INITIATE Lesson Plan: Routes – Investigations for the Smart Car

Lesson plan at a glance...

Name	Routes – Investigations for the Smart Car
Course	Algebra 2, and Conics
Suggested grade	9th to 12th
Prerequisites	Coordinate Geometry
Time	**Preparation:** 5 to 10 minutes **Instruction:** 1 hr 5 mins
Standards	*TPS:* **A.SSE.1:** Interpret expressions that represent a quantity in terms of its context. Students record the equations of the models and sketch graphs of the data points and models. **F.IF.7a, e, A.REI.10:** Graph Linear and showing intercepts. **S.ID.7:** Interpret the slope and intercept in context of the data. **G-CO.1:** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. *CTE:* **5.2.6:** Apply coordinate systems (e.g., absolute, relative, user, cylindrical, cartesian). **5.2.7:** Sketch geometric forms and shapes.

In this lesson plan…

Lesson Overview

In this lesson students will learn some mathematical techniques by which a smart car can navigate itself. They will have an idea how to use mathematical concepts to predict unavailable data. This lesson uses mathematical concepts to find out the coordinates of the most convenient parking spot for a smart car. Also, it trains students to use Bloxter programming for doing mathematical calculations.

Driving Questions

Overarching Driving Question:

- How will autonomous vehicles affect people with disabilities?

Lesson Specific Question:

- What routes need to be planned for these smart buses in Toledo?
    - How can the GoPiGo park do parallel parking?

Materials and Equipment

- For the student:
  - *Required:*
    - A tablet/chromebook
    - Pencil
    - Scratch Paper

Preparation Tasks

	• Check whether all tablets are working, connected to the smart car, and have sufficient battery in them. • To check, whether the smart car is working, try to run the smart car using a single command to make the car move forward for 2 seconds.	5 to 10 minutes

The Lesson

Warm-up Activity: Information Gathering and Brainstorming	10 minutes
Activity: Parking Problem	45 minutes
Wrap-up: Conclusions and Inferences	10 minutes

Warm-up Activity: Information Gathering and Brainstorming (10 minutes)

Activity:
At first a video is shown featuring the automated parking feature of BMW (CARJAMTV, 2017). Real life Smart Car's autonomous valet parking capabilities: https://www.youtube.com/watch?time_continue=29&v=L22S2VGB5Xw

Some questions follow up with them.

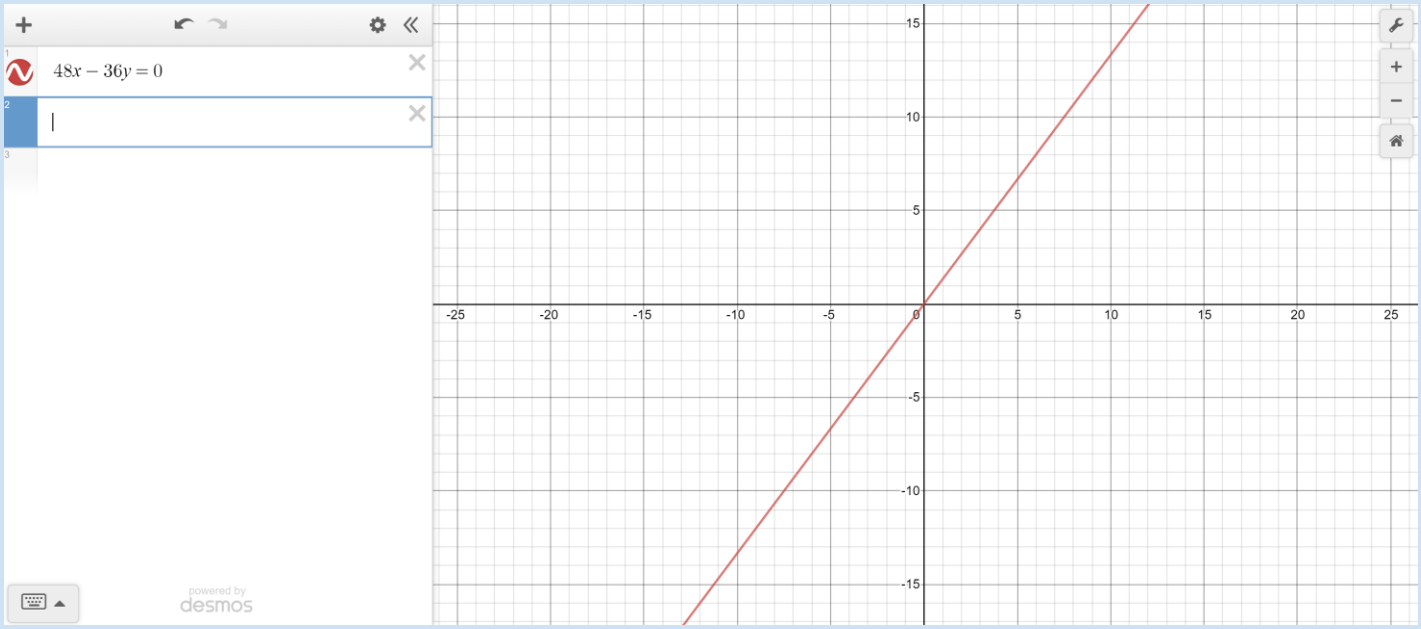
- What did you find interesting in this video?
- How do you think such a technology works?
- Does it involve Math/CTE in anyway?

Question: Imagine your car is at (0,0) facing towards X axis. Now, consider the following tasks:

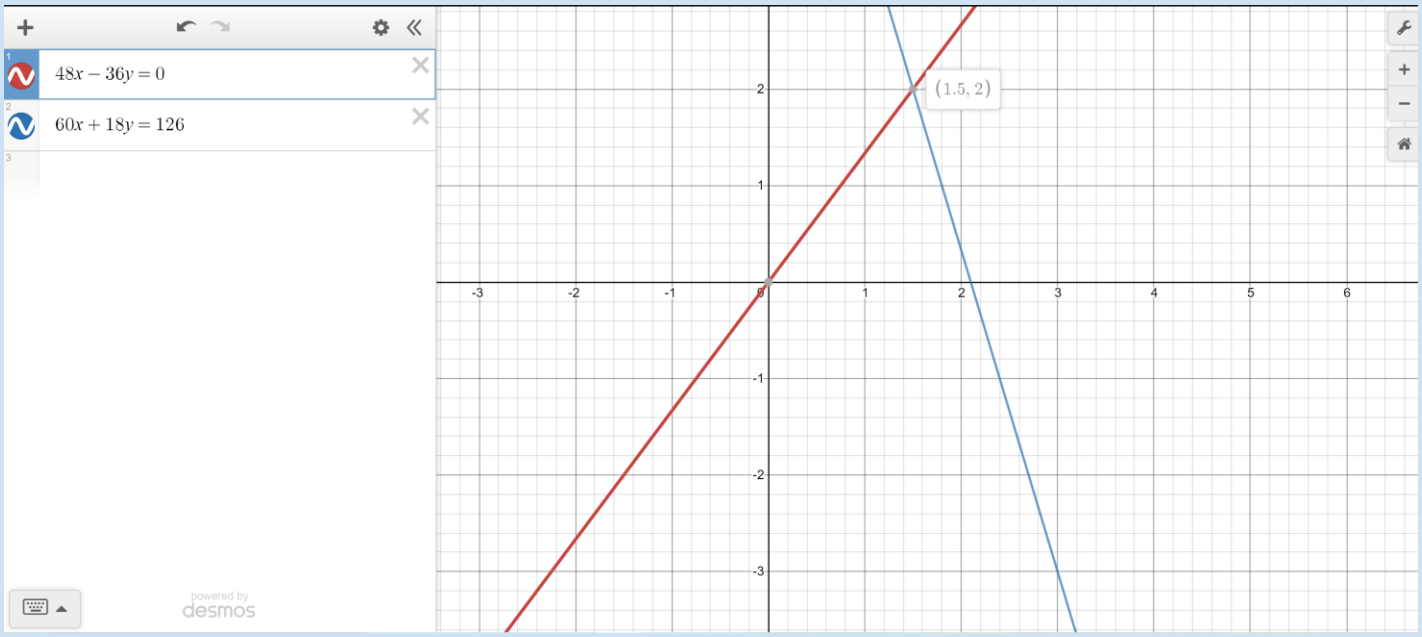
- **Car must run along Path 1 (48x – 36y = 0) till its junction to Path 2 (60x + 18y = 126) arrives.**
- **At the junction, change the car's course to Path 2.**

Solution: Open DESMOS (www.desmos.com) on your Computer/Tablet**:**

- Go to "Start Graphing" option on the screen
- Feed in the equation of Path 1 on the screen.



- On PAPER, Write the equation of Path 1 in slope-intercept form:
  - $y = mx + c$
  - $y = \frac{48}{36}x$
- Find the slope of Path 1 using pen and paper:
  - $m = \frac{48}{36} = \frac{4}{3}$
- Find the angle of turn for the car to follow Path 1 (as car is facing the positive x - axis):
  - $\theta_1 = tan^{-1}\frac{4}{3} = \mathbf{53.13°}$
- Now on DESMOS, write the equation of Path 2:



- Calculate the distance between the origin and point of intersection:
  - $\boldsymbol{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$
  - $\boldsymbol{d = \sqrt{(1.5 - 0)^2 + (2 - 0)^2} = \sqrt{6.25} = 2.5}$

- On PAPER, Write the equation of Path 2 in slope-intercept form:
  - $y = mx + c$
  - $y = -\frac{60}{18}x + 126$
  -
- Find the slope of Path 2 using pen and paper:
  - $m = -\frac{60}{18} = -\frac{10}{3}$
- Find the angle made between the x-axis and the line:
- $\theta_2 = tan^{-1}(-\frac{10}{3}) = 106.72°$
- Let the point on the Origin be O, the point of intersection of Path 1 and 2 be A, and the point of intersection of Path 2 and x-axis be B. Find angle OBA:
  - $\angle OBA = 180 - \theta_2 = 180 - 106.72 = \boldsymbol{73.28}°$
- Now, we know that an exterior angle of a triangle is equal to the sum of opposite internal angles of the triangle. So, find the turning angle of the car from Path 1 to Path 2,
  - $Turning\ Angle = 53.13 + 73.28 = \boldsymbol{126.41}°$

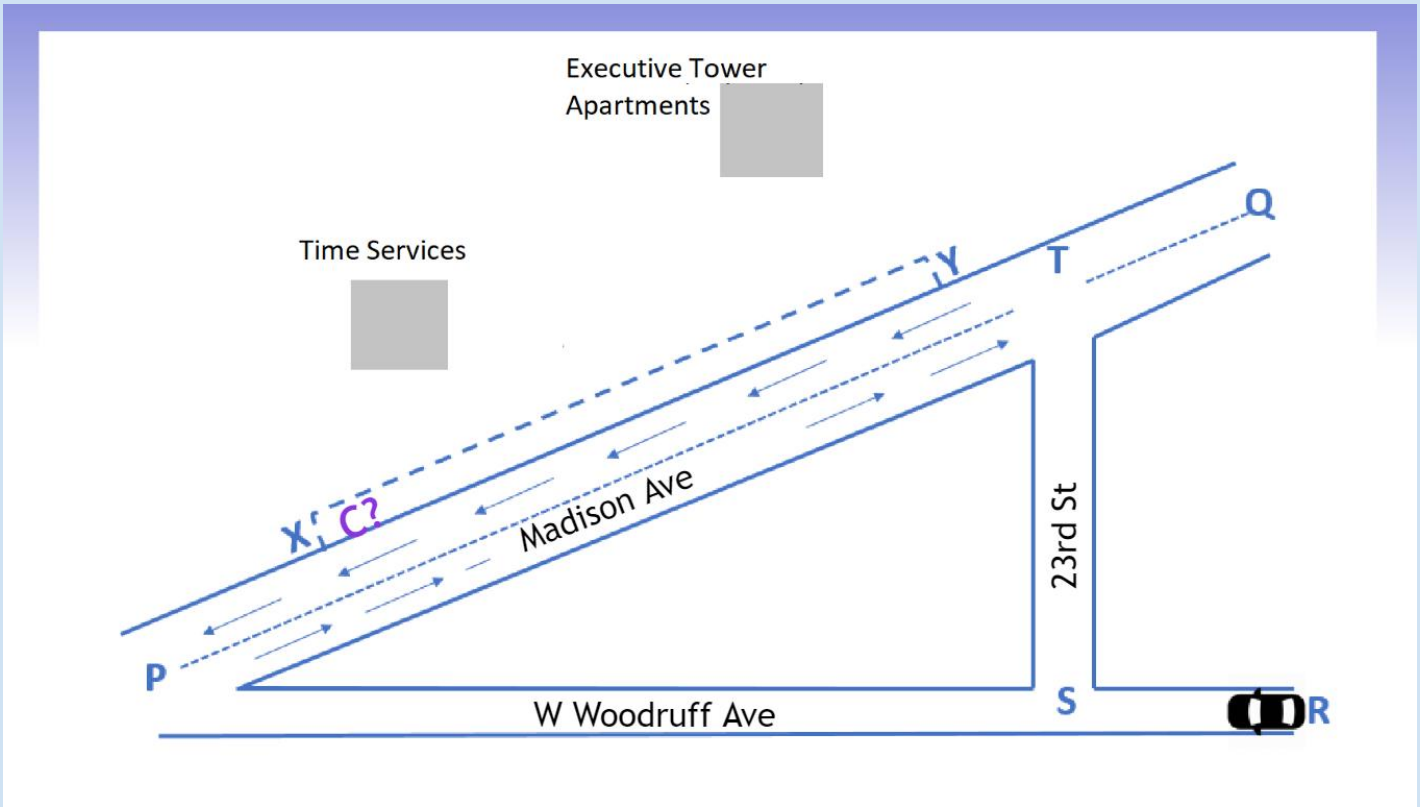
Then another video is shown showing a Tesla doing the same thing. Self-Driving Tesla Drives It's Owner To Work & Then Finds A Parking Spot (TKR Motorsports, 2016): https://www.youtube.com/watch?v=ivTeW4xWQv0&t=115s

Activity 3: Parking Problem (45 minutes)

Activity Overview: A bus needs to drop off two passengers to a common place near two places Executive Tower Apartments and Time Services. The bus must park in the parking zone (XY) located on Madison Avenue. Where should the bus park such that the shortest walking distances for both passengers from the bus to the destinations are the same?

Section 1: Analyzing the Problem and devising an algorithm (10 minutes)

The problem statement will be given to the students. They will carefully analyze the data given in the problem. After analyzing the problem, they will devise the algorithm to solve.



The given Coordinates are:
Time Services (A): (2,3)
Executive Tower Apartments (B): (4,5)
X: (1,1)
Y: (7,3)

Section 2: Calculation on Paper (10 minutes)

After they have devised the algorithm, they will start doing the calculations on paper to get the values which will be used later for programming the smart cars. During this time, the teacher may ask them simultaneously the following questions to give them a hint about the values which must be found:

- How do you find an equation of a line from a given set of coordinates?
- How do you find coordinates of random multiple points located on a line whose equation is given?
- How do you find the midpoint of a line where the coordinates of the end points are given?
- How do you find the coordinate of an intersection of two lines?
- How do you find the distance between two coordinates?

If any students find difficulties with the formulas, the teacher can provide the following formulas:

- The slope of the line segment connecting x_1, y_1 and x_2, y_2 will be $\frac{y_1 - y_2}{x_1 - x_2}$
- If m is the slope of a line, then tan Φ = m, where Φ is the angle of the inclination of the line
- The equation of a line passing through x_1, y_1 and x_2, y_2 is presented by $Y - y_1 = m(X - x_1)$
- The distance between x_1, y_1 and x_2, y_2 is calculated by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- If 'm' is the slope of a line segment AB, the slope of the line perpendicular to AB will be $\frac{1}{-m}$

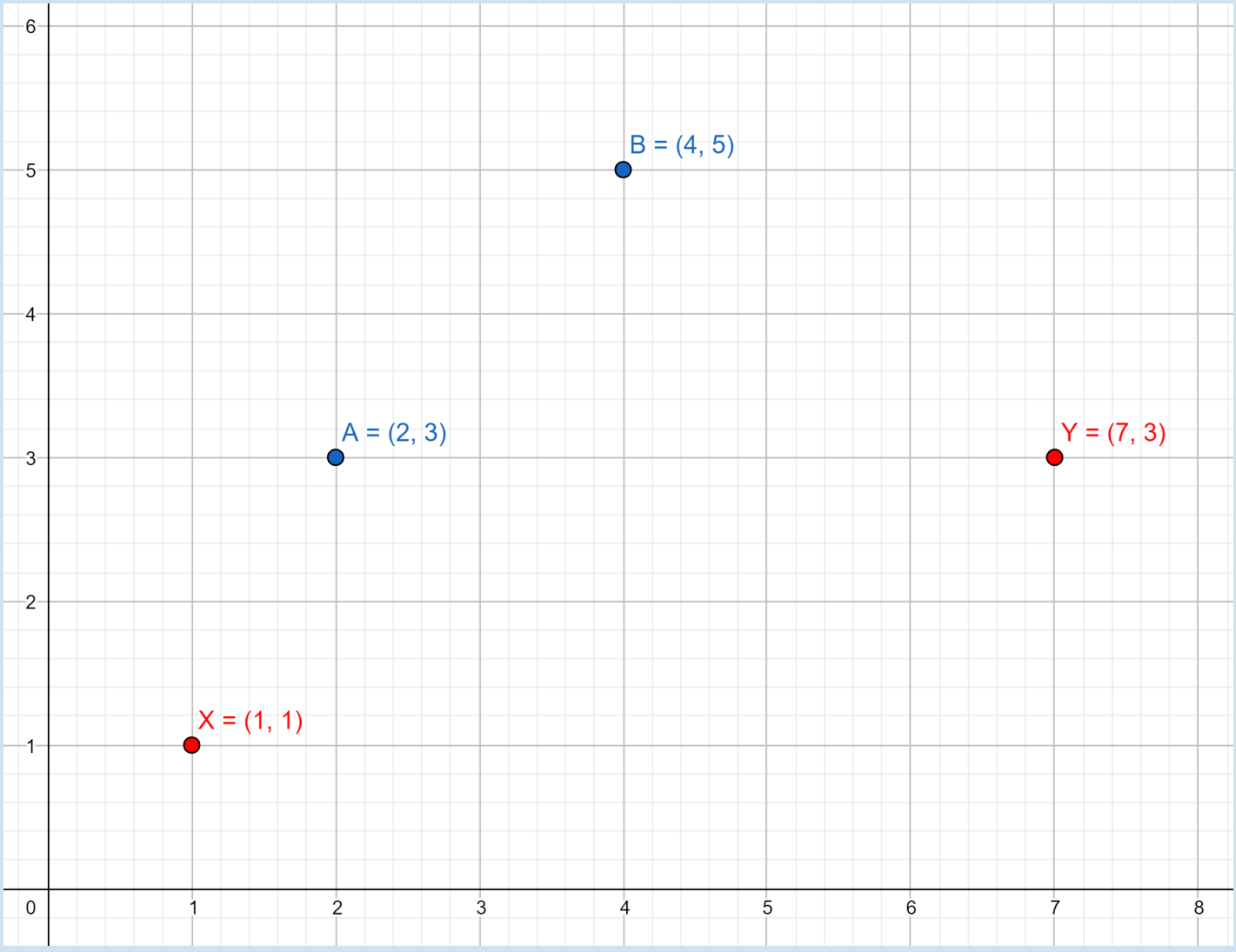
- The X and Y coordinates of the midpoint of a line connecting (x_1, y_1) and (x_2, y_2) are $(x_2 + x_1)/2$ and $(y_2 + y_1)/2$ respectively

Section 3: Sharing with other students (5 minutes)

After doing the calculations on the paper, they will share their answer with another student to check if they have got the same answers or not. In both the cases, they will learn about each other's design of algorithm. If a student feels that the other student's approach is wrong, they will discuss about it and try to come up with a collaborative answer. If they cannot agree with each other even after the discussion, they will wait for the teacher to provide the correct solution with explanation.

Section 4: Comparing with the correct Algorithm and answer (15 minutes)

After all the pairs come up with their answers, they will compare their answers with the presented solution by the teacher. If the answer doesn't match, the teacher will explain each steps of the solution to them. If the answer matches but not the steps of algorithm, the teacher will ask the pair to share and explain their design of algorithm with the others. This way, students will share multiple correct approaches to this problem.

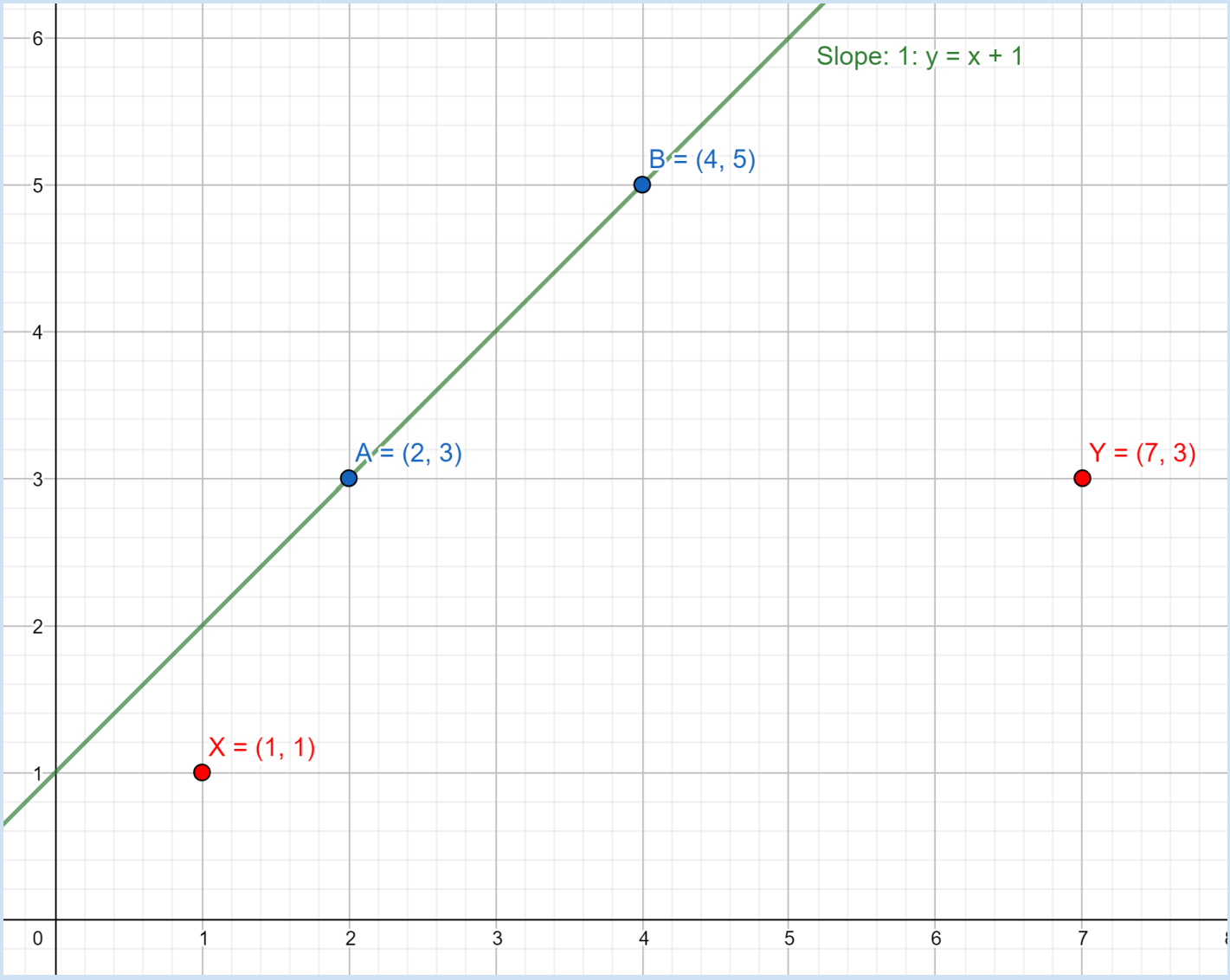


Algorithm: Teacher will explain the following steps of algorithm:

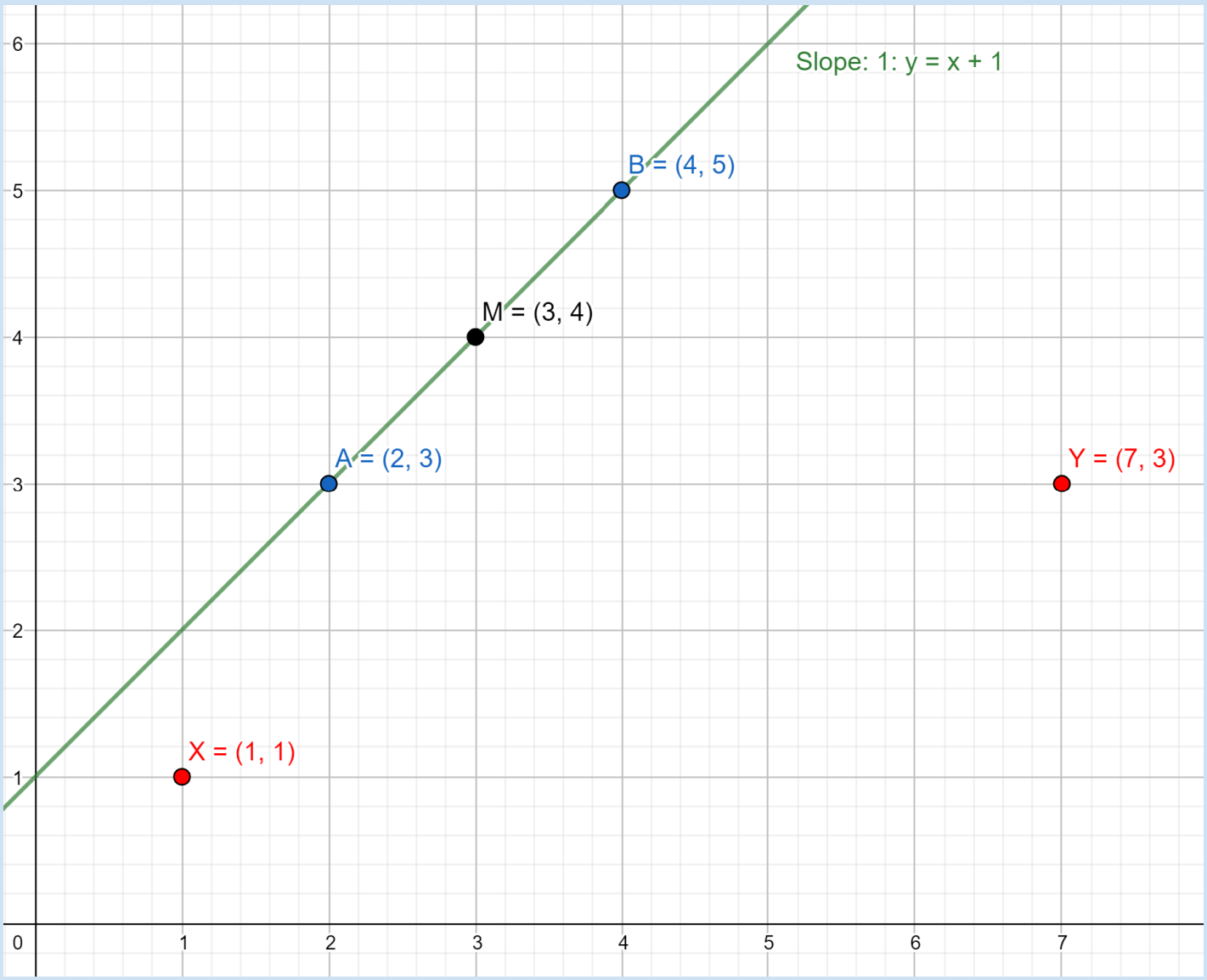
- From the given data, find out the angle of inclination of the line XY on which the car has to park.
- Find out the equation of line XY.
- Find out the slope and equation of line AB.
- Find out the slope of the perpendicular line (EF) passing through the midpoint of AB.
- Find out the Y-intercept of the perpendicular line (EF) passing through the midpoint of AB.
- Find out the equation of the perpendicular line (EF) passing through the midpoint of AB.
- Find out the coordinates of the intersection of EF and XY (C).

Solution: Following steps of the mathematical calculation will be explained by the teacher:

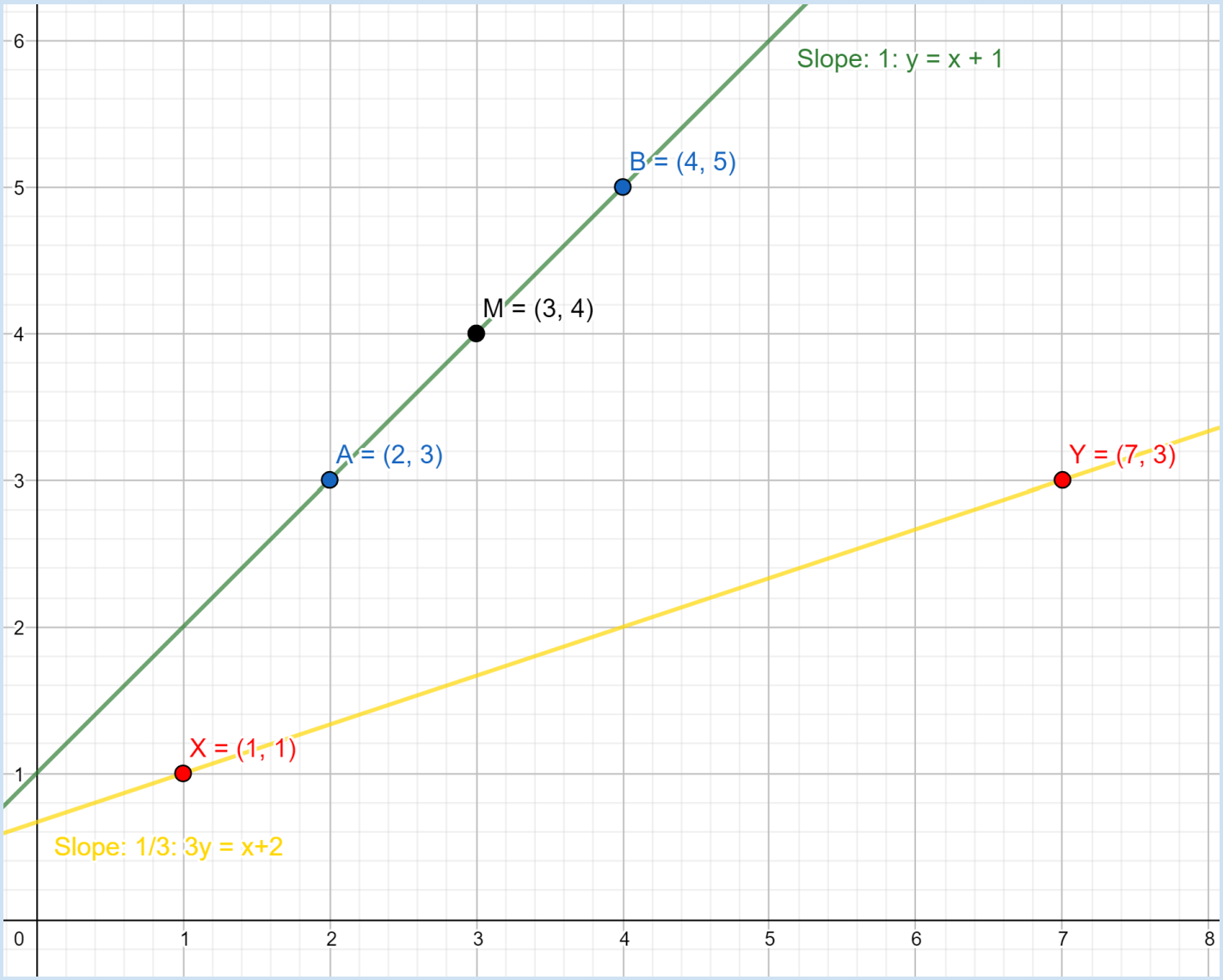
- Slope of AB = $\frac{5-3}{4-2} = \frac{2}{2} = 1$
- Y intercept of AB = $5 - (1 \times 4) = 1$
- Equation of line AB: $y = (1 \times x) + 1 = x + 1$



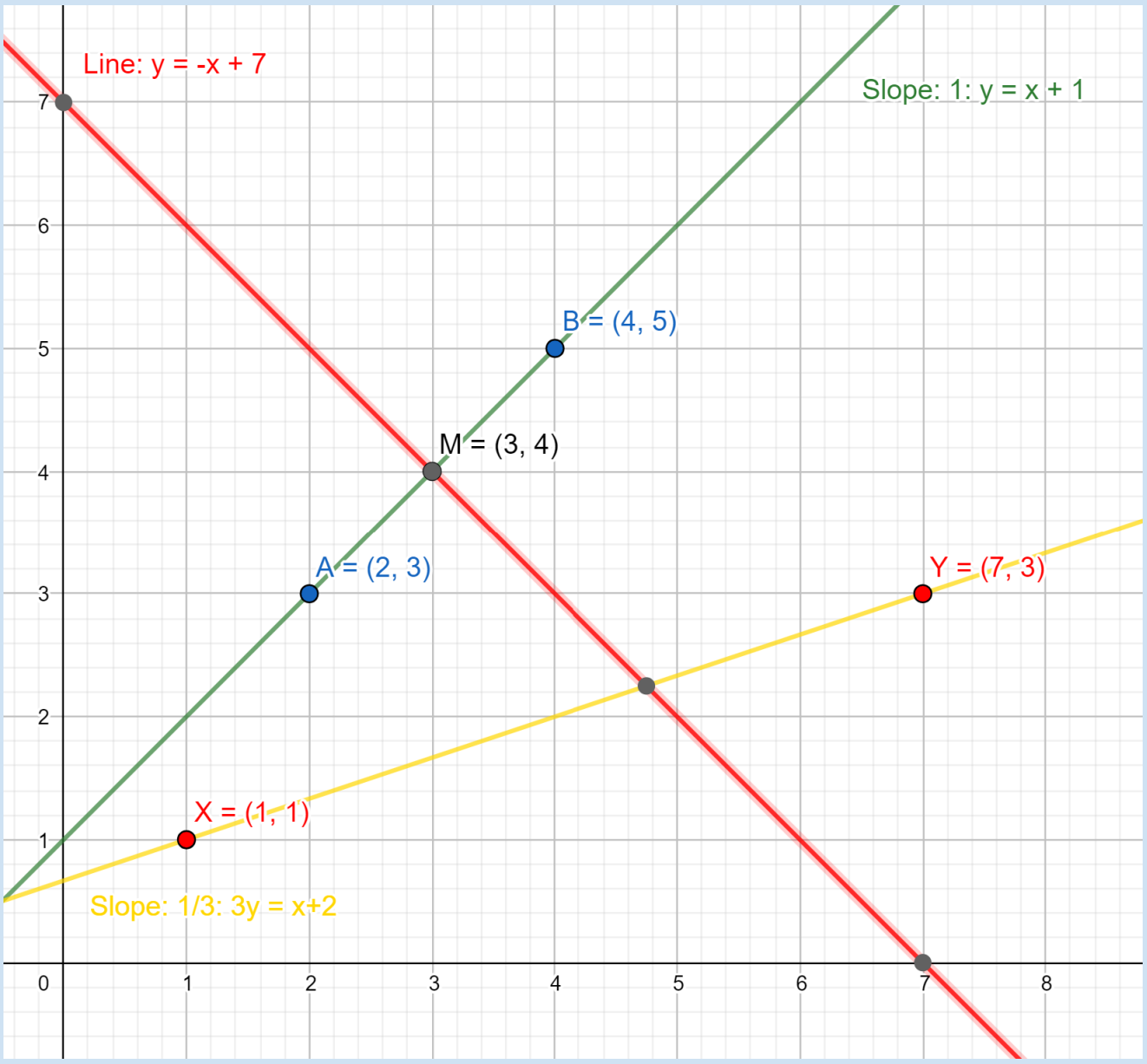
- Coordinate of the midpoint (M) of AB: $\left[\frac{(4+2)}{2}, \frac{(5+3)}{2}\right] = (3,4)$



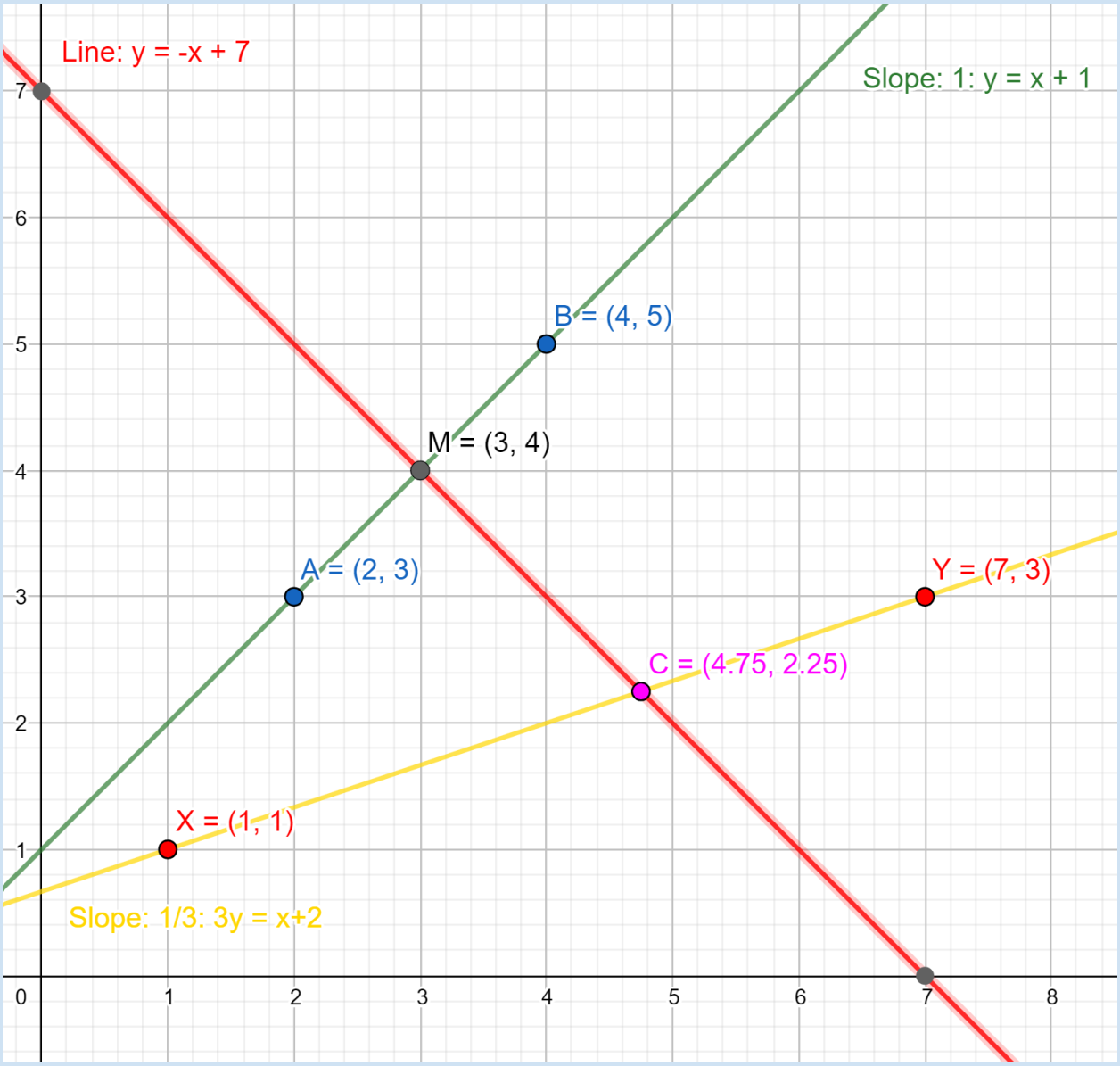
- The slope of the line XY $= \frac{3-1}{7-1} = \frac{1}{3}$
- The Y intercept of line XY $= 1 - \left(\frac{1}{3}\right) * 1 = 1 - \frac{1}{3} = \frac{2}{3}$
- The equation of the line XY: $y = \frac{x}{3} + \frac{2}{3} \rightarrow 3y = x + 2$



- Slope of the line perpendicular to line AB $= [-\left(\frac{1}{1}\right)] = -1$
- The **Y-intercept** of the line perpendicular to AB and passing through M (Line MF): $4 = (-1) * (3) + c \rightarrow c = 4 + 3 \rightarrow c = 7$
- The equation of the line perpendicular to AB: $y = -x + 7$



- The point of intersection of MF and XY, say C, is seen marked on the graph.



The intersection of the line MF and XY is: C (3.25,3.75) [The coordinate of the most optimum parking spot]

Teaching Tips:

- *The teacher should write down one step of the algorithms and do the respective calculations along it.*
- *Even if the students can tell which the better path is by analyzing the figure, the teacher should ask them to do the calculations for all the possible paths.*
- *The teachers can show the live demo on Geogebra* (Hohenwarter, 2016) *while explaining the solution to the class.*

Wrap-up Activity: Analysis and reflection (10 minutes)

Activity Overview: In this activity, students will discuss among themselves and the teachers about what they have learnt and how these new lessons are going to help them.

Teachers will initiate the following topics in which students will do the discussions:

- What were your findings?
- Which components of PBL did you witness here?
- Were there any components of CT here?
- Do you think smart bus will reduce human labor and optimize fuel efficiency while commuting?
- Can you think of a generalized algorithm for the parking problems for a real-life scenario?

Assessment:
Collect students' reflections. Assess for thoughtful, complete responses and experimental understanding. The students' interest in STEM.

Learning Objectives and Standards

Learning Objectives	Standards
LO1: Students will be able to analyze a problem and suggest possible solutions.	*Computer Science* CSTA L1:6.CT.6: Understand the connections between computer science and other fields. CCSS.MATH.PRACTICE.MP1: Make sense of problems and persevere in solving them.
LO2: Students will be able to use and manipulate variables while programming	*Mathematics:* **S.ID.7:** Interpret the slope and intercept in context of the data. **G-CO.1:** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. *CTE:* **5.2.6:** Apply coordinate systems (e.g., absolute, relative, user, cylindrical, cartesian). **5.2.7:** Sketch geometric forms and shapes.
LO3: Students will be able to verbalize a plan (an algorithm) for the whole process.	
LO4: Students will be able to code the smart car to run as per the requirement in the question.	
LO5: Students will be able to see the mathematics behind everyday things.	
LO6: The students should be able to write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	

Additional Information and Resources

Project-based Learning Features

Feature	Where does this occur in the lesson?

Driving Question	The driving question here is what routes need to be planned for the autonomous paratransit buses in Toledo. The students need to do a practical implementation of how the GoPiGo will park in an optimum parking spot.
Investigation & Problem Solving	The students here are investigating the various spots most optimum for parking in a parking space. They are doing this investigation to solve for implementing the routes planned for Toledo on the GoPiGo.
Technology Incorporation	This lesson requires the use of a robot car named GoPiGo (Dexter Industries), and a smart Android Tablet/Chromebook for experimental purposes; and the use of Google Classroom (Google Incorporation, 2018) and other Google products for Evaluation, Cloud Sharing and Online Activities.
Collaborative Opportunities	In all the Activities designed, the people involved in the lesson are working in groups. They share their ideas and knowledge with each other, leading to Collaborative Learning Opportunities.
Assessment techniques	Assessment is done on whether the student was able to find the most optimum parking spot, and which technique he/she applied to get the final result.

Computational Thinking Concepts

Concept	**Where does this occur in the lesson?**
Algorithm Design	In the Activity students are formulating an algorithm to solve a certain math problem.
Decomposition	The Activity requires a smart car to perform certain complex calculations. All these operations are broken down into manageable parts, and each part is dealt one at a time. Only after successfully decomposing the problem to its very end, that we can achieve the desired functionality.

Administrative Details

Contact info:	www.utoledo.edu/research/initiate
Sources:	CARJAMTV. (2017). *Mercedes Self Parking Car Demo Valet Parking Multi Story Car Park Mercedes Autonomous Car CARJAM*. Youtube. https://www.youtube.com/watch?v=L22S2VGB5Xw&t=29s Google Incorporation. (2018). *Google Classroom*. classroom.google.com Hohenwarter, M. (2016). *GeoGebra*. www.geogebra.com TKR Motorsports. (2016). *Self Driving Tesla Drives It's Owner To Work & Then Finds A Parking Spot*. https://www.youtube.com/watch?v=ivTeW4xWQv0&t=115s
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