Master's Level Topology Exam

Trieu Le and Gerard Thompson

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CALCULATORS ARE NOT ALLOWED. DO NO MORE THAN SIX QUESTIONS. 100% WILL BE OBTAINED FOR COMPLETE ANSWERS TO FOUR QUESTIONS.

1. Prove or disprove: a compact metric space is bounded.

2. If (X, d) is a metric space then $\{x \in X : d(x, x_0) \leq \epsilon\}$ is defined to be the closed ball of radius ϵ and center x_0. Prove that a closed ball is a closed set.

3. (a) Give the definition of a topological space.
   (b) Give the definition of the discrete topology on a set.
   (c) Show that any function $f : X \to Y$ where X and Y are topological spaces and X is equipped with the discrete topology is necessarily continuous.

4. Show that in a compact topological space a closed subset is compact. Then show that in a Hausdorff topological space a compact subspace is closed.

5. Explain what it means for a topological space to be *Haudorff*. Show that the product topological space $X \times Y$ is Hausdorff if and only if X and Y are Hausdorff.

6. Let B be an open subset of a topological space X. Prove that a subset $A \subset B$ is relatively open in B if and only if A is open in X.

7. Let X be a topological space. Let $A \subset X$ be connected. Prove that the closure $\overline{A}$ of A is connected.

8. Give the definition of quotient topology and quotient maps for topological spaces. Prove that the composition of two quotient maps is a quotient map.

9. Let γ be a given cover of a topological space X. Assume that for each member $A \in \gamma$, there is given a continuous map $f_A : A \to Y$ such that

   $$f_A \mid A \cap B = f_B \mid A \cap B$$

   for each pair of members A and B of γ. Then we may define a function $f : X \to Y$ by taking

   $$f(x) = f_A(x), \qquad (\text{if } x \in A \in \gamma).$$

   Prove that if γ is a finite closed cover of X, then the function f is continuous.