University of Toledo, Department of Physics and Astronomy

Qualifying Exam

Spring 2005 (Sat 22 January 9 am - 1 pm, MH4009)

Work 2 problems in each category. Be sure to state which problems are omitted.
MECHANICS

1. Compute the motion of a particle of mass m which experiences a force varying sinusoidally with time. Suppose the force is given by $F = F_0 \sin(\omega t)$ and is applied initially at $t=0$ to the particle at rest at the origin. (This could happen to an ion formed in a gas subjected to an oscillating electric field.) Determine the speed and the position of the particle as a function of time.

2. Consider a narrow hole bored through a uniform sphere of mass M and radius R. Show that the force on a particle of mass m at a distance r from the center of the sphere ($r<R$) is given by $F = -\frac{G M m r}{R^3}$. Assume the Earth can be represented as a uniform sphere with such a hole. If the object is released at the top of the hole, compute its velocity when it reaches the center of the Earth. (Given: $M(\text{Earth}) = 6 \times 10^{24}$ kg; $R=6378$ km)

3. When a mass of 5 kg rests on the top of a vertical spring originally 1 m long, the top of the spring is depressed 10 cm. If the mass is now pushed down an additional 20 cm and released, compute how far above the fixed lower end of the spring the mass rises.
1. A point charge \( q \) is placed a small distance \( d \) above a conducting surface.
   a. Find the electric field \( \mathbf{E} \) as a function of position.
   b. What is the force on the charge?
   c. How much work must be performed to remove the charge to infinite distance?
   d. Estimate the work function for the photoelectric effect; i.e., calculate the energy required for a photon to remove an electron from the conducting surface. Give your answer in eV. Hint: what is an appropriate initial value to use for \( d \)?

   Note: \( ke^2 = 14.4 \text{ eV } Å \), where \( k = 1 \) (cgs) or \( k = 1/4\pi\varepsilon_0 \) (MKS).

2. An infinitely long, conducting cylinder of radius \( a \) is oriented along the z-axis and placed in an (initially) uniform external electric field \( \mathbf{E} = E_0 \hat{x} \) (i.e., the axis of the cylinder is perpendicular to the electric field).
   a. Argue that the electrostatic potential \( \Phi \) is independent of \( z \).
   b. Starting from Poisson’s equation in two-dimensional polar coordinates,

   \[
   \nabla^2 \Phi = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \Phi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta^2} = 0,
   \]

   show that the general solution for the electrostatic potential is

   \[
   \Phi = A_0 + B_0 \ln \left( \frac{r}{a} \right) + \sum_{n=1}^{\infty} \left[ A_n \left( \frac{a}{r} \right)^n + B_n \left( \frac{a}{r} \right)^n \right] \left[ C_n \cos(n\theta) + D_n \sin(n\theta) \right].
   \]

   c. Find the electrostatic potential \( \Phi \) and electric field \( \mathbf{E} \) both inside and outside the cylinder. Assume the cylinder is grounded and uncharged.

3. A metallic sphere of radius \( a \) with magnetic permeability \( \mu \) is placed in an initially uniform magnetic field \( \mathbf{H} = H_0 \hat{z} \).
   a. Find the magnetic fields \( \mathbf{B} \) and \( \mathbf{H} \) as a function of position both inside and outside the sphere. Hint: There is no free current (\( \mathbf{J} = 0 \)), so you can use a magnetic scalar potential \( H = -\nabla \Phi_M \).
   b. Determine the magnetization \( \mathbf{M} \) as a function of position.
   c. What is the induced magnetic dipole moment \( \mathbf{m} \) of the sphere?
1. For a particle of spin \( s \) and angular momentum \( \mathbf{l} \), what are the eigenvalues of the operator \( \mathbf{s} \cdot \mathbf{l} \) for the case of \( l=1 \) and \( s=1 \).

2. For the electron in the ground state of the lithium ion \( \text{Li}^{2+} \),
   a. Prove that the wave function has the form
   \[ \psi = A \exp(-r/a), \quad A, a = \text{const} \]
   b. Calculate \( A \), \( a \), and the average potential energy of the electron.
   c. Prove that the mean value of \( 1/r \) is \( 1/a \).
   d. Calculate the average kinetic energy.

3. In a state described by the wave function of the form
\[
\psi = C \exp \left[ \frac{ip_0 x}{\hbar} - \frac{(x-x_0)^2}{2a^2} \right], \quad C, p_0, a = \text{const}
\]
   a. Find the coordinate probability distribution.
   b. Determine the mean values and the fluctuations of the coordinate and momentum.