University of Toledo, Department of Physics and Astronomy

Ph.D. Qualifying Exam

Fall 2016
September 24

Instructions:

- **Do not** write your name on your exam; put your chosen letter on every sheet of paper that you turn in.
- Work 2 out of 3 problems in each category.
- Begin each problem on a new sheet of paper.
- Be sure to state which problems are omitted.

CLASSICAL MECHANICS

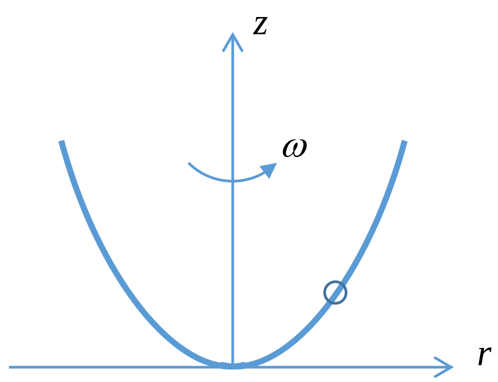
1) On a certain planet, which is perfectly spherically symmetric, the free-fall acceleration has magnitude $g = g_0$ at the North Pole, and $g = \lambda g_0$ at the equator (with $0 \leq \lambda \leq 1$). (a) Prove the general expression for the magnitude of free-fall acceleration $g(\theta)$ as a function of θ can be written as

$$g(\theta) = g_0\sqrt{cos^2\theta + \lambda^2 sin^2\theta}$$

where θ is is the colatitude or polar angle; (b) Determine the constant λ. It can be assumed that the planet is rotating with angular frequency of ω along its North-South axis, and the radius of the planet is R.

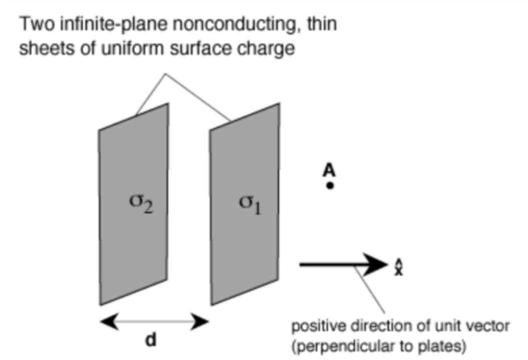
2) A uniform block of mass *m* and dimensions *a* by 2*a* by 3*a* spins about a long diagonal (the body diagonal) with angular velocity ω. Using a coordinates with origin at the center of the block, (a) Find the kinetic energy of the block; (b) Find the angle between the angular velocity vector and the angular momentum vector about the origin.

3) Consider a bead of mass *m* sliding without friction on a wire that is bent in the shape of a parabola and is being spun with constant angular velocity of ω about its vertical axis. Using cylindrical polar coordinates (r, θ, z) and let the equation of the parabola be $z = kr^2$. Use Lagrangian approach to find the equation of motion of the bead and determine whether there are positions of equilibrium, that is, values of r at which the bead can remain fixed, without sliding up or down the spinning wire. Discuss the stability of any equilibrium positions you find.



ELECTRICITY AND MAGNETISM

1. Consider a capacitor consisting of two plates with a positive charge surface density, σ, on the upper plate and negative charge, -σ, on the lower plate. (i.e. in figure $\sigma_1 = +\sigma$ and $\sigma_2 = -\sigma$)



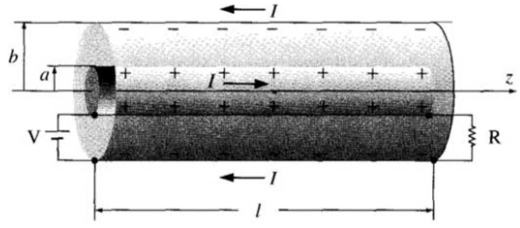
a. Calculate the vector quantities **E** and **D** between the plates in terms of σ and ε_0.
b. Now fill the space between the two plates with water. The dielectric constant of water is 80. Using the solution from part a, what is the electric field in terms of σ, ε_0 and the dielectric constant of water?
c. What is the polarization vector **P** in the water?
d. By what factor is the capacitance changed when we add water to the capacitor?

2. An electron is moving a through a magnetic field **B** = B_0 **z**. It starts with velocity **v** = v_x **x** + v_y **z**. (note **x, y** and **z** are unit vectors in the x, y and z directions).

a. Write an equation for the helical path **X**(t) = x(t) **x** + y(t) **y** + z(t) **z** of the electron in terms of its initial velocity, charge and mass. Assume x(0) = y(0) = z(0) = 0.
b. What is the cyclotron frequency of the electron?
c. How would the path and frequency change if the particle where a proton?

3. Consider an electromagnetic wave moving through a coaxial cable (see figure). Adopt a cylindrical coordinate system where the **z** axis is aligned with the cable**, s** is the radius from the center of the cable, and **φ** is the angular coordinate. Assume the cable has length *L*, that the inner conductor has a radius *s=a* and the outer conductor, *s=b*. A voltage *V* is put between the inner and outer conductor on one side and a resistor with resistance *R* is on the other. The line density of charge is *λ* and the current is *I*. The electric and magnetic fields are:

$$\mathbf{E}(s,\phi,z) = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{s}\hat{s}, \mathbf{B}(s,\phi,z) = \frac{\mu_0}{2\pi}\frac{I}{s}\hat{\phi}.$$



a. Calculate the Poynting vector **S**(z,s,φ) between the conductors.

b. Calculate the energy U(z,s,φ) in the electromagnetic field between the conductors.

From the **E** and **B** fields, calculate the energy flow down the cable (i.e. the energy reaching the end of the cable per unit time). Show that the power is equal to *P = I V*.

QUANTUM MECHANICS

1. A hydrogen atom is in an excited 3d state.
(a) Including spin but ignoring spin-orbit coupling, what are the possible values of *all of the quantum numbers* associated with this state? What is the degeneracy D of this state?
(b) What is the value of the square of its orbital angular momentum L^2 ?
(c) The atom is placed in a very large magnetic field B. Again including spin but ignoring spin-orbit coupling – which is valid for sufficiently large field (Paschen-Back effect) – write down a *general expression* for the energy shift ΔE due to the magnetic field as a function of the magnetic field B, Bohr magneton μ_B. and the relevant (general) quantum numbers describing the 3d state. List the possible values of ΔE. What is the number N_d of distinct shifted energy levels ?
(d) Now consider the transitions from the 3d state to the 2p state. Note that ignoring spin-orbit coupling, and in the *absence* of a magnetic field there is only one spectral line. However, in the presence of a magnetic field, this line will split into several spectral lines. How many spectral lines will be observed in the presence of a very large magnetic field B?

2. An electron in the hydrogen atom has a spatial wavefunction $\psi(\mathbf{r},t) = (\psi_{433} + \psi_{322})/2^{1/2}$ where the indices correspond to *n, l, m*.
(a) What is the expectation value of the z-component of its orbital angular momentum $\langle L_z \rangle$?
(b) What is the expectation value of the square of its orbital angular momentum $\langle L^2 \rangle$?
(c) Its spin wavefunction is given by $\chi = (|+\rangle - 2\,|-\rangle)/5^{1/2}$ where $|+\rangle$ corresponds to spin-up and $|-\rangle$ corresponds to spin-down. What is the expectation value of the square of its spin angular momentum $\langle S^2 \rangle$?
(d) What is the expectation value of the z-component of its spin angular momentum $\langle S_z \rangle$?

3. Consider a one-dimensional potential - corresponding to a potential step at $x = 0$ - of the form $V(x) = 0$ for $-\infty < x \leq 0$ and $V(x) = -V_0$ (with $V_0 > 0$) for $x > 0$. A free particle with mass *m* and energy $E > 0$ approaches the potential step from the left ($x < 0$).
(a) Derive an expression for the reflection probability $R(\alpha)$ as a function of the parameter $\alpha = V_0/E$.
(b) What is the reflection probability $R(0)$ corresponding to $\alpha = 0$? What is the reflection probability $R(\infty)$ corresponding to $\alpha = \infty$?