University of Toledo, Department of Physics and Astronomy

Ph.D. Qualifying Exam

Fall 2007
September 08

Instructions:

- **Do not** write your name on your exam; put your chosen letter on every sheet of paper that you turn in.
- Work 2 out of 3 problems in each category.
- Begin each problem on a new sheet of paper.
- Be sure to state which problems are omitted.
MECHANICS

1. A particle is sitting on top of a smooth, frictionless sphere of radius \( a \). The particle is disturbed slightly (see Figure 1).

![Figure 1]

a) What is the kinetic energy \( T \) and potential energy \( V \)? Specify your coordinate system.

b) Determine the speed of the particle as it slides down the sphere.

c) At what point will the particle leave the sphere?

2. An ant is crawling on a merry-go-round, which has a constant angular velocity \( \omega \) (see Figure 2). The ant is crawling outward along the radial direction at a constant speed \( v \).

![Figure 2. View from above the merry-go-round. Gravity points into the page.]

a. Identify all forces that act on the ant.

b. Write down the equation of motion in terms of the ant’s (moving) coordinates.

c. How far can the ant crawl before it begins to slip, given the coefficient of friction \( m \) between the ant and the merry-go-round.

3. Consider a system where a mass, \( m_1 \), is connected by a light inextensible cord of length \( l \), which passes over a pulley of radius \( R \). Attached to this cord is a second simple pulley of radius \( R \), which supports two weights, \( m_2 \) and \( m_3 \), connected by another cord also of length \( l \).

a. How many degrees of freedom does the system have? State your choice of generalized coordinates.

b. Write down an expression for the kinetic energy \( T \).

c. Write an expression for the potential energy \( V \).

d. What is the Lagrangian?

e. Use Lagrange’s equation to find the equations of motion for the system.
ELECTRICITY & MAGNETISM

1. A concentric pair of spherical conducting shells of radii \( a \) and \( b \) \((a < b)\) are connected across a battery of voltage \( V \). The larger sphere is connected to the negative terminal of the battery and is grounded; the smaller sphere is connected to the positive terminal. The cavity between the shells \((a < r < b)\) is evacuated, and an electron (charge \( q = -e \)) is placed at rest halfway between the spheres at 
\[
r_0 = \frac{(a + b)}{2}.
\]
   a. Find the electric field \( E \) inside the cavity in terms of the potential difference \( V \).
   b. Toward which sphere will the electron move? This sphere is the target.
   c. Obtain an expression for the speed \( v \) of the electron when it hits the target sphere.

2. A non-conducting spherical shell of radius \( b \) has a potential
\[
F = V \cos^2 \theta
\]
maintained on its surface. A second solid conducting sphere of radius \( a \) is placed at its center and grounded.
   a. Find the electrostatic potential \( \Phi \) between the spheres \((a < r < b)\).
   b. What is the electric field \( E \) between the spheres?
   c. Find the total charge \( Q \) on the inner sphere.

3. Two identical concentric circular current loops of radius \( a \) are initially placed in the \( x-y \) plane. Each loop carries a current \( I \) in the \( \hat{f} \) direction. One loop then is rotated about the \( x \)-axis by + 45°, while the other is rotated by - 45°, so that the two loops now are perpendicular with their intersections at \( x = \pm a \).
   a. What is the total magnetic moment \( \mathbf{m} \) (magnitude and direction) of the system? Hint: consider the moments of each loop individually.
   b. What is the magnetic vector potential \( \mathbf{A} \) at large distances \((r > a)\)?
   c. What is the magnetic field \( \mathbf{B} \) at large distances?
2. A particle in a box \([V(x)=0, \, 0 \leq x \leq a; \, V=\infty, \, x<0 \text{ or } x>a]\) is prepared in a state
\[
\psi(x) = \begin{cases} 
\sqrt{\frac{1}{a}} \sin \left( \frac{2\pi}{a} x \right) & 0 \leq x \leq a/2, \\
0 & x < 0; x > a/2 
\end{cases}
\]
[nonzero only over the left half of the box.]

a) If a measurement of energy is conducted, what are the possible outcomes?
b) What are probabilities for finding the particle in the ground (lowest energy) state and the first excited state?

4. The wave function
\[
y = A [2e^{3if} - e^{-if}] 
\]
describes a plane rotator of moment of inertia \(I\) (\(\phi\) is the angle of rotation). An experiment is conducted to determine the angular momentum \(L\) of the rotator.

a. Calculate the coefficient \(A\).
b. What possible values can one find for the measured angular momentum?
c. What are the probabilities of finding these values?
d. What values for the energy of rotation can one find?
e. What is the probability of finding the rotator within 0.1 radians of the position \(\phi = \pi/2\) ?

5. For the electron in the ground state of the lithium ion \(\text{Li}^{2+}\),

a. Prove that the wave function has the form
\[
\psi = A \exp(-r/a), \, A,a = \text{const}
\]
b. Calculate \(A\) and \(a\).
c. Prove that the mean value of \(1/r\) is \(1/a\).
d. Calculate the average kinetic energy.