University of Toledo, Department of Physics and Astronomy

Ph.D. Qualifying Exam

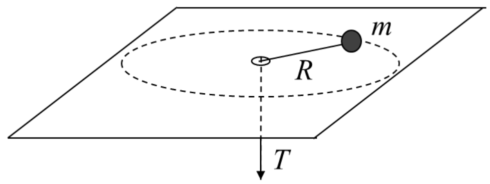
Fall 2015
September 19

Instructions:

- **Do not** write your name on your exam; put your chosen letter on every sheet of paper that you turn in.
- Work 2 out of 3 problems in each category.
- Begin each problem on a new sheet of paper.
- Be sure to state which problems are omitted.

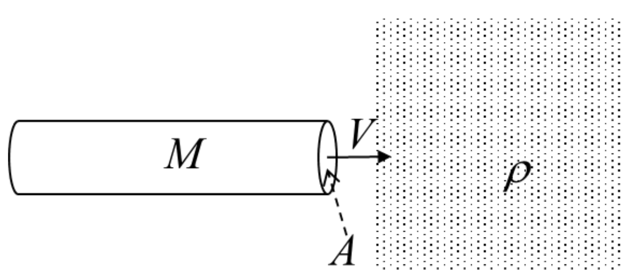
CLASSICAL MECHANICS:

1. A mass *m* moves in a circle on a smooth horizontal table with velocity *v* at a radius *R*. The mass is attached to a string that passes through a smooth hole in the table as shown in the figure.
   (a) What is tension T in the string?
   (b) What is the angular momentum of the mass?
   (c) What is the kinetic energy of the mass?
   (d) The tension in the string is increased gradually and finally m moves in a circle of radius *R/2*. What is the final value of the kinetic energy?



2. A meteorite of mass 1600 kg moves about the earth in a circular orbit in an altitude 4200 km above the surface. It suddenly makes a head-on collision with another meteorite that is much lighter, and loses 2% of its kinetic energy without changing its direction or its total mass.
   (a) What physics principles apply to the motion of the heavy meteorite after its collision?
   (b) Describe the shape of the meteorite orbit after its collision.
   (c) Find the meteorite distance of closest approach to the earth after the collision.

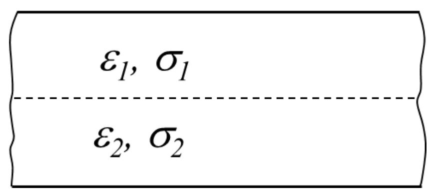
3. Suppose a spacecraft of mass M and cross-sectional area A is coasting with velocity V when it encounters a stationary dust cloud of density ρ as shown in the figure. If the dust sticks to the spacecraft, solve for the subsequent motion of the spacecraft. Assume A is constant over time.



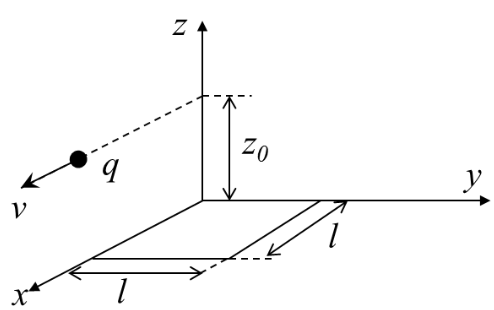
ELECTRICITY AND MAGNETISM

1. An isolated soap bubble of radius 1 cm in a gas with dielectric permittivity ε is at a potential 100 V. If it collapses to a drop of radius 1 mm, what is the change in its electrostatic energy?

2. A space between two infinite flat metal electrodes at distance d is filled with two materials occupying its constituting half-spaces as shown in the figure. The dielectric permittivity and conductivity of these materials are respectively ε_1, ε_2 and σ_1, σ_2. Assume that voltage V is applied between the electrodes and remains constant after that.
   (a) What will be the electric current densities near the electrodes immediately upon the voltage application?
   (b) What will be the electric current densities near the electrodes long time after the voltage application?
   (c) What will be the steady state surface charge density at the materials' interface?



3. Consider a square loop of wire, of side length l lying in the x,y plane as shown in the figure. Suppose a particle of charge q is moving at a constant velocity v, where $v<<c$, at the x,z plane at a constant distance z_0 from the x,y plane. (Assume the particle is moving in the positive x direction.) Suppose the particle crosses the z-axis at t=0. Give the induced electromotive force in the loop as a function of time.



QUANTUM MECHANICS

1. A free particle of mass m moves in one dimension. At time t=0, its wave function is

$$\psi(x,0) = A\exp(-x^2/4a^2)$$

where a and A are two constants.

(a) What is the relation between A and a?

(b) Compute the momentum spread $b = \sqrt{\langle p^2 \rangle - \langle p \rangle^2}$

(c) Show that at time t the coordinate spread is given by $a^2 + b^2 t^2 / m^2$

(d) Interpret the results of parts (b) and (c) in terms of the uncertainty principle.

2. At time t=0 the wave function for hydrogen atom is

$$\psi(\mathbf{r}, 0) = \frac{1}{\sqrt{10}}(2\psi_{100} + \psi_{210} + \sqrt{2}\psi_{211} + \sqrt{3}\psi_{21-1}),$$

where the subscripts are values of the quantum numbers of n,l, and m. Ignore spin and radiative transitions.

(a) What are the expectation values for the energy of that system?

(b) What is the probability of finding the system with l=1 and m=1 as a function of time?

(c) What is the probability of finding the electron within 10^{-10} cm of the proton at t=0?

(d) How does the wave function evolve in time, i. e. what is $\psi(r,t)$?

3. (a) What are the energies and energy eigenfunctions for a nonrelativistic particle of mass m moving on a ring of radius R shown in Figure A

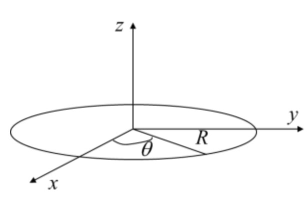


Figure A

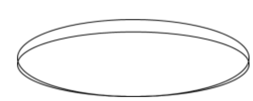


Figure B

(b) What are the energies and energy eigenfunctions if the ring is doubled (each loop still has radius R) as shown in Figure B?