University of Toledo, Department of Physics and Astronomy

Qualifying Exam

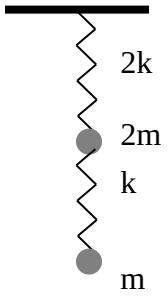
Spring 2016 (February 08)

Work 2 out of 3 in each category. Be sure to state which problems are omitted.

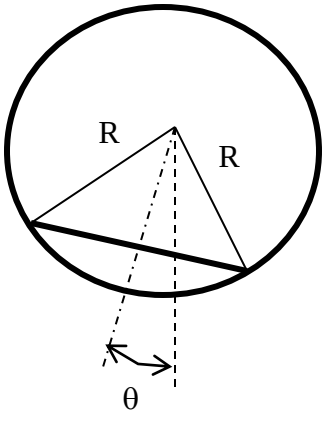
Mechanics

1. A test particle of mass *m* orbits a much more massive object (*M*) in a circular fashion a distance *r* away.
    a) Derive the potential associated with the gravitational force;
    b) Similarly, derive the potential associated with the 'fictional' centrifugal force in terms of angular momentum *L*
    c) Sketch the two potentials as a plot of energy versus *r* as well as the resulting effective potential;
    d) Describe the possible motions for a test particles with energy *E*;
    e) What quantum mechanical system does this central potential resemble?

2. A mass 2*m* is suspended from a fixed support by a spring with spring constant 2*k*. A second mass *m* is attached to the first by a spring with a spring constant *k*. Assuming only vertical small oscillations about the equilibrium position
    a) What is the Lagrangian for the system?
    b) What are the equations of motion?
    c) What are the frequencies of oscillations for the normal modes?



3. A uniform road of length 2*l* and mass *m* slides without friction with its ends moving along a vertical circular path of radius *R*. The center of mass for the rod is a distance *a* from the center of the circle.
    a) What is the equation of motion for the system assuming small oscillations about the equilibrium position (i. e. small angular displacements θ, where θ is the angle between the rod center of mass and the vertical direction? Note that the moment of inertia for the rod is $ml^2/3$.
    b) Let the rod subtend an angle of 120^o relative to the center of the circle. What is the frequency of oscillation?
    c) What is the length of an equivalent simple pendulum?



Electromagnetism

1. A parallel plate capacitor of area A is partially filled with a dielectric layer of permittivity ε. The separation between the plates is d, and the dielectric layer has thickness h, where $h < d$. The capacitor has charge +Q and –Q on the top and bottom plates, respectively. Assume that the plate separation is small.
    a. Find the electric fields **D** and **E** inside the capacitor as a function of position.
    b. Assuming the bottom plate is grounded, what is the electrostatic potential Φ as a function of position?
    c. What is capacitance C of this system?
    d. Determine the forces acting on each plate and on the dielectric layer.

2. A thin non-conducting spherical shell of radius a has a charge density

    placed on its surface.
    a. Using the solution to Laplace's equation, find the electrostatic potential Φ inside and outside the sphere.
    b. What is the electric field **E** inside and outside the sphere?
    c. Evaluate all non-vanishing multipole moments of the charge distribution.
    d. Write down the multipole expansion of the electrostatic potential and show that this agrees with your answer to part (a) for $r > a$.

3. A hollow cylinder of length L and radius a has a uniform charge density σ on its surface. The cylinder rotates with angular frequency ω about its axis (the z-axis).
    a. What is the magnetic field, **B**, as a function of position inside the cylinder? You may ignore end effects; *i.e.*, assume a location far from the ends and that $a << L$.
    b. What is the magnetic dipole moment **m** of the cylinder?
    c. Find the magnetic field **B** at large distances, $r >> L$.

Quantum Mechanics

1. A particle in a box [$V(x)=0,\ 0\le x\le a;\ V=\infty,\ x<0$ *or* $x>a$] is prepared in a state

$$\psi(x)=\begin{cases}\sqrt{\frac{4}{a}}\sin\left(\frac{2\pi}{a}x\right) & 0\le x\le a/2\\ 0 & x<0; x>a/2\end{cases},$$

[nonzero only over the left half of the box.]

   a) If a measurement of energy is conducted, what are the possible outcomes?
   b) What are probabilities for finding the particle in the ground (lowest energy) state and the first excited state?

2. The wave function

$$\psi = A[2e^{3i\phi}-e^{-i\phi}]$$

describes a plane rotator of moment of inertia I (ϕ is the angle of rotation). An experiment is conducted to determine the angular momentum L of the rotator.

   a) Calculate the coefficient A.
   b) What possible values can one find for the measured angular momentum?
   c) What are the probabilities of finding these values?
   d) What values for the energy of rotation can one find?
   e) What is the probability of finding the rotator within 0.1 radians of the position $\phi = \pi/2$?

3. For the electron in the ground state of the lithium ion Li^{2+},
   a. Prove that the wave function has the form
   $$\psi = A\exp(-r/a),\quad A, a = const$$
   b. Calculate A and a.
   c. Prove that the mean value of $1/r$ is $1/a$.
   d. Calculate the average kinetic energy.