M.A. TOPOLOGY EXAM SPRING 1998

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Do no more than five (5) questions. If you think there is a misprint in a question state your query clearly and try to interpret it in a non-trivial manner.

The Examination Committee tries to proofread the exams as carefully as possible. Nevertheless, the exam may contain misprints. If you are convinced a problem has been stated incorrectly, mention this to the proctor and indicate your interpretation in your solution. In such cases do not interpret the problem in such a way that it becomes trivial.

Exam is <u>two</u> hours. Do five (5) problems.

1. Define what it means for a topological space to be *disconnected.* Prove that a space is disconnected if and only if there is a continuous map from the space onto the discrete two point space {0,1}.

2. State whether the following propositions are true or false. If they are true prove them. If they are false give a counterexample.

   (a) In a compact topological space a closed subspace is compact.

   (b) In a metric space a compact subspace is closed.

   (c) In a Hausdorff space a compact subspace is closed.

3. Prove that a topological space X is Hausdorff if and only if the diagonal $\Delta = \{(x, x) \in X \times X : x \in X\}$ is closed in $X \times X$ where $X \times X$ has the product topology induced by X.

4. Prove that a continuous bijection from a compact topological space onto a Hausdorff topological space is necessarily a homeomorphism.

5. Define the product topology $X \times Y$ on topological spaces X, Y. Prove from your definition that each of the projections from $X \times Y$ to X and Y, respectively, are both continuous and open maps.

6. Define what it means for $p : X \to Y$ to be a quotient map of topological spaces or, equivalently, for p to be an identification map. Verify in detail that $\exp : \mathbb{R} \to \mathbb{R}^2$ given by $\exp(t) = (\cos t, \sin t)$ gives an identification map of $\mathbb{R}$ onto the unit circle.

7. Prove that as subspaces of $\mathbb{R}$ with the usual topology the open interval (0,1) is not homeomorphic to the half-open interval [0,1).