Charged-Particle and Radiation Equilibria

Chapter 4

F.A. Attix, Introduction to Radiological Physics and Radiation Dosimetry

Outline

- Radiation equilibrium
- Charged particle equilibrium
- Causes of CPE failure
- Transient CPE
- Summary

Introduction

- The concepts of radiation equilibrium (RE) and charged-particle equilibrium (CPE) are useful in radiological physics as a means of relating certain basic quantities
- CPE allows the equating of the absorbed dose $D$ to the collision kerma $K_c$ and exposure $X$
- Radiation equilibrium makes dose $D$ equal to the net rest mass converted to energy per unit mass at the point of interest

Radiation equilibrium

- Radiation equilibrium (RE) exists for the volume $v$ if the following four conditions exist throughout $V$ (in the nonstochastic limit):
  a. The atomic composition of the medium is homogeneous
  b. The density of the medium is homogeneous
  c. The radioactive source is uniformly distributed
  d. There are no electric or magnetic fields present to perturb the charged-particle paths, except the fields associated with the randomly oriented individual atoms

Radiation equilibrium

- Consider an extended volume $V$ containing a distributed radioactive source with a smaller internal volume $v$ about a point of interest, $P$
- Radioactivity is emitted isotropically on average
- $V$ is required to be large enough so that the maximum distance of penetration $d$ of any emitted ray and its progeny (i.e., scattered and secondary rays) is less than the minimum separation $s$ of the boundaries of $V$ and $v$

- This will be true for all possible orientations of tangent planes around the volume $v$; therefore, in the nonstochastic limit, for each type and energy of ray entering $v$, another identical ray leaves
- This condition is called radiation equilibrium (RE) with respect to $v$
Radiation equilibrium

- As a consequence of radiation equilibrium the energy carried in and that carried out of \( v \) are balanced for both indirectly and directly ionizing radiation:
  \[
  \langle R_{\text{in}} \rangle = \langle R_{\text{out}} \rangle, \quad \langle R_{\text{in}} \rangle = \langle R_{\text{out}} \rangle.
  \]
- The energy imparted can then be simplified to
  \[
  \bar{\varepsilon} = \sum Q
  \]
- Therefore under RE conditions the expectation value of the energy imparted to the matter in the volume \( v \) is equal to that emitted by the radioactive material in \( v \).

Radiation equilibrium

- In non-stochastic consideration the volume \( v \) can be reduced to infinitesimal \( dv \), then RE exists at the point P
- Since \( D = \frac{ds}{dm} \), under condition of radiation equilibrium at a point in a medium, the absorbed dose is equal to the expectation value of the energy released by the radioactive material per unit mass at that point, \( \frac{d}{dm} \sum Q \)

Radiation equilibrium

- Presence of homogeneous constant magnetic and/or electric field throughout \( V \) can make radiation field isotropic
- RE will still be satisfied if the flow of radiation is balanced
  - Consider flows of particles between \( dv', dv, \) and \( dv' \): \( a+B=A+b \)
  - If anisotropy is homogeneous everywhere in \( V \), the same balance of flow is held

Charged-particle equilibrium

- Charged-particle equilibrium (CPE) exists for the volume \( v \) if each charged particle of a given type and energy leaving \( v \) is replaced by an identical particle of the same energy entering, in terms of expectation values
- If CPE exists,
  \[
  \langle R_{\text{in}} \rangle = \langle R_{\text{out}} \rangle
  \]
- RE condition is sufficient for CPE to exist

Charged-particle equilibrium

- In many practical cases RE condition is not satisfied, but can be adequately approximated if CPE condition exists
- Consider two general situations: distributed radioactive sources and indirectly ionizing radiation from external sources
CPE for distributed sources – case 1

- For the trivial case of a distributed source emitting only charged particles, in a system where radiative losses are negligible
- The dimension $s$ (the minimum separation of $v$ from the boundary) to the is taken to be greater than the maximum range $d$ of the particles

  - If all of the four conditions $a$ through $d$ are satisfied, both RE and CPE exist (they are identical for this case)

CPE for distributed sources – case 2

- Consider now the case where both charged particles and relatively more penetrating indirectly ionizing radiation are emitted
- Let the distance $d$ be the maximum range of the charged particles only, and distance $s > d$

  - Conditions $a$ through $d$ are satisfied
  - Only CPE exists in this case
  - RE is not attained since rays escaping from the volume $v$ are not replaced.

  $\left( \bar{R}_{in} \right)_u > \left( \bar{R}_{out} \right)_u$

CPE for distributed sources – case 2

- The equation for the expectation value of the energy imparted
  $\mathcal{E} = \left( \bar{R}_{in} \right)_u - \left( \bar{R}_{out} \right)_u + \sum Q$

  - Since the indirectly ionizing rays are so penetrating that they do not interact significantly in $v$, $\mathcal{E}$ is equal to the kinetic energy given to charged-particles by the radioactive source in $v$, less any radiative losses by those particles while in $v$
  - The average absorbed dose in $v$ is thus $\bar{D} = \mathcal{E} / m$ for CPE condition

CPE for distributed sources – case 3

- A distributed source emitting penetrating indirectly ionizing radiation
- Achieving CPE will also require that RE is attained
  - The following equations are applicable
    $\left( \bar{R}_{in} \right)_c = \left( \bar{R}_{out} \right)_c$ and $\left( \bar{R}_{in} \right)_u = \left( \bar{R}_{out} \right)_u$
    $\mathcal{E} = \sum Q$

CPE for distributed sources

- The calculation of the absorbed dose is straightforward for either of these limiting cases (CPE or RE)
  - Intermediate situations are more difficult to deal with, i.e., when the volume $V$ is larger than necessary to achieve CPE in $v$, but not large enough for RE
  - In that case some fraction of the energy of the indirectly ionizing radiation component will be absorbed, and it is relatively difficult to determine what that fraction is
CPE for indirectly ionizing radiation from external sources

- A volume $V$ contains a smaller volume $v$
- The boundaries of $v$ and $V$ are required to be separated by at least the maximum distance of penetration of any secondary charged particle present
- If the following conditions are satisfied throughout $V$, CPE will exist for the volume $v$:
  - The atomic composition of the medium is homogeneous
  - The density of the medium is homogeneous
  - There exists a uniform field of indirectly ionizing radiation (rays are only negligibly attenuated passing through the medium)
  - No inhomogeneous electric or magnetic fields are present

CPE for indirectly ionizing radiation from external sources

- These conditions are similar to those of RE, except for:
  - Condition $c$: uniform field of radiation replaces the uniform radioactive source
  - The separation of boundaries of $v$ and $V$ are required to be at least the maximum distance of penetration of any secondary charged particle, rather than that of the most penetrating radiation (indirectly ionizing)
  - The last condition $d$ has been shown to be a sufficient substitute for the requirement of a complete absence of electric or magnetic fields

CPE for indirectly ionizing radiation from external sources

- Analogy with the distributed source case: since the radiation field is uniform, the number of charged particles produced per unit volume in each energy interval and element of solid angle will be uniform
- However, since neutron and photon interactions generally result in anisotropic angular distributions of secondary and scattered radiations, these particles are not emitted isotropically as in the case of radioactive point sources
- This anisotropy will be homogeneous throughout $V$

CPE for indirectly ionizing radiation from external sources

- For CPE conditions
  \[ \mathbf{E}''_v = \mathbf{E} + (\mathbf{R}_\text{out})_u - (\mathbf{R}_\text{out})_a + \mathbf{R}_v \]
- However, under those same conditions we may also assume that any radiative interaction by a charged particle after it leaves $v$ will be replaced by an identical interaction inside of $v$, resulting in
  \[ (\mathbf{R}_v)_u = (\mathbf{R}_v)_a + \mathbf{R}_v \]

CPE for indirectly ionizing radiation from external sources

- Homogeneous anisotropy, together with a uniform medium in which the charged particles can slow down throughout $V$ (as guaranteed by the first two conditions) is sufficient to produce CPE for the volume $v$
- Consider, for example, a simplified case of straight charged particle tracks, all emitted at angle $\theta$ through identical interactions at different points
  - Three charged particles $e_1$-$e_2$ will contribute the same kinetic energy as $e_1$ alone would, if its entire track remained inside of $v$
  - Thus CPE exists inside of $v$

CPE for indirectly ionizing radiation from external sources

- FIGURE 44. Illustrating Eqs. (4.4b) and (4.4c). CPE exists (in the non-relativistic limit) because whenever $e_u$ moves the volume $\nu$ with a kinetic energy $\mathbf{E}$ equal to that carried out by electron $e_v$, then unless we $e_u$ will also pick up $e_\nu$ and $e_v$, the outgoing $e$ and $e_v$ have $\mathbf{E}$ and $\nu$ 0. If $e_u$ captures an $e_v$ then $(\mathbf{R}_\nu)_u = (\mathbf{R}_\nu)_a + \mathbf{R}_v$ and since $\mathbf{E}''_\nu = 0$. If $e_u$ is not captured $\nu = 0$, $\nu = 0 = 0$, and $\nu = 0$. If $e_u$ is captured $\nu = 0$. If $e_u$ is captured $\nu = 0$. If $e_v$ is captured $\nu = 0$. If $e_v$ is captured $\nu = 0$. If $e_v$ is captured $\nu = 0$.

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CPE for indirectly ionizing radiation from external sources

- Provided that the volume $v$ is small enough to allow radiative-loss photons to escape $\bar{\varepsilon} = \bar{\varepsilon}_n$
- Reducing $v$ to the infinitesimal volume $dv$, containing mass $dm$ about a point of interest $P$, we can write
  \[
  \frac{d\bar{\varepsilon}}{dm} = \frac{d\bar{\varepsilon}_n}{dm}
  \]
  hence under CPE:
  \[
  D = K_c
  \]

CPE in measurement of exposure

- Exposure $X$ (which is only defined for $x$ and $\gamma$ rays) is equal to the product of $K_c$ and $e/W$ for air
- This poses a practical difficulty in the measurement of $X$, since collision kerma ($K_c$) cannot be readily measured by any direct means
- The attainment of CPE in an ionization chamber, however, allows the measurement of the ionization collected within a defined volume and mass of air, in place of the ionization produced everywhere by all the secondary electrons that start within the defined volume, as called for by the exposure definition

CPE for indirectly ionizing radiation from external sources

- If the same photon energy fluence $\Psi$ is present in media A and B having two different average energy absorption coefficients, the ratio of absorbed doses under CPE conditions in the two media will be given by
  \[
  \frac{D_A}{D_B} = \frac{(K_c)_A}{(K_c)_B} = \left(\frac{\mu_{en}}{\rho}\right)_A \left(\frac{\mu_{en}}{\rho}\right)_B
  \]
  - Doses $D_A$ and $D_B$ can differ due to either different atomic compositions A and B, or radiation spectra

CPE for indirectly ionizing radiation from external sources

- The derivation of this equation proves that under CPE conditions at a point in a medium, the absorbed dose is equal to the collision kerma there
- That is true irrespective of radiative losses
- This relationship equates the measurable quantity $D$ with the calculable quantity $K_c = \Psi \cdot \mu_{en}/\rho$

CPE for indirectly ionizing radiation from external sources

- It is possible for CPE to exist in a volume without satisfying all the requirements, $a$ through $d$, under certain geometrical conditions
- The ion-collecting region of a free-air chamber represents such a situation, to be discussed in Chapter 12
- Another example is the trivial case of a point source within a volume large enough so the radiation cannot reach the boundary surface, hence no replacement particles are required

Ion chamber operation

- $v$ must be small enough to allow the escape of radiative losses
- Corrections must be made for larger volumes

*FIGURE 4.5. The role of CPE in the measurement of exposure $X$. The average exposure in the finite air volume $v$ equals the total charge of either sign released in air for all electrons $(e)$ that originate in $v$, divided by the air mass $\rho$. If CPE exists, each electron carrying an energy $(e)$ is accompanied by another electron $(\alpha)$ carrying the same energy $E$. This then ionization occurs in $v$ if all electrons $e$, remained there. The measurement of that charge divided by $v$ is then equivalent to a measurement of the exposure $X$ as $\alpha$; Radiative losses are assumed to escape from $v$, and any ionization they produce is not to be included in $X$. 
Relating absorbed dose to exposure

• It is sometimes useful to know how much absorbed dose would be deposited at some point in air as a result of an exposure X
• The relationship is indeterminate in the absence of CPE, since

\[ D_{\text{air}}^{\text{CPE}} = (K_e)_{\text{air}} = X \left( \frac{\gamma}{e} \right) \]

where the first equality is valid only if CPE exists at the point in question

CPE failure: Proximity to the source

• If the volume V is too close to the source of the indirectly ionizing radiation, then the energy fluence will be significantly nonuniform within V, being larger on the side nearest the source
  • Thus there will be more particles (e_3) produced at points like P_3 than particles e_1 at P_1, and more particles will enter v than leave it
  • CPE consequently fails for v

CPE failure: Proximity to a boundary of inhomogeneity in the medium

• If the volume V is divided by a boundary between dissimilar media, loss of CPE may result at v, since the number of charged particles arriving at v will generally be different than would be the case for a homogeneous medium
• This difference may be due to a change in charged-particle production, or a change in the range or geometry for scattering of those particles, or a combination of these effects

Causes of CPE failure in a field of indirectly ionizing radiation

There are four basic causes for CPE failure in an indirectly ionizing field:

a. Inhomogeneity of atomic composition within volume V
b. Inhomogeneity of density in V
c. Non-uniformity of the field of indirectly ionizing radiation in V
d. Presence of a non-homogeneous electric or magnetic field in V

CPE failure: Proximity to a boundary of inhomogeneity

• Consider a case of a beam of MV photons incident on a solid unit-density air-equivalent phantom
• The photon beam is not contaminated by secondary electrons from the photon source or associated hardware
  • The absorbed dose D in the phantom first increases steeply up to a maximum, then decreases more gradually in a condition called transient charged particle equilibrium (TCPE)
We temporarily consider TCPE as being ~ the same as CPE
CPE failure: proximity to a boundary of inhomogeneity

- The spherical volume \( V \), having radius \( d \) equal to the maximum range of secondary electrons, must contain a uniformly irradiated homogeneous medium throughout if TCPE is to be produced at point \( P \).
- If \( P \) is too close to the surface, the portion \( V' \) of \( V \) will project out of the phantom.
- To replace the solid missing from that volume, a 1000-fold larger volume \( V'' \) of air is required (distance \( ac = 1000ab \)).
- But \( V'' \) is not homogeneously (nor even completely) irradiated.

CPE failure: High energy radiation

- As the energy of indirectly ionizing radiation increases, the penetrating power of the secondary charged particles increases faster than the penetrating power of the primary radiation.
- For example, a 7% attenuation of \( \gamma \)-rays would occur in a water layer equal in thickness (\( \approx 5 \) cm) to the maximum range of secondary electrons produced by 10-MeV \( \gamma \)-rays.
- The neutron effect is much smaller (1%) at that energy, assuming hydrogen-recoil proton secondaries.

CPE failure: High energy radiation

- As a result, there is a significant attenuation of primary radiation over the distance \( d \).
- The radiation field is not uniform anymore (failure of condition c in CPE definition).
- CPE fails due to the difference in the number of charged particles produced.
  - Due to attenuation fewer photons reach \( P_1 \) than \( P_3 \).
  - The number of charged particles generated at \( P_3 \) is greater than at \( P_1 \).

Transient charged-particle equilibrium

- TCPE is said to exist at all points within a region in which \( D \) is proportional to \( K_c \), with the constant of proportionality being greater than unity.
- Consider two cases where a broad “clean” beam of indirectly ionizing radiation (i.e., unaccompanied by charged particles) is falling perpendicularly on a slab of material:
  - Negligible radiative losses from secondary charged particles.
  - Significant radiative losses; photons are allowed to escape from the phantom.
- Assume negligible radiative losses from secondary charged particles \( (K_r = 0) \):
  - This would be strictly true only for incident neutrons.
  - Also in carbon, water, air, and other low-Z media \( K_r = K - K_c \) remains less than 1% of \( K \) for photons up to 3 MeV.
- The absorbed-dose curve is rising with depth near the surface as the population of charged particles flowing toward the right is augmented by more interactions of indirectly ionizing rays.

The dose curve reaches a maximum \( (D_{max}) \) at the depth where the rising slope due to buildup of charged particles is balanced by the descending slope due to attenuation of the indirectly ionizing radiation.
Transient charged-particle equilibrium

- For a "clean" beam of indirectly ionizing radiation $D_{\text{max}}$ occurs at approximately the same depth as where the $D$-curve crosses the $K_c$-curve
- The presence of charged-particle "contamination" in the beam is often observed to shift the depth of $D_{\text{max}}$ closer to the surface, where it no longer approximates the depth at which $D = K_c$
- Thus one should not assume that $D = K_c$ at $D_{\text{max}}$

Transient charged-particle equilibrium

- Roesch suggest a relationship between the $D$- and $K$-curves for TCPE conditions, in assumption of no radiative interactions, and ignoring scattered photons:

$$D = K_c e^{\mu' \gamma TCPE} = K_c \left(1 + \mu' x + \frac{(\mu' x)^2}{2!} + \ldots\right)$$

$$= K_c \left(1 + \mu' x\right)$$

- Here $D$ and $K_c$ are for the same given depth, at which TCPE is required, $\mu'$ is the common slope of the $D$, $K$, and $K_c$ curves at that depth; and $x$ is the mean distance the secondary charged particles carry their kinetic energy in the direction of the primary rays while depositing it as dose.

Summary

- Radiation equilibrium
- Charged particle equilibrium
- Causes of CPE failure
- Transient CPE