

here $C \equiv \pi(N_A Z/A)r_0^2 = 0.150Z/A$ cm^2/g; in which $N_A Z/A$ is the number of electrons per gram of the stopping medium, and $r_0 = e^2/m_0c^2 = 2.818 \times 10^{-13}$ cm is the classical electron radius

- For either electrons or heavy particles (z - elem. charges)
- Based on Born approximation: particle velocity is much greater than that of the atomic electrons ($v = \beta c >> u$)
- Verified with cyclotron-accelerated protons

Soft-Collision Term

- Can further simplify the expression by introducing

$$k = \frac{2Cm_0c^2z^2}{\beta^2} = 0.1535\frac{Zz^2}{A\beta^2}\frac{MeV}{g/cm^2}$$

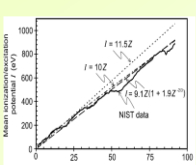
$$\beta = v/c$$

- The factor k controls the dimensions in which the stopping power is to be expressed

Soft-Collision Term: Excitation Potential

- The mean excitation potential I is the geometric-mean value of all the ionization and excitation potentials of an atom of the absorbing medium
- In general I for elements cannot be calculated
- Must instead be derived from stopping-power or range measurements
  - Experiments with cyclotron-accelerated protons, due to their availability with high β-values and the relatively small effect of scattering as they pass through layers of material
- Appendices B.1 and B.2 list some I-values

Soft-Collision Term: Excitation Potential



Mean ionization/excitation potential I against atomic number Z. Also plotted are three empirical relationships

Table 6.3. Mean ionization/excitation potential I for various absorbing materials (from the ICRU Report 37)

Element	H	C	Al	Cu	Ag	W	Pb	Ra	U	Cf
Z	1	6	13	29	47	74	82	88	92	98
I (eV)	19.2	78	167	322	470	727	823	826	890	966

Table 6.4. Mean ionization/excitation potential I for various compounds of interest in medical physics (from the ICRU Report 37)

Compound	I (eV)	Compound	I (eV)
Air (dry)	85.7	Lithium fluoride	94
Water (liquid)	75	Photographic emulsion	331
Water (vapor)	71.6	Sodium iodide	452
Muscle (skeletal)	75.3	Polystyrene	68.7
Bone (compact)	91.9	A-150 plastic	65.1

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Hard-Collision Term

- The form of the hard-collision term depends on whether the charged particle is an electron, positron, or heavy particle
- For heavy particles, having masses much greater than that of an electron, and assuming that $H << T'_{\max}$, the hard-collision term may be written as

$$\left(\frac{dT_h}{\rho dx}\right)_c = k\left[\ln\left(\frac{T'_{\max}}{H}\right) - \beta^2\right]$$

Mass Collision Stopping Power for Heavy Particles

$$\left(\frac{dT}{\rho dx}\right)_c = 0.3071\frac{Zz^2}{A\beta^2}\left[13.8373+\ln\left(\frac{\beta^2}{1-\beta^2}\right)-\beta^2-\ln I\right]$$

- Combines both soft and hard collision contributions
- No dependence on particle mass
- Depends on particle properties:
  - z - particle charge
  - particle velocity through $\beta = v/c$ (not valid for very low β)

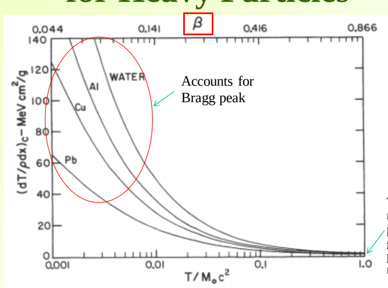
Mass Collision Stopping Power for Heavy Particles

$$\left(\frac{dT}{\rho dx}\right)_c = 0.3071\frac{Zz^2}{A\beta^2}\left[13.8373+\ln\left(\frac{\beta^2}{1-\beta^2}\right)-\beta^2-\ln I\right]$$

- Depends on stopping medium properties through Z/A, which decreases by ~20% from C to Pb
- The term –ln *I* provides even stronger variation with *Z* (the combined effect results in $(dT/\rho dx)_c$ for Pb less than that for C by ≅40-60 % within the β-range 0.85-0.1)

Overall trend for heavy charged particles: stopping power decreases with Z

Mass Collision Stopping Power for Heavy Particles



The kinetic energy required by any particle to reach a given velocity is proportional to its rest energy, M_0c^2

FIGURE 8.2. Mass collision stopping power ($dT/\rho dx$), for singly charged heavy particles, as a function of β (upper scale) or of their kinetic energy T normalized by the rest mass M_0c^2. For protons $T/M_0c^2 = 1$ at $T = 938$ MeV. For particles with z charges, multiply ordinate by z^2. From data of Bichsel (1968).

Mass Collision Stopping Power for Electrons and Positrons

$$\left(\frac{dT}{\rho dx}\right)_c = k\left[\ln\left(\frac{\tau^2(\tau+2)}{2\left(I/m_0c^2\right)^2}\right)+F^{\pm}(\tau)-\delta-\frac{2C}{Z}\right]$$

$$\tau \equiv T/m_0c^2$$

- Combines both soft and hard collision contributions
- $F(\tau)$ term – depends on β and τ
- Includes two corrections (introduced by U. Fano):
  - shell correction 2C/Z
  - correction for polarization effect δ

Shell Correction

- When the velocity of the passing particle is *not* much greater than that of the atomic electrons in the stopping medium, the mass-collision stopping power is over-estimated
- Since *K*-shell electrons have the highest velocities, they are the first to be affected by insufficient particle velocity, the slower *L*-shell electrons are next, and so on
- The so-called “shell correction” is intended to account for the resulting error in the stopping-power equation
- The correction term *C/Z* is the same for all charged particles of the same β, and is a function of the medium

Shell Correction

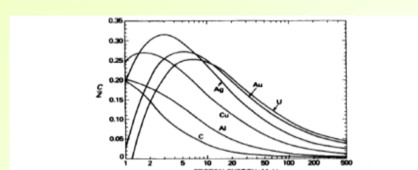


FIGURE 8.3. Semiempirical shell corrections of Bichsel for selected elements, as a function of the proton energy (ICRU, 1984a). Reproduced with permission from H. Bichsel and the International Commission on Radiation Units and Measurements.

- The correction term C/Z is a function of the medium and the particle velocity for all charged particles of the same velocity, including electrons

Polarization Effect

- Atoms near the particle track get polarized, decreasing the Coulomb force field and corresponding interaction (and energy loss)
- Introduce density-effect correction influencing soft collisions
- The correction term, δ, is a function of the composition and density of the stopping medium, and of the parameter

$$\chi \equiv \log_{10}(p/m_0c) = \log_{10}\left(\beta/\sqrt{1-\beta^2}\right)$$

for the particle, in which p is its relativistic momentum mv, and m_0 is its rest mass

- Mass collision stopping power *decreases in condensed media*
- Relevant in measurements with ion chambers at energies > 2 MeV
- Effect is always present in metals (δ ~0.1 even at low energies)

Polarization Effect

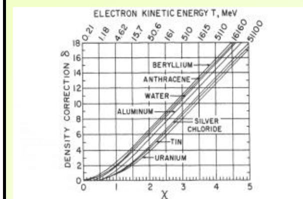


FIGURE 8.4. Density-effect correction δ as a function of χ and electron kinetic energy T. After Sternheimer (1952). Reproduced with permission from R. M. Sternheimer and the American Physical Society.

TABLE 8.2. Polarization Effect for Electrons

	Effect[a] (%)		
T (MeV)	C	Cu	Au
0.1	0	0	0
1.0	3	1.5	0.7
5	9	7	5
10	12	10	8
50	20	18	15

[a]Decrease in mass collision stopping power for condensed media vs. gases.

Appendix *E* contains tables of electron stopping powers, ranges, radiation yields, and density-effect corrections δ

- δ increases almost linearly as a function of χ above χ ≅ 1 for a variety of condensed media (*relativistic* effect)
- It is somewhat larger for low-Z than for high-Z media

Polarization Effect

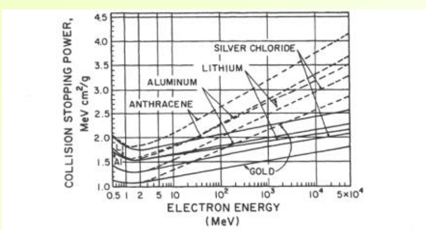


FIGURE 8.5. Mass collision stopping power for electrons in anthracene, Al, Li, AgCl, and Au, with (solid curves) and without (dashed curves) correction for polarization effect. After Sternheimer (1952). Reproduced with permission from R. M. Sternheimer and the American Physical Society.

The steep rise in collision stopping power for $\beta < m_0c^2$ is not shown, but the minimum at ≅ 3 m_0c^2 is evident, as is the continuing rise at still higher energy

Mass Radiative Stopping Power

- Only electrons and positrons are light enough to generate significant bremsstrahlung ($1/m^2$ dependence for particles of equal velocities)
- The rate of bremsstrahlung production by electrons or positrons is expressed by the *mass radiative stopping power* (in units of MeV cm²/g)

$$\left(\frac{dT}{\rho dx}\right)_r = \sigma_0 \frac{N_A Z^2}{A}\left(T + m_0c^2\right)\overline{B}_r$$

here the constant $\sigma_0 = {}^1/_{137}(e^2/m_0c^2)^2 = 5.80 \times 10^{-28}$ cm²/atom, T is the particle kinetic energy in MeV, and $\overline{B}_r$ is a slowly varying function of Z and T

Mass Radiative Stopping Power

- The mass radiative stopping power is proportional to N_AZ^2/A, while the mass collision stopping power is proportional to N_AZ/A, the electron density
- Ratio of radiative to collision stopping power

$$\frac{(dT/\rho dx)_r}{(dT/\rho dx)_c} \cong \frac{TZ}{n}$$

T – kinetic energy, Z – atomic number, n ~700 or 800 MeV

Radiation Yield

- The *radiation yield* $Y(T_0)$ of a charged particle of initial kinetic energy T_0 is the total fraction of that energy that is emitted as electromagnetic radiation while the particle slows and comes to rest
- For heavy particles $Y(T_0) \approx 0$
- For electrons the production of bremsstrahlung x-rays in radiative collisions is the only significant contributor to $Y(T_0)$
- For positrons, in-flight annihilation would be a second significant component, but this has typically been omitted in calculating $Y(T_0)$

Mass Stopping Powers vs. Energy and Z

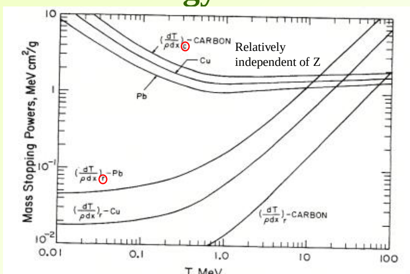


FIGURE 8.6. Mass radiative and collision stopping powers for electrons (and approximately for positrons) in C, Cu, and Pb. (From data of Bichsel, 1968).

Restricted Stopping Power

- Energy cutoff allows to account for escaping delta-rays with energy $\geq\Delta$ (not depositing energy locally)
- Linear Energy Transfer (radiobiology and microdosimetry)

$$L_\Delta(keV/\mu m) = \frac{\rho}{10}\left[\left(\frac{dT}{\rho dx}\right)_\Delta (MeV\ cm^2/g)\right]$$

- If the cut-off energy Δ is increased to $T_{\max}$ – use unrestricted *LET*:

$$\left(\frac{dT}{\rho dx}\right)_\Delta = \left(\frac{dT}{\rho dx}\right)_c \quad \text{and} \quad L_\Delta \equiv L_\infty$$

Restricted Stopping Power

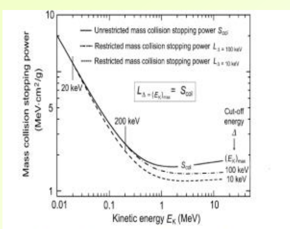


Fig. 6.16. Unrestricted mass collision stopping power S_{col} and restricted mass collision stopping power L_Δ with thresholds Δ = 10 keV and Δ = 100 keV for electrons in carbon against kinetic energy E_K. Data are based on the ICRU Report 37

- Restricted stopping power L_Δ is smaller than unrestricted
- For air-filled ion chamber with electrode separation 2mm Δ=10keV is reasonable (the range of a 10 keV electron in air ~2mm)

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Range

- The *range* $\Re$ of a charged particle of a given type and energy in a given medium is the expectation value of the pathlength *p* that it follows until it comes to rest (discounting thermal motion)
- The *projected range* <*t*> of a charged particle of a given type and initial energy in a given medium is the expectation value of the <u>farthest depth of penetration</u> t_f of the particle in its initial direction
- Both are non-stochastic quantities
- Can introduce range based on starting kinetic energy

CSDA Range

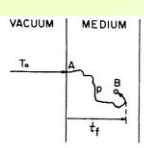


FIGURE 8.7. Illustrating the concepts of *pathlength p* and *farthest depth of penetration*, t_f, for an individual electron. *p* is the total distance along the path from the point of entry A to the stopping point B. Note that t_f is not necessarily the depth of the terminal point B.

$$\Re_{CSDA} \equiv \int_0^{T_0} \left(\frac{dT}{\rho dx}\right)^{-1} dT$$

Continuous slowing down approximation

- T_0 – starting energy of the particle
- Units: g/cm²
- Appendix E

CSDA Range: Protons

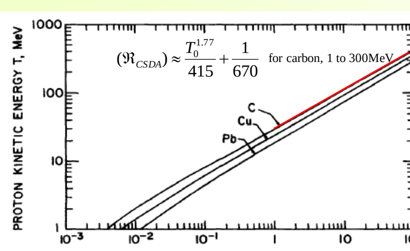


FIGURE 8.8. CSDA range (abscissa) vs. proton kinetic energy (ordinate) for C (graphite), Cu, and Pb.(From data of Bichsel, 1968).

Range is ***greater*** for higher Z due to decrease in stopping power

CSDA Range: Other Heavy Particles

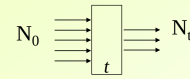
For particles with the same velocity $T = M_0 c^2 \left[\frac{1}{\sqrt{1-\beta^2}} - 1 \right]$

- Kinetic energy of a particle ~ to its rest mass
- Stopping power for singly charged particle is independent of mass
- Consequently, the range is ~ to its rest mass
- Can calculate the range for a heavy particle based on CSDA range values for protons at energy $T_0^P = T_0 M_0^P / M_0$

$$\Re_{CSDA} = \frac{\Re_{CSDA}^P M_0}{M_0^P z^2}$$

z – charge of heavy particle

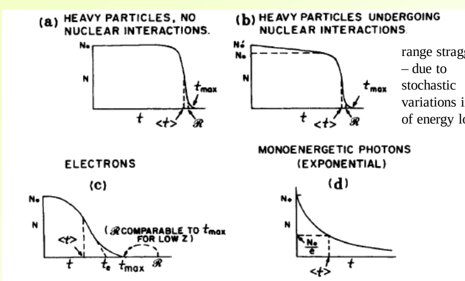
Projected Range



- Count the number of particles that penetrate a slab of increasing thickness (disregard nuclear reactions)
- N_0 number of incident mono-energetic particles in a beam perpendicular to the slab

$$\langle t \rangle \equiv \frac{\int_0^\infty t \cdot t_f(t) dt}{\int_0^\infty t_f(t) dt} = \frac{\int_0^\infty t \cdot \frac{dN(t)}{dt} dt}{\int_0^\infty \frac{dN(t)}{dt} dt} = -\frac{1}{N_0} \int_0^\infty t \cdot t_f(t) dt$$

Projected Range



range straggling – due to stochastic variations in rates of energy loss

FIGURE 8.9. Numbers of monoenergetic charged particles or photons penetrating through a slab thickness t of absorbing medium. Scattered photons are assumed to be ignored in d. $\langle t \rangle$ is the projected range, t_e is the extrapolated penetration depth, t_{max} is the maximum penetration depth, and R is the range ($\cong R_{CSDA}$).

Electron Range

- Electrons typically undergo multiple scatterings
- Range straggling and energy straggling due to stochastic variations in rates of energy loss
- Makes range less useful characteristic, except for low-Z materials, where range is comparable to max penetration depth t_{max}
- For high-Z range increases, t is almost independent

Electron Range

TABLE 8.4. Comparison of Maximum Penetration Depth t_{max} with CSDA Range[a] for Electrons of Energy T_o

T_o (MeV)	Z	t_{max} (mg/cm^2)	$\Re_{CSDA}$ (mg/cm^2)	$t_{max}/\Re_{CSDA}$
.05	13 (Al)	5.05	5.71	.88
.10	13 (Al)	15.44	18.64	.83
.15	13 (Al)	31.0	36.4	.85
.05	29 (Cu)	5.42	6.90	.79
.10	29 (Cu)	17.1	22.1	.77
.15	29 (Cu)	34.0	42.8	.79
.05	47 (Ag)	5.04	7.99	.63
.10	47 (Ag)	15.6	25.2	.62
.15	47 (Ag)	30.2	48.4	.62
.05	79 (Au)	4.73	9.88	.48
.10	79 (Au)	14.3	30.3	.47
.15	79 (Au)	27.6	57.5	.48

[a] After Bichsel (1968), based on experimental results of Gubernator and Flammersfeld, and CSDA ranges of Berger and Seltzer (1964). Reproduced with permission from H. Bichsel and Academic Press, Inc.

- For low-Z media, t_{max} is comparable to R, which increases as a function of Z
- t_{max} is almost independent of Z

Calculation of Absorbed Dose

Parallel beam of charged particles of kinetic energy T_0 perpendicularly incident on a foil Z

Assumptions:

- Collision stopping power is constant and depends on T_0
- Scattering is negligible
- Effect of delta rays is negligible

Calculation of Absorbed Dose

Energy lost in collision interactions (energy imparted)

$$E = \Phi\left(\frac{dT}{\rho dx}\right)_c \rho t \quad \left[\frac{\mathrm{MeV}}{\mathrm{cm}^2}\right]$$

ρt – mass thickness of foil, t – mean electron pathlength

Absorbed dose:

$$D = \frac{\Phi(dT/\rho dx)_c \rho t}{\rho t} = \Phi\left(\frac{dT}{\rho dx}\right)_c \quad \left[\frac{\mathrm{MeV}}{\mathrm{g}}\right]$$

mass per unit area of foil

$$D = 1.602\times 10^{-10}\,\Phi\left(\frac{dT}{\rho dx}\right)_c \quad [\mathrm{Gy}]$$

Dose in the foil is independent of its thickness (under our assumptions)

Dose from Heavy Particles

Based on range can find the residual kinetic energy of exiting particle

$$\Delta T = T_0 - T_{ex}$$

$$E = \Phi\,\Delta T$$

$$D = 1.602\times 10^{-10}\,\frac{\Phi\,\Delta T\cos\theta}{\rho t}$$

If beam is not perpendicular – $\rho t/\cos\theta$ accounts for angle

Dose in Gray

Dose from Electrons

- Need to account for path lengthening due to scatter
- Need to account for bremsstrahlung production, consider radiation yield
- Energy spent in collisions:

$$\Delta T_c = (T_0 - T_{ex})_c$$

- Average dose:

$$\overline{D} = 1.602\times 10^{-10}\,\frac{\Phi\,\Delta T_c}{\rho t}$$

Electron Backscattering

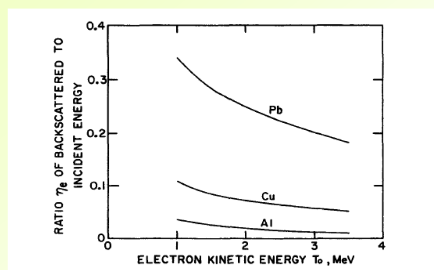


FIGURE 8.12. Fraction η_e of incident energy flux carried away by backscattered electrons. Primary electrons are perpendicularly incident, with individual kinetic energy T_0, on infinitely thick ($> t_{max}/2$) layers of the indicated scattering materials. After Wright and Trump (1962).

The Bragg Curve

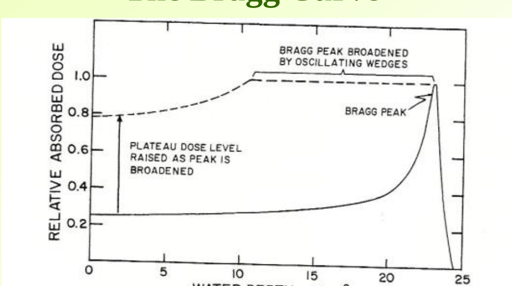


FIGURE 8.13a. Dose vs. depth for 187-MeV protons in water, showing Bragg peak. The dashed curve demonstrates the effect of passing the beam through optimally designed, variable-thickness absorbers such as oscillating wedges. (After Karlsson, 1964. Reproduced with permission from Strahlentherapie.)

Dose vs. Depth for Electron Beams

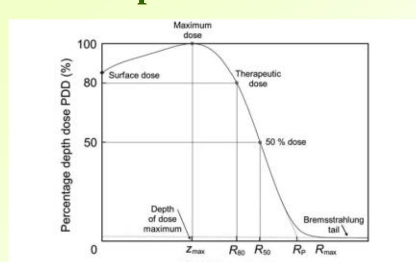


Fig. 6.15. Typical electron beam depth dose curve (dose against depth in water) normalized to 100 at the depth of dose maximum z_{max}. Several ranges of interest in radiotherapy and dosimetry, such as R_{80}, R_{50}, R_p, and R_{max}, are identified on the curve

Dose vs. Depth for Electron Beams

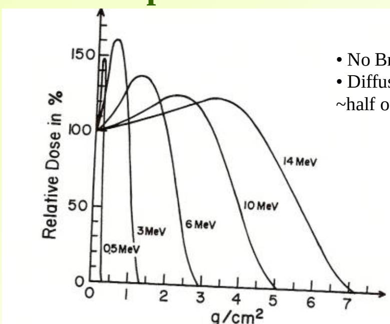


FIGURE 8.14a. Dose vs. depth in water for broad electron beams of the indicated incident energies. (After Holm, 1969. Reproduced with permission of N. W. Holm and Academic Press.)

- No Bragg peak
- Diffused peak at ~half of t_{max}

Calculation of Absorbed Dose at Depth

$$D_w = 1.602 \times 10^{-10} \int_0^{T_{max}} \Phi_x(T) \left(\frac{dT}{\rho dx} \right)_{c,w} dT$$

- At any point P at depth x in a medium w for know fluence spectrum
- For x below particle range

$$D_x = 1.602 \times 10^{-10} \Phi_0 \left(\frac{dT}{\rho dx} \right)_{c,w}$$

Summary

- Types of charged particle interactions
- Stopping power
- Range
- Calculation of absorbed dose