University of Toledo, Department of Physics and Astronomy

Ph.D. Qualifying Exam

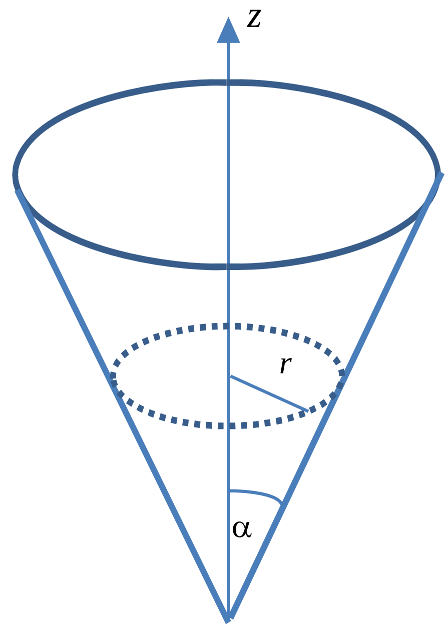
Fall 2018
September 22

Instructions:

- **Do not** write your name on your exam. On every sheet of paper that you turn in, put your assigned letter code at the upper left corner and a label identifying the problem (such CM1, EM2, QM3) at the upper right corner.
- Work 2 out of 3 problems on each subject. Staple together solutions for each subject and turn them in separately. Indicate clearly which problems are to be graded and which are to be omitted.
- Begin each problem on a new sheet of paper.

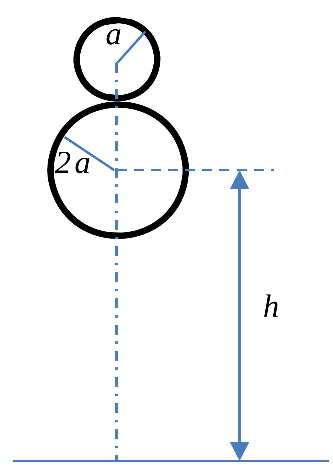
CLASSICAL MECHANICS (CM):

1. A particle of mass m is constrained to move on the inside surface of a smooth cone of half-angle α. The particle is subject to a gravitational force. With proper initial conditions, the particle can move in a stable oscillatory circular orbit with an angular momentum of l about the vertical axis with the plane of orbit at a constant height above the horizontal plane passing through the apex of the cone. (a) Use Lagrange method to find the equations of motion; (b) determine the radius of the stable circular orbit r_0 ; (c) Determine the oscillatory frequency of the radius r that varies around the r_0.



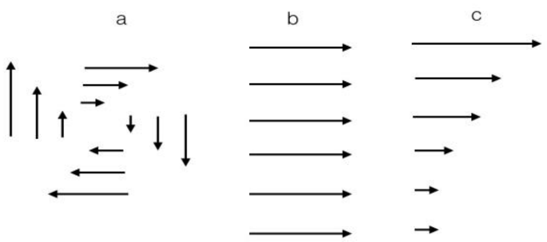
2. A bullet of mass m is fired straight up with an initial speed of v_0. Assuming that the air drag force on the bullet varies quadratically with speed (i.e $\vec{F} = -cv\vec{v}$), find (a) the maximum height the bullet can reach; (b) the speed when it returns to ground.

3. A small steel ball with radius a is on top of a similar ball with radius $2a$. They are dropped together from a height of h above the ground measured from the center of the larger ball. Assume all collisions are elastic, and the centers of the balls are always aligned along the vertical line, find the maximum height the small ball may reach after the collisions.



ELECTRICITY AND MAGNETISM (E&M):

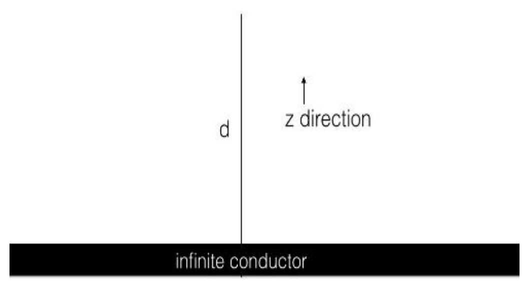
1. a. Below are drawings of electric fields. Which of the three can be created by electrostatic charges? Which ones cannot? Why?



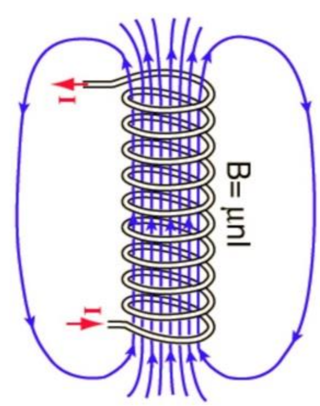
b. Use Gauss's law to determine the electric field surrounding an infinite line charge with charge density λ. Think carefully about the coordinate system you adopt.

2. a. There are no electric field within a perfect conductor: $E= 0$. Given this condition, describe why the electric field at the surface of a conductor has to be perpendicular to the surface.

b. Imagine a line of charge perpendicular to a conductor in the x, y plane and extending from the conductor at $z = 0$ to a height $z = d$. The charge density is λ. What is the corresponding charge density and position of the mirror charge needed for the method of images? Draw the mirror charge and resulting field lines.



3. The magnetic field in a solenoid is $\boldsymbol{B}= B_z\,\hat{z}$ (this is a constant) inside the very long solenoid and $B = 0$ outside. The radius of the solenoid is R. Use the curl theorem to derive $\boldsymbol{A}$ outside the solenoid ($s>R$) as a function of radius from the center of the solenoid (remember that $\boldsymbol{A}$ is the vector potential, where $\boldsymbol{B} = \nabla\times\boldsymbol{A}$). Use cylindrical coordinates.



QUANTUM MECHANICS (QM):

1. Assume that Ψ_{nlm} denotes an eigenfunction of the hydrogen atom with principal quantum number *n*, and angular momentum quantum numbers *l* and *m*. The hydrogen atom is in a state described by the wave function $\Psi = C\left(\Psi_{100} + 2\Psi_{111} + \Psi_{210} + 4\Psi_{21-1}\right)$
   - **a.** Find a normalization constant *C*.
   - **b.** What is the expectation value of the energy?
   - **c.** What is the expectation value of $\boldsymbol{L}^2$?
   - **d.** What is the expectation value of L_z?

2. For the electron in the ground state of the Li ion Li^{2+},
   - **a.** Prove that the wave function has the form $\psi = A\exp(-r/a), \quad A, a = const$
   - **b.** Calculate *A*, *a*, and the average potential energy of the electron.

3. A particle in a box [$V(x)=0,\ 0\le x\le a;\ V=\infty,\ x<0$ *or* $x>a$] is prepared in a state

$$\psi(x) = \begin{cases} \sqrt{\dfrac{4}{a}}\sin\left(\dfrac{2\pi}{a}x\right) & 0 \le x \le a/2 \\ 0 & x<0; x>a/2 \end{cases},$$

[nonzero only over the left half of the box.] If a measurement of energy is conducted, what are the possible outcomes? What are probabilities for finding the particle in the ground (lowest energy) state and the first excited state?