Complete three of the following five problems. In the next five problems X is assumed to be a topological space. All "maps" given in both sections are assumed to be continuous although in a particular problem you may need to establish continuity of a particular map.

1. A subset $A \subset X$ is said to be locally closed if for any $x \in X$ there is a neighborhood U of x such that $A \cap U$ is relatively closed. Prove that a locally closed set is closed.

2. (a) If X is infinite and has the finite complement topology show that the diagonal $\Delta \subset X \times X$ $\Delta = \{(x,x) | x \in X\}$ is not closed.

   (b) If X is an arbitrary topological space what is the necessary and sufficient condition for Δ to be closed? Prove your conjecture.

3. X is said to be locally connected if any neighborhood of any point contains a connected neighborhood. Prove that the connected components of a locally connected space are both open and closed.

4. Suppose that $f : X \longrightarrow Y$ is continuous and for any $y \in Y$, $f^{-1}(y)$ is compact. Suppose that $\{A_j\}_{j=1}^{\infty}$ is a decreasing sequence of subsets of X such that for any j, $f(A_j) = Y$ show that $f(\bigcap_{j=1}^{\infty} A_j) = Y$.

5. A space X is said to be completely normal if for any subsets A and B of X such that $\bar{A} \cap B = \phi$ and $A \cap \bar{B} = \phi$, there are disjoint open sets U_A and U_B containing A and B respectively. Prove that any subspace of a completely normal space is normal.