Classical Mechanics

1. Find the inertia matrix for a uniform solid rectangular parallelopiped which has mass M. One of its vertices is at the origin (0, 0, 0) and its three neighboring vertices are at (a, 0, 0), (0, b, 0) and (0, 0, c).

2. A scientist in a train sets up a simple pendulum of length b from the ceiling of train in which he is travelling. In each of the following cases find the angle the pendulum makes with a vertical line when it is in equilibrium and the period of small oscillations if the pendulum is set in motion. All measurements are conducted on the train. The magnitude of the acceleration due to gravity is g.
(a) The train is moving on a straight track with constant speed v_0.
(b) The train then undergoes a constant acceleration to reach a higher speed v_1 in time t_1.
(c) It then moves at the same speed on a circular portion of radius R of the track.
(d) After the circular motion is competed it moves in a straight line with the same speed.
(e) Finally, it decelerates uniformly for a distance d_1 and returns to its original speed v_0.

3. Figure 1 shows a crude model of a yoyo. A massless string is suspended vertically from a fixed point and the other end is wrapped several times around a uniform cylinder of mass *m* and radius *R.* When the cylinder is released, it moves vertically down, rotating as the string unwinds.

(a) Calculate the moment of inertia of a thin solid cylinder of total mass m and radius R about its axis of symmetry.
(b) Calculate the length x(t) that the string unwinds as a function of time until it is fully unwound.

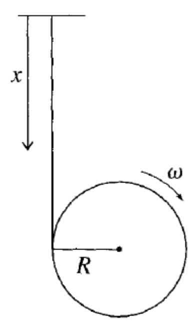

Figure 1.

A useful formula $I_{p,q} = \int \left[\left[\sum_{j=1}^{3} x_j^2\right]\delta_{p,q} - x_p x_q\right] \rho dV$

Quantum Mechanics Section

Problem 1

At time $t = 0$ a particle is represented by the wave function, $\Psi(x, t = 0) = \Psi(x)$ described below:

$$\begin{cases} \Psi(x) = A \cdot (\frac{x}{a}) & 0 \leq x \leq a \\ \Psi(x) = A \cdot (\frac{b-x}{b-a}) & a \leq x \leq b \\ \Psi(x) = 0 & otherwise \end{cases}$$

(a) Normalize the wave function (that is, find A, in terms of a and b).where A, a, and b are (positive) constants.

(b) Sketch $\Psi(x)$, as a function of x.

(c) Where is the particle most likely to be found, at $t = 0$?

(d) What is the probability of finding the particle to the left of a?

(e) What is the expectation value of x, $\langle x \rangle$?

Problem 2

Consider a system whose intial state $|\Psi(t = 0)\rangle$ and Hamiltonian, H, are given by:

$$|\Psi(t = 0)\rangle = \frac{1}{5} \begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix}, \quad H = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 5 \\ 0 & 5 & 0 \end{bmatrix}$$

(a) If a measurement of the energy is carried out, show the possible values obtained are $E_1 = -5$, $E_2 = 3$, $E_3 = 5$.

(b) Show the respective orthonormal eigenvectors from part a for each eigenvalue are:

$$|\Phi_1\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, |\Phi_2\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, |\Phi_3\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

(c) Determine the probabilities to obtain each of the eigenvalues (in part a). One thing to note, if you do not do part b, you can still use the inforamtion in part b to help determine the probabilities requested.

Problem 3

A particle in a 1-D infinite square well of width, a, has as its initial wave function at time $t = 0$ an even mixture of the first two stationary states:

$$\Psi(x, 0) = \Psi(x) = A(\Psi_1(x) + \Psi_2(x))$$

(a) Determine the expression for $\Psi(x)^* \cdot \Psi(x)$.

(b) Normalize $\Psi(x)$ using only the orthonormality properties of Ψ_1 and Ψ_2 where $\langle\Psi_i|\Psi_i\rangle = 1$ and $\langle\Psi_i|\Psi_j\rangle = 0$ where $i \neq j$.

(c) When a measurement of the energy of this particle is made, do not forget to consider the following equation for the energy eigenvalue Schrodinger Equation equation for this square well: $\hat{H}\Psi_n(x) = E_n\Psi_n(x)$ where $n = 1, 2, 3, ...$ With this in mind what values of energy might be obtained in terms of E_n?

(d) What is the probability of measuring each of the values?

(e) Determine the expectation value of the energy, $\langle E\rangle$ in terms of E_n.