THE UNIVERSITY OF TOLEDO
Topology M.A. Comprehensive Examination
April , 2010

This exam has been checked carefully for errors. If you find what you believe to be an error in a question, report this to the proctor. If the proctor's interpretation still seems unsatisfactory to you, you may alter the question so that in your view it is correctly stated, but not in such a way that it becomes trivial.

100% will be awarded for *complete* answers to 6 problems. Indicate carefully which six problems you wish to have graded.

1. Give the definition of continuity for a function from one topological space to another. *Use your definition* to show that $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = \frac{1}{x^2}$ is not continuous.
2. Prove that the space $(-1, 1)$ is not homeomorphic to $[-1, 1]$.
3. Prove that a closed subset of a compact topological space is compact.
4. Prove that in a Hausdorff topological space a compact subset is closed.
5. Prove that in a metric space a compact subset is closed and bounded.
6. Prove that a closed and surjective map is a quotient map.
7. Prove that for any given subset E in a metric space X, the function $f : X \to \mathbb{R}$ defined by
$$f(x) = d(x, E), \qquad (x \in X),$$
is continuous.
8. Let A be a subspace of the topological space X. Define what it means for a subset B of A to be closed in A in the subspace topology. Prove carefully from your definition that B is closed in A if and only if B is closed in X.
9. Define compactness for a topological space. A collection of subsets is said to have the finite intersection property if every finite class of those subsets has non-empty intersection. Prove that a topological space is compact iff every collection of closed subsets that has the finite intersection property itself has non-empty intersection.
10. Let $f : X \mapsto Y$ be a quotient map, with Y connected. Show that if $f^{-1}(y)$ is connected for all $y \in Y$, then X is connected.
11. Prove that the intersection of two open dense subsets of a topological space is dense.
12. Let X be a topological space. Let $A \subset$ X be connected. Prove that the closure $\overline{A}$ of A is connected.