University of Toledo, Department of Physics and Astronomy

Ph.D. Qualifying Exam

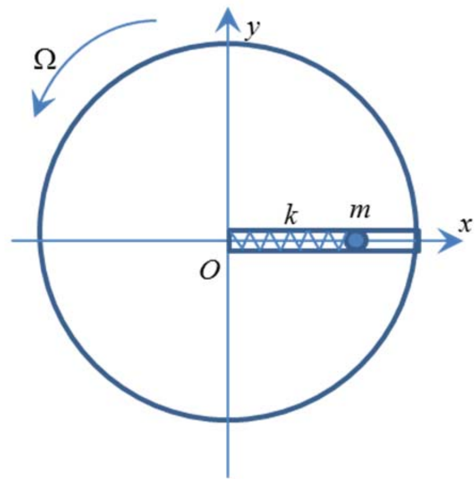
Spring 2018
February 10

Instructions:

- **Do not** write your name on your exam. On every sheet of paper that you turn in, put your assigned letter code at the upper left corner and a label identifying the problem (such CM1, EM2, QM3) at the upper right corner.
- Work 2 out of 3 problems on each subject. Staple together solutions for each subject and turn them in separately. Indicate clearly which problems are to be graded and which are to be omitted.
- Begin each problem on a new sheet of paper.

Classical Mechanics

1. A disc is laid on a horizontal *xy*-plane. A slot is cut parallel to its diameter. Along the slot, a mass *m* with a spring is installed. The spring has a spring constant of *k*, and is fixed to the center of the disc on one end, and connected to the mass *m* on the other end. The spring and the mass are restricted to move along the slot only. Now let the disc rotate about its center *O* along the vertical z-axis that passes through the center with an angular velocity of Ω. If an *xy*-coordinate system is attached to the disc as shown, derive the equation of motion of the mass *m* in the non-inertial reference frame *Oxyz*.. Find the condition under which the mass *m* may oscillate? And if it does oscillate under the condition, what is the oscillation frequency?



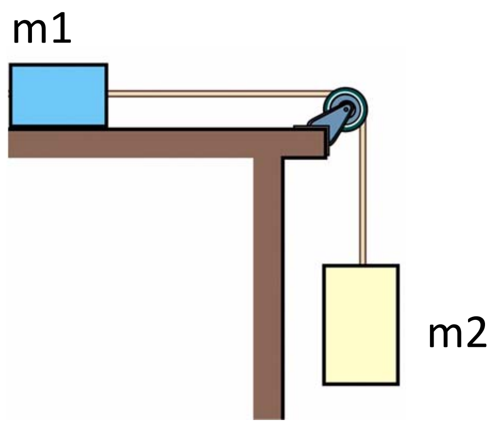
(Top view of the disc)

2. A mass *m* doing one-dimensional motion is subject to a force given by:

$$F(x) = \lambda^2 \left(\frac{1}{x^2} - \frac{2}{a^2} \right)$$

Show that the mass will oscillate around an equilibrium position x_0. Find the x_0 and the oscillating angular frequency ω. Express your answers in terms of λ, a, and m.

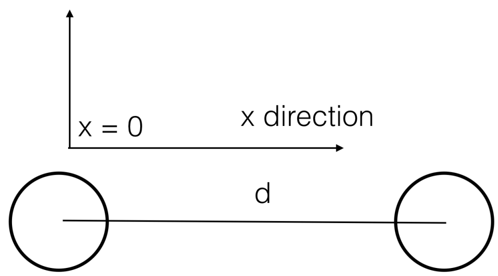
3. Two blocks masses, m_1 and m_2, are connected by a FLEXIBLE HEAVY UNSTRETCHABLE cord. The length of the cord is L and its mass is m_3. The block of m_1 is placed on a smooth frictionless table, and the other block of m_2 hangs over the edge through a mass-less small pulley. Find the acceleration of the blocks.



0.1. E & M problems

1. Imagine two cylinders, both with a charge surface density of σ and radius a. A "top" view looking down on the two cylinders is shown below. The center of the left cylinder is at $x = 0$. The electric field and potential of a *single* cylinder are given by:

$$\vec{E}(s) = \frac{1}{\epsilon_0}\frac{\sigma a}{s}\hat{s}, \qquad V(s) = \frac{1}{\epsilon_0}a\sigma_a[ln(a) - ln(s)] \tag{1}$$



Write down the equations for the electric field along the line connecting the centers of the two cylinders Where is the field weakest?

Write down the equations for the electric potential along the line connecting the centers of the two cylinders as a function of x. Where is the electric potential at a minimum?

2. An electromagnetic wave traveling through a gas composed of difference species of molecules/atoms can be written as:

$$\tilde{\mathbf{E}}(z,t) = \tilde{\mathbf{E}}e^{-\kappa z}e^{i(kz-wt)}$$

where

$$\kappa \cong \frac{Nq^2\omega^2}{2m\epsilon_0 c}\sum_i \frac{f_i\gamma_i}{(w_j^2 - w^2)^2 + \gamma^2 w^2}$$

and N is the number density of atoms/molecules, q is the charge of an electron, m is the mass of an electron, w_j is the natural frequency for species j (in most cases, these frequencies correspond to the UV), f_j is the fraction of molecules of species j and γ_j is the damping coefficient. How could this explain a red sunset?

3. Calculate the Poynting vector of a wave propagating in coaxial cable. The electric and magnetic field are given by:

$$\mathbf{E}(s,\phi,z,t) = \frac{A\cos(kz - wt)}{s}\hat{s}, \mathbf{B}(s,\phi,z,t) = \frac{A\cos(kz - wt)}{cs}\hat{\phi}.$$

The conductor in the center of the coaxial cable has a radius a, the radius of the inner edge of the outer conductor is b (see Figure). What is the time averaged flow of energy along the z axis, i.e. the amount of power that passes through the cable?

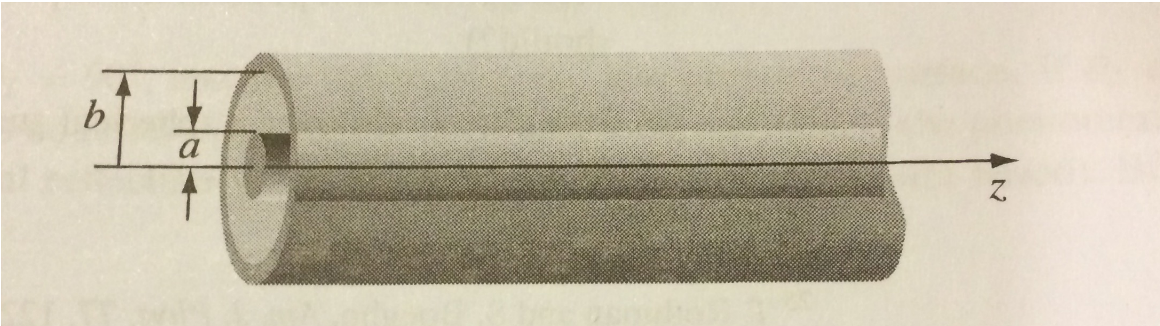


FIGURE 9.26

Quantum Mechanics

1. Consider a particle of mass m in a 1D finite square-well of width a and depth $-V_0$ where $V_0 > 0$, e.g. $V(x) = -V_0$ for $|x| < a/2$ (region 1) and $V(x) = 0$ for $|x| > a/2$ (region 2).

(a) Make a graph of the potential $V(x)$. Label the graph showing region 1 and region 2.

(b) Write down the corresponding 1D Schrodinger equation in region 1 and use it to obtain the form of the most general bound-state solution (in region 1) $\psi_1(x)$ with energy $E < 0$. Identify any constants (e.g. k_1) in your solution.

(c) Write down the corresponding 1D Schrodinger equation in region 2 and use it to obtain the most general, normalizable bound-state eigenfunction solution $\psi_2(x)$ for $x \geq a/2$ with energy $E < 0$.

(d) What are the boundary conditions that $\psi_1(x)$ and $\psi_2(x)$ must satisfy in order to form a valid solution?

(e) Using your result in (b) write down expressions for the (unnormalized) ground-state wavefunction $\psi_{1,g}(x)$ in region 1 and first excited state wavefunction $\psi_{1,exc}(x)$ in region 1.

2. The Hamiltonian for a two-level system is represented by the matrix $H = \hbar\omega \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$

(a) If the system is initially in the state $\psi = \begin{pmatrix} \sqrt{4/9} \\ \sqrt{5/9} \end{pmatrix}$ what is the expectation value of the energy $\langle E \rangle$?

(b) If you measure the energy of this state what values might you get and with what probabilities?

(c) Now consider another observable A represented by the Hermitian matrix $\mathbf{A} = \begin{pmatrix} 0 & 2\gamma i \\ -2\gamma i & 0 \end{pmatrix}$ where γ is real. Find the eigenvalues and (normalized) eigenvectors of $\mathbf{A}$.

(d) Assume that the system is in the state $\psi = \begin{pmatrix} 3/5 \\ 4i/5 \end{pmatrix}$ and the observable A is measured. What possible values of A will be obtained and what is the probability of obtaining each value?

3. Consider a system with two orthonormal basis functions $|1\rangle$ and $|2\rangle$ whose Hamiltonian is given by:

$$H = \epsilon \; \{3 \; |1\rangle\langle 1| + 3 \; |2\rangle\langle 2| + |1\rangle\langle 2| + |2\rangle\langle 1|\}.$$

(a) Write down the Hamiltonian in matrix form.

(b) Determine the eigenvalues E_+ and E_- and corresponding normalized eigenvectors $|e_+\rangle$ and $|e_-\rangle$ of the Hamiltonian.

(c) Assuming that $|\psi(t = 0)\rangle = |1\rangle$ obtain an expression for $|\psi(t)\rangle$.