University of Toledo, Department of Physics and Astronomy

Qualifying Exam

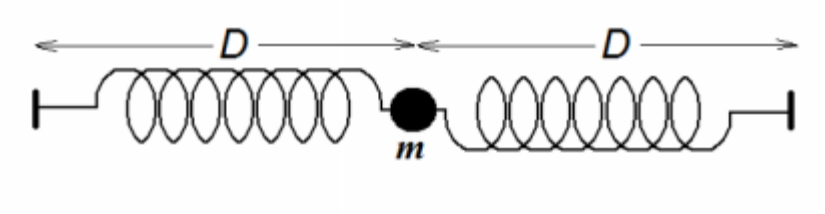
Spring 2016 (February 08)

Work 2 out of 3 in each category. Be sure to state which problems are omitted.

Mechanics

1. A test particle of mass *m* orbits a much more massive object (*M*) in a circular fashion a distance *r* away.
    a) Derive the potential associated with the gravitational force;
    b) Similarly, derive the potential associated with the 'fictional' centrifugal force in terms of angular momentum *L*
    c) Sketch the two potentials as a plot of energy versus *r* as well as the resulting effective potential;
    d) Describe the possible motions for a test particles with energy *E*;
    e) What quantum mechanical system does this central potential resemble?

2. Two identical, massless springs of spring constant k and equilibrium length l are joined together and stretched between two fixed points separated by a distance 2D where D > l. A mass m is attached to the point where the springs are joined. Find the frequencies of small oscillation for the system, neglecting the effects of gravity. Note that the mass, m, is free to move in three dimensions.

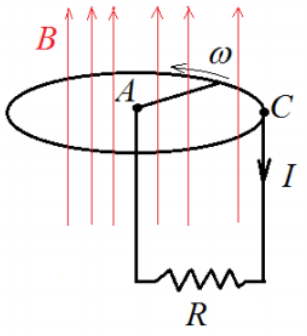


3. Consider a bucket of radius R that is hanging from a rope. The rope is twisted so that the bucket is spinning with a constant angular velocity ω about the rope, which defines a vertical axis through the center of the bucket. Write an equation which describes the shape of the surface of the water in the bucket as a function of r, the distance from the center of the bucket.

Electromagnetism

1. A parallel plate capacitor of area *A* is partially filled with a dielectric layer of permittivity ε. The separation between the plates is *d*, and the dielectric layer has thickness *h*, where $h < d$. The capacitor has charge +Q and –Q on the top and bottom plates, respectively. Assume that the plate separation is small.
    a. Find the electric fields **D** and **E** inside the capacitor as a function of position.
    b. Assuming the bottom plate is grounded, what is the electrostatic potential Φ as a function of position?
    c. What is capacitance *C* of this system?
    d. Determine the forces acting on each plate and on the dielectric layer.

2. One end of a conducting rod rotates with angular velocity ω in a circle of radius r making contact with a horizontal, conducting ring of the same radius. The other end of the rod is fixed. Stationary conducting wires connect the fixed end of the rod (A) and a fixed point on the ring (C) to either end of a resistance R. A uniform vertical magnetic field B~ passes through the ring. (a) Find the current I flowing through the resistor. (b) What is sign of the current if positive I corresponds to flow in the direction of the arrow in the figure? (c) What torque must be applied to the rod to maintain its rotation at the constant angular rate ω?



3. A hollow cylinder of length *L* and radius *a* has a uniform charge density σ on its surface. The cylinder rotates with angular frequency ω about its axis (the *z*-axis).
    a. What is the magnetic field, **B**, as a function of position inside the cylinder? You may ignore end effects; *i.e.*, assume a location far from the ends and that $a << L$.
    b. What is the magnetic dipole moment **m** of the cylinder?
    c. Find the magnetic field **B** at large distances, $r >> L$.

Quantum Mechanics

1. A particle in a box [$V(x)=0,\ 0\le x\le a;\ V=\infty,\ x<0$ *or* $x>a$] is prepared in a state

$$\psi(x)=\begin{cases}\sqrt{\frac{4}{a}}\sin\left(\frac{2\pi}{a}x\right) & 0\le x\le a/2\\ 0 & x<0; x>a/2\end{cases},$$

[nonzero only over the left half of the box.]

   a) If a measurement of energy is conducted, what are the possible outcomes?
   b) What are probabilities for finding the particle in the ground (lowest energy) state and the first excited state?

2. The wave function

$$\psi = A[2e^{3i\phi}-e^{-i\phi}]$$

describes a plane rotator of moment of inertia I (ϕ is the angle of rotation). An experiment is conducted to determine the angular momentum L of the rotator.

   a) Calculate the coefficient A.
   b) What possible values can one find for the measured angular momentum?
   c) What are the probabilities of finding these values?
   d) What values for the energy of rotation can one find?
   e) What is the probability of finding the rotator within 0.1 radians of the position $\phi = \pi/2$?

3. For the electron in the ground state of the lithium ion Li^{2+},
   a. Prove that the wave function has the form
   $$\psi = A\exp(-r/a),\quad A, a = const$$
   b. Calculate A and a.
   c. Prove that the mean value of $1/r$ is $1/a$.
   d. Calculate the average kinetic energy.