University of Toledo, Department of Physics and Astronomy

Ph.D. Qualifying Exam

SPRING 2017
FEBRUARY 2017

Instructions:

- **Do not** write your name on your exam; put your chosen letter on every sheet of paper that you turn in.
- Work 2 out of 3 problems in each category.
- Begin each problem on a new sheet of paper.
- Be sure to state which problems are omitted.

MECHANICS

1. A particle is sitting on top of a smooth, frictionless sphere of radius *a*. The particle is disturbed slightly (see Figure 1).

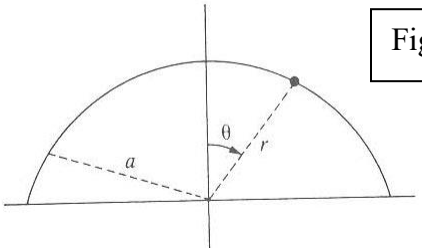

Figure 1

a) What is the kinetic energy *T* and potential energy *V*? Specify your coordinate system.
b) Determine the speed of the particle as it slides down the sphere.
c) At what point will the particle leave the sphere?

2. An ant is crawling on a merry-go-round, which has a constant angular velocity ω (see Figure 2). The ant is crawling outward along the radial direction at a constant speed *v*.

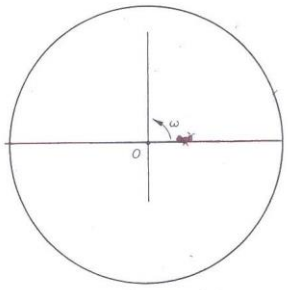

Figure 2. View from above the merry-go-round. Gravity points into the page.

a. Identify all forces that act on the ant.
b. Write down the equation of motion in terms of the ant's (moving) coordinates.
c. How far can the ant crawl before it begins to slip, given the coefficient of friction m between the ant and the merry-go-round.

3. Consider a system where a mass, m_1, is connected by a light inextensible cord of length l, which passes over a pulley of radius *R*. Attached to this cord is a second simple pulley of radius R, which supports two weights, m_2 and m_3, connected by another cord also of length l.

a. How many degrees of freedom does the system have? State your choice of generalized coordinates.
b. Write down an expression for the kinetic energy *T*.
c. Write an expression for the potential energy *V*.
d. What is the Lagrangian?
e. Use Lagrange's equation to find the equations of motion for the system.

ELECTRICITY AND MAGNETISM

1. Apply Gauss's Law to a stationary point charge to derive an expression for the force between two point charges (i.e., Coulomb's Law).

2. A wire loop of radius a carries a current I_1. Let the center of the loop be the coordinate origin, and let the loop axis be the z-axis. A second test loop of radius b ($b << a$) is oriented parallel to the first loop and placed on the z-axis a distance d above the first loop. The test loop is then placed into motion, moving upward with velocity v along the z-axis.

   a) Find the magnetic field **B** everywhere on the z-axis.

   b) If the test loop has resistance R, find the current I_2 induced in the test loop.

3. a) Determine the electric field **E** at a point P a distance z along the central axis of a uniform ring of positive charge of radius R.

   b) Evaluate and explain in words the result in part (a) when z >> R.

   c) Now consider a uniform disk of positive charge of radius R, and write the expression for **E** at P.

QUANTUM MECHANICS

1. The potential energy of a particle is
   $V = \infty$ for $x < 0$ and $x > a$, and $V = 0$ for $0 < x < a$.
   a) Determine the energy levels and normalized wave functions of the particle in its second and third energy states.
   b) Show that the average value of x is $a/2$.
   c) Show that the dispersion of x is approximately $a^2/12$ for the high-energy states of the particle. What is the criterion of applicability of this result?
   d) Find the average value of square of the momentum in the n*th* energy state.

2. For the electron in the ground state of the helium ion He^+,
   a) Prove that the wave function has the form
   $\psi = A\exp(-r/a), \quad A, a = const$
   b) Calculate A, a, and the average potential energy of the electron.
   c) Prove that the mean value of $1/r$ is $1/a$.

3. Given the electron kinetic energy 100 eV,
   a) Evaluate its de Broglie wavelength.
   b) Under what circumstances will the electron show distinctly wave properties when it is scattered by a periodic structure of linear period d?
   c) The electron beam of the above energy is incident on a single slit to form an image on a remote screen (l=1 m; see Fig. 1). Based on the uncertainty principle,
      1. Estimate the transversal component of the electron momentum after it passed through the slit.
      2. Estimate the difference between the slit width $d=10^{-8}$ m and the image size D.