THE UNIVERSITY OF TOLEDO
Topology M.A. Comprehensive Examination
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This exam has been checked carefully for errors. If you find what you believe to be an error in a question, report this to the proctor. If the proctor's interpretation still seems unsatisfactory to you, you may alter the question so that in your view it is correctly stated, but not in such a way that it becomes trivial.

Do any 5 of the 11 problems.

Let $\Phi = \Pi_{\mu \in M} X_\mu$ be the product of the topological spaces $(X_\mu)_{\mu \in M}$ and with Φ having the product topology.

1. Prove that the projection $p_\mu : \Phi \to X_\mu$ is an open map from Φ onto X_μ for each $\mu \in M$.

2. Prove that a function $f : W \to \Phi$ from a space W into the topological product Φ is continuous iff, for each $\mu \in M$, the composition $p_\mu \circ f$ is continuous.

3. Prove: Let γ be a given cover of a topological space X. Let us assume that, for each member $A \in \gamma$, there is given a continuous map $f_A : A \to Y$ such that

$$f_A \mid A \cap B = f_B \mid A \cap B$$

for each pair of members A and B of γ. Then we may define a function $f : X \to Y$ by taking

$$f(x) = f_A(x), \qquad (\text{if } x \in A \in \gamma).$$

Prove that if γ is a finite closed cover of X, then the combined function f is continuous.

4. Prove that every compact space is normal.

5. Prove that every continuous mapping of a compact space into a Hausdorff space is closed.

6. Prove that for any given subset E in a metric space X, the function $f : X \to R$ defined by

$$f(x) = d(x, E), \qquad (x \in X),$$

is continuous.

7. Prove that every continuous image of a connected set is connected.

8. Prove that if two connected sets A and B in a space X have a common point p, then $A \cup B$ is connected.

9. Prove that a point p of a space X belongs to the closure $C1(E)$ of a set E in X iff there exists a net Φ in E which converges to p.

10. Prove that every closed subspace of a locally compact space is locally compact.

11. Prove that every compact set K in a Hausdorff space X is closed.